Universidad de Navarra

Facultad de Ciencias Económicas y Empresariales

Working Paper nº 06/11

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Working Paper No.06/11
February 2011

ABSTRACT

The problem of non-existence of perfect equilibrium in the original model of Harold Hotelling and the principle of minimum differentiation he suggested have been tackled on different grounds. This paper provides a survey on the address approach to horizontal product differentiation and the different ways that works after Hotelling solved the problem of non-existence of perfect equilibrium by changing some of the assumptions of the model.

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The problem of non-existence of perfect equilibrium in the original model of Harold Hotelling and the principle of minimum differentiation he suggested have been tackled on different grounds. This paper provides a survey on the address approach to horizontal product differentiation and the different ways that works after Hotelling solved the problem of non-existence of perfect equilibrium by changing some of the assumptions of the model.

Key Words: Oligopolistic Competition, Horizontal Product Differentiation, Hotelling
JEL Classification: D43, L10, L13

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1. Introduction

The analysis of competition and market structure when products are horizontally differentiated has attracted the interest of economists due to two main implications of product differentiation: first, firms do not take into account the decrease in rivals' output caused by an increase in its own output; and, second, firms do not appropriate the whole surplus that the introduction of their products generates. As a consequence, social and private (firms') incentives will differ. The economic analysis of horizontal product differentiation has mainly had two main focuses: first, determine the degree of product differentiation in a market; and second, whether there is too much or too little product diversity according to social desirability.

There are two main approaches to the analysis of product differentiation: the address approach and the non-address approach.[1] Here in this work I revise the address approach by summarising the main items in the analysis of Hotelling (1929) and highlighting the main works that tried to solve the problem of non-existence of perfect equilibrium and the principle of minimum product differentiation. The assumption of quadratic transportation costs benefited the existence of a perfect equilibrium, yielding maximum product differentiation. Relaxing other assumptions, such as the perfect inelasticity of demand, the one-dimensional differentiation and the duopolistic market structure, yield perfect equilibria where the degree of differentiation is intermediate between maximum and minimum differentiation.

The remainder of this paper is organised as follows: Section 2 presents the main items of Hotelling's original model. Section 3 develops the literature review of the address approach to horizontal product differentiation after Hotelling and Section 4 concludes.

2. Original Hotelling Model

H. Hotelling (1929) pioneered the address approach to horizontal product differentiation analysis. Unlike in the non-address approach, consumers in the address (or location) models of horizontal product differentiation are heterogeneous in their preferences for the differentiated goods. The differentiated products are defined by their characteristics.

[1] See Bowley (1924) and Singh and Vives (1984) for the seminal analysis of the non-address approach with fixed viarieties and Chamberlin (1933) and Dixit and Stiglitz (1977) for the seminal analysis of the non-address approach with endogenous varieties.

Hotelling follows P. Sraffa (1926) in his view that not only products may be different from each other (as Bowley had already modelled) but also different consumers may have different preferences for a firm's product:

> The causes of the preference shown by any group of buyers for a particular firm are of the most diverse nature, and may range from long custom, personal acquaintance, confidence in the quality of the product, proximity […][2]

Harold Hotelling's main claim was that the implication in the models of Cournot and Bertrand that a firm can get the entire market by slightly undercutting its rival's price is not plausible. In Hotelling's words:

> If a seller increases his price too far he will gradually lose business to his rivals, but he does not lose all his trade instantly when he raises his price only a trifle. Many customers will still prefer to trade with him because they live nearer to his store than to the others, or because they have less freight to pay from his warehouse to their own, or because his mode of doing business is more to their liking […][3]

In order to make the analysis as simple as possible, H. Hotelling (1929) illustrates a model of competition in a differentiated products' market, where goods are differentiated in a single characteristic. That characteristic has typically been defined as *physical location*, but it may also be regarded as *location* in the space of any other characteristic. [4]

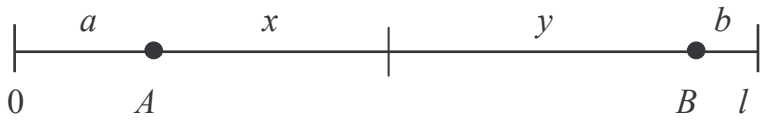

Figure 1

Figure 1 illustrates Hotelling's example. Let us assume with him that consumers are uniformly distributed along a linear segment of length l. Consumer located at 0 is the one whose most preferred version of the differentiated good is the one with the lowest amount of the characteristic (in Hotelling's example, the cider with the lowest possible degree of sourness), whereas consumer located at l is the one who prefers the good with the highest degree of the characteristic.

There are two firms, A and B. Firm A is located a units of distance from the left end of the market and firm B is located b units of distance from the right end of the

[2] In P. Sraffa (1926), p. 544.
[3] In H. Hotelling (1929), p. 44.
[4] In H. Hotelling (1929)'s original example, the differentiated product is cider, that is assumed to be differentiated in its degree of sourness alone.

market. Firm A's price and quantity are respectively p_1 and q_1, and firm B's price and quantity are respectively p_2 and q_2. Production is assumed to take place at no cost. Consumers' transportation cost is equal to c per unit of distance and each consumer will purchase one unit of the good from the firm that has the lowest sum of price plus transportation cost to her.

In this model, firms' locations are fixed and each firm chooses the price at which it sells its product. If firm B sets a price p_2 higher than $p_1 + c(l - a - b)$, all consumers will buy from firm A as A's delivered price would be lower than B's delivered price for every consumer in the linear market. Therefore, in equilibrium it must hold that:

$$p_1 \leq p_2 + c(l - a - b) \text{ and } p_2 \leq p_1 + c(l - a - b)$$

There is one consumer who is indifferent between buying the good from either firm A or firm B. Let us assume that this *marginal consumer* is the one whose location is depicted in Figure 1, located at a distance x from firm A and a distance y from firm B. The delivered price for this marginal consumer is the same for products of both firms: $p_1 + cx = p_2 + cy$. From this condition and the equality condition $a + b + x + y \equiv l$ we get the following values for x and y:

$$x = \frac{1}{2}\left(l - a - b + \frac{p_2 - p_1}{c}\right), \; y = \frac{1}{2}\left(l - a - b + \frac{p_1 - p_2}{c}\right)$$

Firm A will sell a quantity $q_1 = a + x$ and firm B will sell a quantity $q_2 = b + y$. Each firm's profit maximisation condition leads to equilibrium prices and quantities:

$$p_1^* = c\left(l + \frac{a-b}{3}\right), \; p_2^* = c\left(l - \frac{a-b}{3}\right), \; q_1^* = \frac{1}{2}\left(l + \frac{a-b}{3}\right), \; q_2^* = \frac{1}{2}\left(l - \frac{a-b}{3}\right)$$

Firms' equilibrium profits are:

$$\pi_1^* = \frac{c}{2}\left(l + \frac{a-b}{3}\right)^2, \; \pi_2^* = \frac{c}{2}\left(l - \frac{a-b}{3}\right)^2.$$

H. Hotelling (1929) makes three main observations: First, equilibrium profits increase with transportation costs. Hotelling concludes from this observation:

> These particular merchants would do well, instead of organising improvement clubs and booster associations to better the roads, to make transportation as difficult as possible. Still better would be their situation if they could obtain a protective tariff to hinder the transportation of their commodity between them. […] the object of each is merely to attain something approaching a monopoly.

The second observation, whose implication has been extremely relevant for the posterior literature on horizontal product differentiation, relates to the location decisions by firms:

> […] suppose that A's location has been fixed but that B is free to choose his place of business. Where will he set up shop? Evidently he will choose b so as to make
>
> $$\pi_2 = \frac{c}{2}\left(l - \frac{a-b}{3}\right)^2$$
>
> as large as possible. […] for all values of b within the conditions of the problem, π_2 increases with b. […] This means that he will come just as close to A as other conditions permit.

The third observation concerns symmetry of firms. When firms' locations are perfectly symmetric, such that $a = b$, each firm earns an identical profit:

$$\pi_1(a=b) = \pi_2(a=b) = \frac{c}{2}l^2 .$$

Firm A's profit is maximised when $(a-b)$ is highest and minimised when $(a-b)$ is lowest. Firm B's profit is maximised when $(b-a)$ is highest and minimised when $(b-a)$ is lowest.

In any case, given a firm's location, its rival has an incentive to approach it as much as possible. The symmetry of the problem and the dynamics of it would lead the firms to end up sharing the centre of the market, where $a = b = l/2$. According to the game's solutions, each firm would earn a profit

$$\pi_1(a=b) = \pi_2(a=b) = \frac{c}{2}l^2 .$$

If both firms are located in the centre of the market, a firm cannot unilaterally increase profits by changing location (as it would lose customers to the benefit of its rival). Nevertheless, it can increase profits by decreasing its price by an infinitesimal amount and get the whole market. At this point, a Bertrand-type competition in prices would be triggered, until both firms charged a price $p_i = 0 \quad \forall i = 1,2$, resulting in $\pi_i = 0$ $\forall i = 1,2$, so each firm would make zero profits. In Hotelling's own words:

> […] if b increases so that B approaches A, both q_2 and p_2 increase while q_1 and p_1 diminish. From B's standpoint the sharper competition with A due to proximity is offset by the greater body of buyers with whom he has an advantage.

> But the danger that the system be overturned by the elimination of one competitor is increased. The intermediate segment of the market acts as a cushion as well as a bone of contention; when it disappears we have Cournot's case and Bertrand's objection applies.

This implies that there is no locational equilibrium in the original Hotelling model, so its analysis cannot describe the equilibrium level of product differentiation.

When trying to describe competition with differentiated brands, aiming, among other things, to solve Bertrand's paradox, Harold Hotelling falls in that paradox too. In spite of that, he contributes to the theory of competition by describing a model lying between perfect competition and monopoly, and in which a slight price cut does not very likely imply that the firm gets the whole market.

In next section, I will review the different approaches that economists have taken in order to solve the problem of inexistence of locational equilibrium in Hotelling model.

3. The address-approach after Hotelling

All the assumptions used by H. Hotelling (1929) are listed by A. Lerner and H. Singer (1937). They also analyse the implications of each assumption one by one.

In this section, we will focus our attention on four of these assumptions:

(a) Costs of transportation are linear.

(b) Consumers' demand is completely price-inelastic, such that each consumer buys one unit of the good irrespective of the price.

(c) Products are differentiated in only one characteristic.

(d) The market is a duopoly.

The reason why I chose those four assumptions is because they have crucial implications for the equilibrium degree of product differentiation. The equilibrium degree of product differentiation (or the location of firms in equilibrium) is the unresolved problem in H. Hotelling (1929)'s analysis and the most relevant question for my problem of analysing supply-side substitution, that can be described as the incentive of a firm to relocate after an increase in rivals' prices.

I will face each of the previous four assumptions in each of the following subsections.

3.1. Transportation costs

As it has been explained above, H. Hotelling (1929) found that a firm in his linear market had an incentive to locate as close as possible to its rival firm. Following K.

Boulding (1966), this result has been called in the literature *the principle of minimum differentiation*.

C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) show, on one hand, that H. Hotelling (1929)'s model does not have a price equilibrium when firms are not far from each other; and, on the other hand, when transportation costs are quadratic in distance, the model has an equilibrium price solution for each possible pair of locations. In the (long-run) location equilibrium, firms choose maximum product differentiation.

I will first illustrate the problem of existence of equilibrium in H. Hotelling (1929) analysed by C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979):

In the model described by H. Hotelling (1929) illustrated in Figure 1 above, for $a+b=l$ (i.e. when firms choose the same location), the unique equilibrium point is $p_1^* = p_2^* = 0$.[5] For $a+b<l$ (where firms have different locations), there is an equilibrium point if and only if

(1) $$\left(l+\frac{a-b}{3}\right)^2 \geq \frac{4}{3}l(a+2b)$$

(2) $$\left(l-\frac{a-b}{3}\right)^2 \geq \frac{4}{3}l(2a+b)$$

and, whenever it exists, the equilibrium is uniquely described by prices:

$$p_1^* = c\left(l+\frac{a-b}{3}\right) \text{ and } p_2^* = c\left(l-\frac{a-b}{3}\right)$$

If we consider only symmetric locations for the equilibrium ($a=b$), conditions (1) and (2) reduce to $a=b\leq l/4$. Therefore, both duopolists must be located outside the quartiles for the price equilibrium described by H. Hotelling (1929) to exist.

C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) conclude:

> If conditions (1) and (2) are strictly verified, then, as noted by Hotelling, both $\partial\pi_1(p_1^*, p_2^*)/\partial a$ and $\partial\pi_2(p_1^*, p_2^*)/\partial b$ are strictly positive, which implies a tendency of both sellers towards the centre. But a major consequence of the preceding proposition is that, as far as the Cournot equilibrium is taken as the market solution, nothing can be said of this solution when conditions (1) and (2) are violated.

[5] This is the Bertrand result.

Concluding, when the firms are located within the quartiles, they start undercutting each other's prices, resulting in a process of price cuts that do not converge to an equilibrium. In order for a price equilibrium to exist, the firms have to be located outside the quartiles. But, when firms are located outside the quartiles, firms have an incentive to relocate closer to each other, so there is no location equilibrium in the original Hotelling game.

A remedy to this problem is found by C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) using quadratic transportation costs. When costs of transportation are quadratic, the total cost for a consumer located a distance x from firm i when buying the good from firm i is $p_i + cx^2$.

Let us have a look back to Figure 1. The marginal consumer is now that for whom $p_1 + cx^2 = p_2 + cy^2$, with $a + b + x + y \equiv l$.

Equilibrium prices to this game are:

$$p_1^* = c(l - a - b)\left(l + \frac{a-b}{3}\right) \text{ and } p_2^* = c(l - a - b)\left(l - \frac{a-b}{3}\right)$$

We can observe from those prices that, when firms choose the same location ($a + b = l$), then Bertrand result holds ($p_1^* = p_2^* = 0$).

Equilibrium quantities are:

$$q_1^* = \frac{1}{2}\left(l + \frac{a-b}{3}\right) \text{ and } q_2^* = \frac{1}{2}\left(l - \frac{a-b}{3}\right)$$

yielding equilibrium profits:

$$\pi_1^* = \frac{c(l-a-b)}{2}\left(l + \frac{a-b}{3}\right)^2 \text{ and } \pi_2^* = \frac{c(l-a-b)}{2}\left(l - \frac{a-b}{3}\right)^2.$$

The differentials of those profits with respect to the location variable of each firm are:

$$\frac{\partial \pi_1}{\partial a} = -\frac{1}{3}(3a + b + l) \text{ and } \frac{\partial \pi_2}{\partial b} = -\frac{1}{3}(a + 3b + l)$$

that are both strictly negative for each value of a and b. This implies that firms will choose to locate in each of the edges of the market, yielding $a = b = 0$.

Therefore, C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) solved the problem of existence of subgame-perfect equilibrium in prices in H. Hotelling (1929) and also found that theory may not support the *principle of minimum differentiation*. In authors' words:

> […] one should expect intuitively that product differentiation must be an important component of oligopolistic competition. It seems to be clear that oligopolists should gain an advantage by dividing the market into submarkets in each of which some degree of monopoly power would reappear.

D. Neven (1985) formalises the model sketched by d'Aspremont, J.-J. Gabszewicz and J. Thisse (1979) in a two-stage game in which two firms simultaneously choose their locations in the first stage and simultaneously set prices in the second stage. D. Neven (1985) uses quadratic transportation costs and supports C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979)'s result that a perfect equilibrium exists where the firms locate at the opposite ends of the market and charge the same price. In Neven's words:

> Firms tend to relax price competition through product differentiation. This result appears to be consistent with Hotelling's basic observation that firms effectively avoid the zero-profit Bertrand solution through some kind of differentiation.

After the divergence of the results found for linear and quadratic transportation costs, J.-J. Gabszewicz and J.-F. Thisse (1986) and S. Anderson (1988) explore the existence of equilibria when transportation costs have both a linear as well as a quadratic term. In J.-J. Gabszewicz and J.-F. Thisse (1986), the transportation cost function between locations s and s' is defined as

$$t(s,s') = c|s-s'| + d(s-s')^2 .$$

A similar expression for transportation cost is used by S. Anderson (1988).

Both J.-J. Gabszewicz and J.-F. Thisse (1986) and S. Anderson (1988) find that there does not exist a pure-strategy price equilibrium in the two-stage game of horizontal product differentiation when firms' locations are close to each other. We can thus observe that Hotelling's problem of non-existence of pure-strategy price equilibrium, with linear transportation cost, for every location cannot be solved by adding a quadratic component to the linear transportation cost.[6]

The conclusion of our review of the role of transportation costs in spatial competition equilibrium so far is that there is no pure-strategy equilibrium in every subgame when transportation costs are linear in distance. There is a perfect equilibrium in every subgame in the model with quadratic transportation costs where the duopolists choose

[6] See S. Anderson (1988) for a mixed-strategy price equilibrium solution.

maximum product differentiation.[7] A very illustrative explanation of the role played by the shape of the transportation cost function on the existence of price equilibrium and on equilibrium locations is offered by N. Economides (1986b).

N. Economides (1986b) explores the regions of existence of equilibrium in the price subgame and the equilibrium degree of product differentiation for intermediate types of utility functions between the linear and the quadratic utilities. The transportation cost function is defined as $f(d) = d^{\alpha}$, $1 \le \alpha \le 2$, with d being the distance between two locations in the linear market of length 1.[8] Figure 2 summarises the regions of existence of price equilibrium and the equilibrium locations for each value of α. Firm 1 is located at point x and firm 2 is located at y, therefore locations x in Figure 2 correspond to firm 1's locations. Locations $\bar{x}$ correspond to firm 1's equilibrium locations.

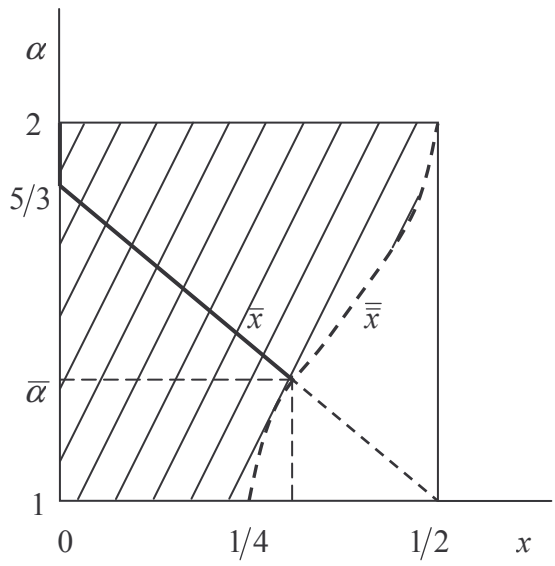

Figure 2

Locations in the shaded are in Figure 2 (West of $\bar{\bar{x}} = 1.4 - 2.76\alpha + 2.08\alpha^2 - 0.46\alpha^3$), are those for which there exists a Nash equilibrium in prices. Thus, for linear transportation

[7] J. Friedman and J. Thisse (1993) study the existence of equilibria in a spatial duopoly with quadratic transportation costs where the firms simultaneously choose locations at the beginning of time and choose prices in each of a countably infinite succession of time periods. They identify a trigger strategy price equilibrium, characterised by partial collusion: firms will non-collusively choose locations in the middle of the market segment and they will choose collusive prices in each of the infinite-horizon periods. The location equilibrium in this partial collusion game (with each firm in the middle of the market) yields different locations than in a full collusion game (where firms collude both in locations and in prices), with firms symmetrically located at the quartiles.

[8] Note that $\alpha = 1$ corresponds to the linear utility function considered by H. Hotelling (1929), and $\alpha = 2$ corresponds to the quadratic utility function considered by C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979).

costs ($\alpha = 1$), there exists an equilibrium in the price subgame for locations $x \leq 1/4$. For values of $\alpha < \overline{\alpha} \approx 1.26$, there does not exist an equilibrium in the location subgame. For values $\overline{\alpha} \approx 1.26 \leq \alpha \leq 5/3$, there exists an equilibrium in the location stage with firms choosing interior locations $\bar{x} = 5/4 - (3/4)\alpha$. When $5/3 \leq \alpha \leq 2$, firms choose maximum differentiation in equilibrium. There only exists a price equilibrium for all possible firms' locations when transportation costs are quadratic in distance ($\alpha = 2$), as C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) had pointed out.

3.2. Elasticity of demand

In the original Hotelling model, every consumer was bound to buy one unit of the good irrespective of price, so demand was completely price-inelastic.

Some extensions of the model have relaxed this assumption in two ways. The first way is to assume that consumers have a reservation price, such that they don't purchase the good if delivered price exceeds that reservation price. The second procedure is to assume that demand is price elastic at each point of the demand curve.

Let us start with the assumption that demand is price inelastic up to a reservation price, such that each consumer will purchase one unit of the good if the delivered price (price plus transportation cost) is lower than reservation price. If the delivered price exceeds reservation price, the consumer does not buy the good at all. This approach was firstly used by A. Lerner and H. Singer (1937). They show that the equilibrium location of firms (and thus the degree of product differentiation) depends on the relationship between three quantities: the length of the market, the cost of transport and the reservation price. They also showed that, with a finite reservation price, even with linear transportation costs, the *minimum differentiation* result implied by H. Hotelling (1929) did not necessarily hold. Two more recent works by N. Economides (1984) and J. Hinloopen and C. van Marrewijk (1999) study the existence of equilibria in the two-stage game where firms choose locations in the first stage and set up prices in the second stage.

Let us first analyse the equilibrium in the price subgame (second stage of the game) in N. Economides (1984): There are two firms, one located at x and the other located at y in a linear market of length equal to 1.[9] The reservation price is equal to k for all

[9] In N. Economides (1984)'s model, both x and y are defined as the distance from the origin, i.e. from point 0 of the market segment.

consumers in the market. The transportation cost is assumed to be linear in distance and thus the utility that the consumer located at w gets from the consumption of good z is $V_w(z) = k - |z - w|$. Consumer w will not buy the differentiated good at all if $V_w(z) = k - |z - w| < 0 \ \ \forall z \in [0,1]$.

When the reservation price k is sufficiently high as to induce all consumers in $[0,1]$ to buy one unit of the good, a Nash equilibrium in prices for different firms' locations ($x \neq y$) exists if and only if

$$x^2 + y^2 + 2xy - 8x + 28y - 20 > 0 \text{ and } x^2 + y^2 + 2xy - 32x + 4y + 4 > 0$$

and is described by prices:

$$p_x^* = \frac{1}{3}(2 + x + y), \ p_y^* = \frac{1}{3}(4 - x - y).$$

As it happened in H. Hotelling (1929), the conditions for the existence of equilibrium are fulfilled for distant locations and fail for close locations by firms.[10]

If firms have identical locations ($x = y$), a Nash equilibrium in prices exists and equilibrium prices are $p_x^* = p_y^* = 0$.

We can observe that, when the reservation price is very high, such that all consumers buy one unit, we have the same problem as in H. Hotelling (1929) where there didn't exist a Nash equilibrium in prices for close locations. N. Economides (1984) solves this problem by considering a reservation price that is binding, such that not all consumers purchase the good. N. Economides (1984) describes three different types of equilibria that are reached for three different ranges of values of the reservation price k:

(i) With different locations by firms, when reservation price k is very small, specifically $k < \min(x, 1 - y, y - x)$, there is a unique Nash equilibrium in prices with equilibrium prices $p_x^* = p_y^* = k/2$, equilibrium quantities $q_x^* = q_y^* = k$ and equilibrium profits $\pi_x^* = \pi_y^* = k^2/2$. This equilibrium is illustrated in Figure 3. It is called by N. Economides (1984) *local monopolistic* equilibrium as firms in this case are local monopolies. An increase in a firm's own price does not lead any of its consumers to start purchasing the rival's good. This is because there is a set of consumers (x_2, y_1) located between x and y that do not consume the differentiated good.

[10] The similarity of these results to those in H. Hotelling (1929) are due to the fact that, in this preliminary case of N. Economides (1984), reservation price is not binding and, in both cases, transportation costs are linear in distance.

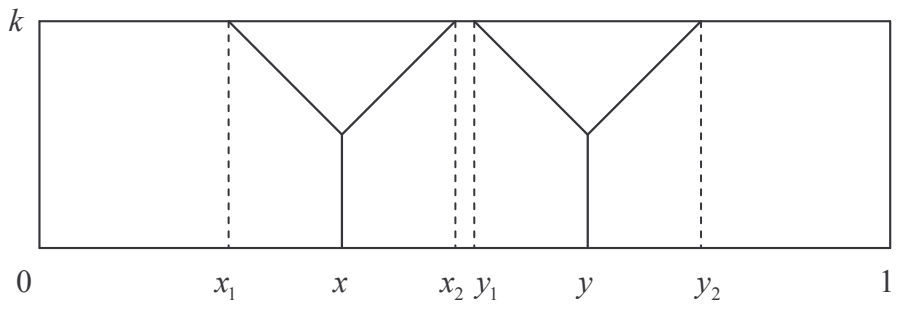

Figure 3

(ii) For intermediate values of k, specifically for $y-x<k<\min(x,1-y,7/6(y-x))$, there is a continuum of equilibria called *touching* equilibria, where the marginal consumer who is indifferent between buying the good from x or from y is also indifferent between buying the differentiated good or not buying at all (see Figure 4).

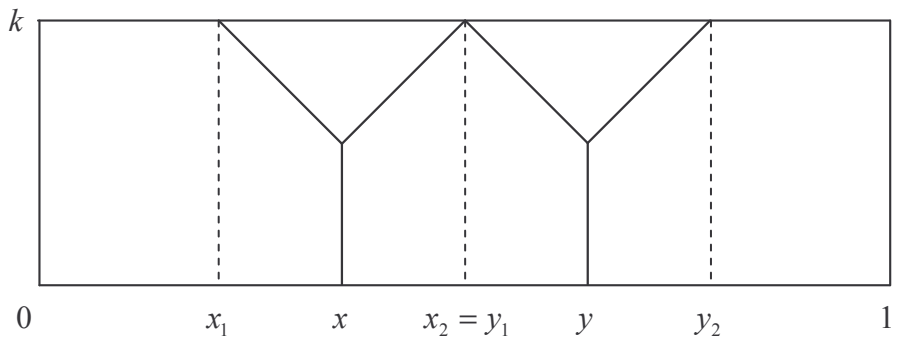

Figure 4

Equilibrium prices in this case are $p_x^* = p_y^* = k-(y-x)/2$, equilibrium quantities are $q_x^* = q_y^* = y-x$ and equilibrium profits are $\pi_x^* = \pi_y^* = (y-x)[k-(y-x)]/2$.

(iii) For a slightly higher reservation price, the marginal consumer between x and y is strictly better by consuming the differentiated good (from either firm) than not buying the good at all. This happens for $7/6(y-x)<k<\min(x,1-y,(7+5\sqrt{10})(y-x)/6)$. In this range of reservation prices, the two firms are directly competing with each other, and that is why N. Economides (1984) calls this the *competitive* equilibrium. Figure 5 illustrates this type of equilibrium, that is characterised by equilibrium prices $p_x^* = p_y^* = (2k+y-x)/5$, equilibrium quantities $q_x^* = q_y^* = (3/2)[(2k+y-x)/5]$ and equilibrium profits $\pi_x^* = \pi_y^* = (3/2)[(2k+y-x)/5]^2$.

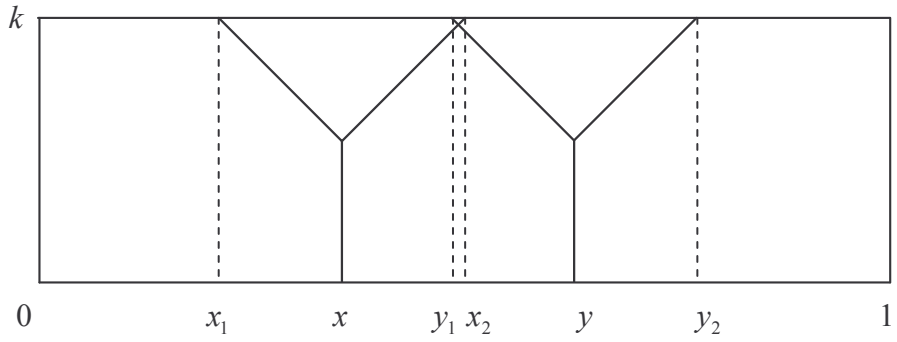

Figure 5

In this linear transportation cost case, Nash equilibrium prices are lowest for the *local monopolistic* equilibrium and are highest for the *competitive* equilibrium. In Economides's own words:

> This result is surprising at first glance. […] A closer look at the situation reveals that the result is not so surprising. In the 'touching' region competition is more fierce than in the 'competitive' region. This is because in the 'touching' and 'local monopolistic' regions the firms cannot increase their prices expecting all consumers located between x and y to continue buying. […] On the other hand, in the 'competitive case', all consumers between x and y can be expected to continue buying at slightly higher prices.

The second part of the analysis in N. Economides (1984) is to determine the existence of equilibria in the varieties (or location) subgame (first stage of the game). The author starts this section by stressing that the introduction of a finite reservation price reduces the incentive of firms to locate closer to each other described by H. Hotelling (1929). In Hotelling's setting, all consumers in the edges consumed the good, so the local firm had a monopoly over a wide range of customers for a large range of prices. This increased the incentive of a firm to approach the other firm in order to increase its monopoly fringe and also increase demand by consumers in-between firms. Firms might also have an incentive to undercut the other firm to capture the local enclave of the opponent.

With a not-too-high reservation price, local enclaves are smaller (as consumers at the edges may prefer not to buy the good at all) and the firm has to compete in price to gain their business. With smaller local enclaves, there are lower incentives to undercut, and this implies better chances for existence of Nash equilibrium. Competition is easier at the enclaves than at the region between firms, so firms have some incentives to move towards the edges.

Let us now analyse the relocation tendencies in each of the three price Nash equilibrium situations:

(i) In the *competitive* region, the differential of equilibrium profits with respect to location for firms located at x and y are, respectively,

$$\frac{\partial \pi_x^*}{\partial x} = -\frac{3}{5} p_x^* = -\frac{3}{5}\left(k - \frac{y-x}{2}\right) < 0 \text{ and } \frac{\partial \pi_y^*}{\partial y} = \frac{3}{5} p_y^* = \frac{3}{5}\left(k - \frac{y-x}{2}\right) > 0$$

so firms will relocate marginally away from each other and reach the *touching* region.

(ii) In the *touching* region, the values of those differentials are

$$\frac{\partial \pi_x^*}{\partial x} = -k + (y-x) < 0 \text{ and } \frac{\partial \pi_y^*}{\partial y} = k - (y-x) > 0,$$

so, again, firms have incentives to relocate away from each other and reach the *local monopolistic* region.

(iii) In the *local monopolistic* region, $\partial \pi_x^* / \partial x = \partial \pi_y^* / \partial y = 0$, so firms in this region will not relocate.

From this analysis we can conclude that, in the varieties (location) subgame, a perfect Nash equilibrium exists if there exist x and y (with $0 < x < y < 1$), such that $k < \min(x, 1-y, y-x)$. In case it exists, the long-run equilibrium is a *local monopolistic* equilibrium.

A revision and extension of the analysis in N. Economides (1984) is made by J. Hinloopen and C. van Marrewijk (1999). They summarise the three variables whose relationship determines market equilibrium (length of the market, reservation value and transportation costs) in a single parameter α:

$$\alpha \equiv l \frac{t}{\upsilon}.$$

α is the size of the market relative to the effective reservation price υ / t, where l is the length of the market, υ is the reservation value and t is the (linear) transportation cost per unit of distance. J. Hinloopen and C. van Marrewijk (1999) show that the existence of equilibria depends on the parameter value α [11]:

(i) For $0 < \alpha < 8/7$, the problem simplifies to the one in H. Hotelling (1929), where reservation price is very high and there is no pure strategy symmetric equilibrium in both prices and locations.

[11] J. Hinloopen and C. van Marrewijk (1999) restrict themselves to the search for symmetric equilibria.

(ii) When reservation price is low, specifically for $2 \le \alpha < \infty$, the problem simplifies to the one in N. Economides (1984) and the *local monopolistic* equilibrium is reached. Equilibrium is characterised by price $p_i^* = \upsilon/2$ and locations[12]:

$$\frac{l}{2\alpha} \le h_i^* \le \frac{l(\alpha-1)}{2\alpha}.$$

But there are also equilibria in the two-stage game when the reservation price is *intermediate*:

(iii) For $4/3 \le \alpha \le 2$, there exist equilibria in which the degree of differentiation is called by the authors *intermediate differentiation*. The symmetric location equilibrium is $h_i^* = l/4$ with price equal to $p_i^* = \upsilon(l-(\alpha/4))$.

(iv) Finally, for $8/7 \le \alpha \le 4/3$, equilibria are called *approaching minimum differentiation* equilibria. The symmetric location equilibrium is characterised by prices $p_i^* = \alpha\upsilon/2$ and locations

$$h_i^* = l\frac{2-\alpha}{2\alpha}.$$

When $\alpha < 4/3$, firms are located between the quartiles and the centre of the market.

We can thus conclude that, for given values of the transportation cost and the length of the market, the higher the reservation value, the closer the firms will be to each other. Analogously, the lower the reservation value, the closer the firms will be to the ends of the market.

Further to considering the existence of a reservation value as an upper limit to individual demand, A. Smithies (1941) analyses the location and price equilibrium when demand is elastic at every point of the demand curve, so the quantity demanded is a continuous and decreasing function of price. In the simplest scenario, demand function is linear and can be written as:

$$p = aq + b, \text{ with } a < 0,\ b > 0,$$

where p is the delivered price at each point of the linear market (i.e., mill price plus transportation cost).[13] A. Smithies (1941) shows that duopolists in equilibrium will be located between the centre of the market and the quartiles. The exact location depends

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[12] With h_1 being the distance of firm 1 to the left end of the market and h_2 the distance of firm 2 to the right end of the market.

[13] A. Smithies (1941) assumes linear transportation costs, and he further assumes that transportation cost is a lump-sum that doesn't depend on the number of units transported.

on the conjectures about rival's reactions to changes in price and location and on the ratio of the unit transportation cost to the demand intercept b.

K. Rath and G. Zhao (2001) modify the model in A. Smithies (1941) by using a quadratic, rather than a linear, transportation cost. They find that a Nash equilibrium in prices exists for each possible pair of locations, x_1 and x_2. A is the demand intercept (or reservation value) in K. Rath and G. Zhao (2001), and b is the transportation cost parameter. There is a unique equilibrium pair of locations (x_1^*, x_2^*), that are symmetric, $x_1^* = 1 - x_2^*$. Those equilibrium locations (and, thus, the extent of product differentiation) are determined by the value of A^2/b. There is a constant C (approx. 5.45708) such that, if $A^2/b \geq C$, the equilibrium locations are the extreme end points of the market, $x_1^* = 0$. If $A^2/b < C$ then the equilibrium locations are interior, $x_1^* \in (0, 1/2)$.

The explanation of this result is straightforward. As reservation price A rises, the model in K. Rath and G. Zhao (2001) gradually approximates the two stage model with inelastic demand and quadratic transportation costs in C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979). So for large reservation values (i.e. $A^2/b \geq C$), we obtain extreme locations, i.e. maximum product differentiation. As A^2/b decreases, the locations gradually move towards the centre of the market. At the extreme case when A^2/b tends to zero, the firms approach the market centre.

K. Rath and G. Zhao (2001) give us a useful explanation of the main implication of the use of an elastic demand with quadratic transportation costs. When demand is inelastic, the revenue of firm i is equal to $p_i z$ (where z is each firm's market share, considering symmetric equilibrium locations). If one of the firms gradually moves itself towards its end of the market, its price increases and its market share decreases, but the effect of higher revenue due to higher price exceeds the effect of lower revenue due to lower market share, and this will result in firms choosing extreme locations.[14] When consumers' demand is linear in price, the revenue of firm i is equal to $p_i(A - p_i)z$.[15] The negative effect of an increase in product differentiation by firm i is double: via lower $A - p_i$ and via lower market share z. This example depicts the lower incentive of firms to locate at the ends of the market when demand is elastic.

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[14] This phenomenon is particular of quadratic transportation costs.

[15] Where $A - p_i$ is the quantity of the good demanded by each consumer.

After reviewing the works that analyse the existence of equilibria in the two-stage spatial model of competition when the demand is not completely price-inelastic, we can conclude that elastic demand tends to enhance the range of locations for which we have a (first-stage) location equilibrium. Those equilibria tend to be characterised by intermediate levels of product differentiation.[16]

3.3. Multi-dimensional differentiation

We have assumed so far that products were defined by a single characteristic and product differentiation was described by products having different values of that characteristic.

N. Economides (1986a) analyses the existence of price equilibria in the analogy of Hotelling's duopoly model when products are defined by two characteristics. In his model, consumers are assumed to be uniformly distributed (according to their most preferred variety) on a unit disk. Utility functions are linear in the Euclidean distance in the product space and reservation prices of all consumers are set sufficiently high so as to ensure that all consumers buy the differentiated product. N. Economides (1986a) restricts his analysis to the search of price equilibria when the locations of the two competitors are symmetric with respect to the centre of the market, specifically at locations $x = (c,0)$ for firm 1 and location $y = (-c,0)$ for firm 2. A consumer located at $w = (w_1, w_2)$ will purchase the good from firm 1 if $p_1 + r\|(c,o) - (w_1, w_2)\| < p_2 + r\|(-c,0) - (w_1, w_2)\|$. The consumers who are indifferent between buying from firm 1 or from firm 2 are located on a hyperbola, that is the boundary between the locations of all consumers who buy from firm 1 and the locations of all consumers who buy from firm 2.

The unit disk in Figure 6 is the whole market area and the shaded area represents the market for firm 1. The case in the figure is one where $p_1 > p_2$ and, as a consequence, firm 2's clientele is larger than firm 1's.

N. Economides (1986a) proves that there exists an equilibrium in prices for every possible $0 < c \leq 1$ in the two-firm, two-dimensional product-differentiation model when the two firms are symmetrically located on an axis through the market centre.

[16] With linear transportation costs, differentiation increases as reservation value decreases. With quadratic transportation costs, differentiation increases as reservation value increases.

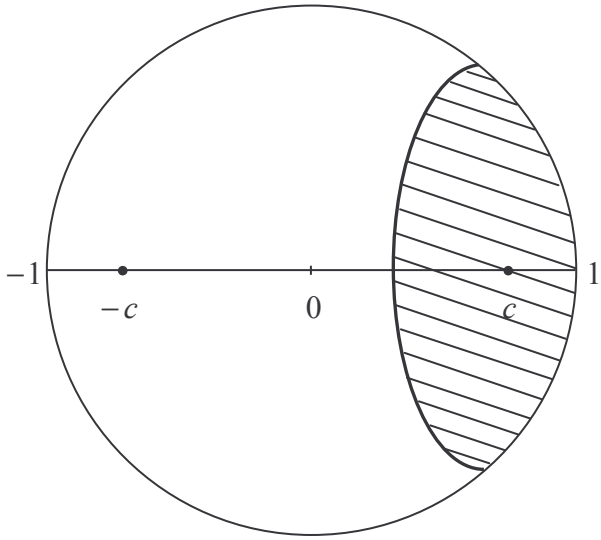

Figure 6

Let us now compare the results of the two-dimensional product differentiation model to those for the one-dimensional model, both with quadratic transportation costs. In both cases, equilibrium prices and profits decrease as the distance between firms decreases. In N. Economides (1986a)'s notation, $p^* \to 0$ as $c \to 0$. The percentage price cut necessary to undercut the opponent is significantly higher with two characteristics. Equilibrium profits are larger than undercutting profits.

N. Economides (1986a)'s work was focused on the search for price equilibria given locations. T. Tabuchi (1994), E. Vendorp and A. Mayeed (1995), A. Ansari, N. Economides and J. Steckel (1998) and A. Irmen and J.-F. Thisse (1998) search for the location equilibrium in the two-stage model of competition, where firms simultaneously choose locations in the first stage and prices in the second stage. They all reach a subgame perfect equilibrium by using a quadratic transportation cost.[17]

The main finding in this branch of literature is that, in a multi-dimensional duopoly, firms tend to choose maximum differentiation in one dimension and minimum differentiation in all other dimensions. More specifically, in a two-dimensional model firms try to maximise differentiation in one dimension and minimise differentiation in the other. A. Ansari, N. Economides and J. Steckel (1998) call this the *principle of maximum-minimum differentiation*. In a three-dimensional model, firms in equilibrium

[17] T. Tabuchi (1994) and E. Vendorp and A. Mayeed (1995) conduct their analysis for two product characteristics, whereas A. Ansari, N. Economides and J. Steckel (1998) and A. Irmen and J.-F. Thisse (1998) allow for higher dimensions.

differentiate maximally in one dimension and minimally in the remaining two. They call this the *principle of max-min-min differentiation*.

In A. Ansari, N. Economides and J. Steckel (1998), the attribute in which producers locate most distantly is the one that consumers value the most. In their setup, the disutility of distance function has different weights for the different attributes, measuring the importance that each attribute has for consumers. In the two-dimensional model, when consumers care a lot about the attribute of the first dimension, the *max-min* equilibrium holds where firms maximally differentiate in the first dimension only. Analogously, when consumers place a high weight on the second attribute, the *min-max* equilibrium holds, with firms maximally differentiated in the second dimension. When weights are relatively similar, both the *max-min* and the *min-max* equilibria exist.[18]

Let us briefly reproduce A. Ansari, N. Economides and J. Steckel (1998)'s model of two-dimensional duopoly, with firms 1 and 2. The space of consumers' and products' locations is a unit square. Firm 1 has a product defined by the characteristics vector (x_1, y_1) and firm 2 has a product defined by (x_2, y_2). Let us assume that all consumers have an equal vector of weights $w = (w_1, w_2)$, where w_1 is the weight attached to the first attribute and w_2 is the weight attached to the second attribute. A consumer located at (a,b) who consumes product i will enjoy a utility

$$U_i(a,b;x_i,y_i,p_i) = Y - w_1(a-x_i)^2 - w_2(b-y_i)^2 - p_i$$

where Y is a constant reservation value assumed to be sufficiently high so as to ensure that all consumers purchase the good. We observe that transportation costs are quadratic in distance. Demand for firm 1's good is the set of all consumers for whom $p_1 - p_2 < w_1\left[x_2^2 - x_1^2 - 2a(x_2 - x_1)\right] + w_2\left[y_2^2 - y_1^2 - 2b(y_2 - y_1)\right]$. Changing the sign in the inequality gives us firm 2's demand. The locus of consumers who are indifferent between buying from either firm is represented by a straight line which partitions the (unit square) market into the two firms' demand areas. Figure 7 depicts the unit square market, where x is the characteristic that is drawn in the horizontal axis and y is

[18] The same pattern holds for the three-dimensional model. A. Ansari, N. Economides and J. Steckel (1998) find a *max-min-min* equilibrium, with products maximally differentiated in the first dimension and minimally differentiated in the remaining two, when the weight consumers place in the first attribute is large. When the weight placed in the second attribute is large as well, the *min-max-min* equilibrium occurs as well. And when the three attributes have similar weights, the *min-min-max* equilibrium exists in addition to the previous two.

depicted in the vertical axis. The area above the partition line is firm 1's market and the area below the line is firm 2's market.

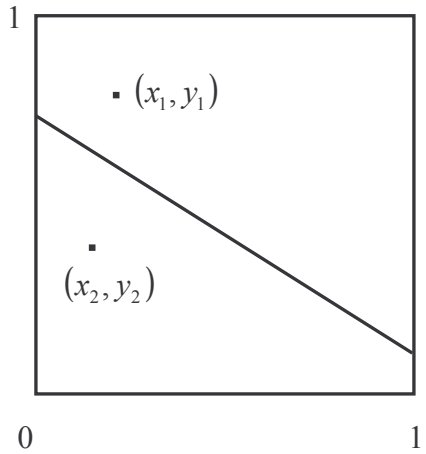

Figure 7

A. Ansari, N. Economides and J. Steckel (1998) show that there exists a price equilibrium for any pair of product positions chosen by the two firms in the first stage in both two and three dimensions.

In the second stage of the two-dimensional game, the existence of either one or two location equilibria depends on the value of the ratio of attribute weights $w \equiv w_1/w_2$:

(i) For $w < 0.406$ only the *max-min* equilibrium exists. This equilibrium is characterised by locations $(x_1^*, y_1^*) = (0, 1/2)$, $(x_2^*, y_2^*) = (1, 1/2)$.

(ii) For $w > 1/0.406 = 2.46$ only the *min-max* equilibrium exists, described by locations $(x_1^*, y_1^*) = (1/2, 0)$, $(x_2^*, y_2^*) = (1/2, 1)$.

(iii) Finally, for $0.406 < w < 2.46$ both the *max-min* and the *min-max* equilibria exist.

The location equilibrium results in A. Ansari, N. Economides and J. Steckel (1998) somehow resemble the *maximum differentiation* result of C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979). In the latter, the use of a quadratic transportation costs led us to find a subgame perfect equilibrium with first-stage locations characterised by maximum differentiation. If we add a second attribute in that setup whose value for consumers is negligible, firms would still produce maximising their distance according to the relevant attribute and will produce the same degree of the negligible attribute. That is the *max-min* product differentiation result in A. Ansari, N. Economides and J. Steckel (1998).

3.4. Number of firms

The early analysis of equilibrium in spatial competition models with more than two competitors focused on the search for locational equilibria when firms' prices were all the same. With identical (and exogenous) prices, the equilibrium in the two-firm game was characterised by minimum differentiation with the two firms located at the centre of the market. Chamberlin (1933, p. 261) was the first to notice that there was no pure-strategy equilibrium in the pure location game with three firms.[19] A. Lerner and H. Singer (1937) and B. Eaton and R. Lipsey (1975) extend and formalise the analysis to higher number of firms. In the equilibrium of the four-firm game, two firms will be located in pairs at the quartiles. In the equilibrium of the five-firm game, two firms will be located at $1/6$, two at $5/6$ and one at $1/2$. With more than five firms, the equilibrium ceases to be unique.

In an endogenous price framework, S. Salop (1979) looks for price equilibria in a market where n firms are symmetrically located on a circumference of unit perimeter. The equilibrium concept used is the *Chamberlinian* zero-profit monopolistically competitive equilibrium. With equidistant firms, there are three types of price equilibria and the resulting equilibrium will depend on the relative values of fixed cost F, marginal cost m, transportation cost parameter c (assuming linear transportation costs), number of consumers in the circular market L and effective reservation value υ (that is equal to the reservation value of the differentiated good minus the surplus attached to the homogeneous outside good). When $\upsilon - m = \sqrt{2cF/L}$, there is a *monopoly* equilibrium, where the marginal consumer between two neighbouring brands is strictly better by not consuming the good, characterised by prices $p_m = m + c/2n_m$, with an equilibrium number of firms $n_m = \sqrt{cL/2F}$. When $\upsilon - m \geq (3/2)\sqrt{cF/L}$, there is a *competitive* equilibrium, where the marginal consumer between two neighbouring brands is strictly better by consuming the good, characterised by prices $p_c = m + c/n_c$ and number of firms $n_c = \sqrt{cL/F}$. The third type of equilibrium takes place when $\sqrt{2cF/L} < \upsilon - m < (3/2)\sqrt{cF/L}$ and is called *kinked* equilibrium. At this equilibrium, the marginal consumer between two brands is indifferent between consuming the good

[19] See A. Shaked (1982) for a mixed-strategy location equilibrium in the three-firm case.

from either of them and not consuming the good at all.[20] Kinked equilibrium prices are $p_k = \upsilon - c/n_k$ and the number of firms is given by the condition $n_k F/L + c/n_k = \upsilon - m$.

N. Economides (1989) looks for a perfect equilibrium in the circular market game played in three stages: firms enter the market in the first stage, choose locations in the second stage and set up prices in the third stage. As in S. Salop (1979), there are n firms in the market and preferences are defined on a circumference of unit perimeter. Instead of linear transportation costs, Economides (1989) assumes quadratic transportation costs. The subgame perfect equilibrium in N. Economides (1989) corresponds to a *competitive* equilibrium, with the marginal consumer strictly consuming the differentiated good. The equilibrium is symmetric, with firms charging equal prices $p_j^* = c + (1/n^*)^2$ and equidistantly located at a distance $x_j^* - x_{j-1}^* = 1/n^*$ from each of the two immediate neighbours. The equilibrium number of firms is $n^* = I\left[(\mu/F)^{1/3}\right]$, c is the marginal cost, equal for all firms, $I[x]$ is the integer part of x (i.e. the largest integer smaller or equal to x), μ measures the density of consumers in the unit circumference and F is the firm's fixed cost.

Once the equilibrium in a spatial competition model with more than two firms has been described for the circular city, the interest has to be turned to the linear market again, as remarked by N. Economides (1993):

> The circumference model is a good paradigm for some characteristics, such as color. In the locational interpretation, the circumference model describes well the choices of consumers distributed along the coastline of a lake. For most goods, however, it is appropriate to use a line interval $[a,b]$ as the space of potential products. For a serious study of the spatial economy, the simplicity and symmetry of the circular model cannot compensate for its lack of the appropriate structure.

N. Economides (1993) studies the existence of price equilibria in the game where $n \geq 3$ firms are located on the line interval $[0,1]$ and the cost of transportation is linear in distance. He proves the existence of a Nash equilibrium in prices in the second stage of the game, but there doesn't exist an equilibrium in the first (location) stage for every possible pattern of locations. When reservation price *k* is sufficiently high so as to make every consumer to purchase the good, firms in the linear market have an incentive to relocate towards the central firm.

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[20] This *kinked* equilibrium is analogous to the *touching* equilibrium in N. Economides (1984).

As there does not exist a locational equilibrium corresponding to the *competitive* price equilibrium, N. Economides (1993) describes the *competitive* price equilibrium when firms are symmetrically located and equally spaced along the unit line.[21] That competitive price equilibrium is characterised by a convex, U-shaped price structure, where the firm at the centre charges the lowest price and the firms at the edges charge the highest price. The reason of this price configuration is that the firms at the edges enjoy monopoly power over those consumers between the firm's own location and the edge. Some of this monopoly power is transferred to their neighbours and this transmission of monopoly power is decreasingly passed on to the firms with more interior locations.

When reservation price k is low, specifically for $k \leq m + \lambda/n$ (where m is the marginal cost of production, λ is the unit transportation cost and n is the number of firms), there exist an infinity of subgame perfect equilibria in the two-stage game, with equilibrium locations between two neighbouring firms satisfying the condition $x_j^* - x_{j-1}^* > (k-m)/\lambda$. These equilibria are *local monopolistic* equilibria as there are some consumers between any two firms who do not buy the differentiated product.

N. Economides (1993) looks for the existence of *kinked* price equilibria for the symmetric locational configuration. He proves the existence of price equilibria for intermediate reservation prices, i.e. for $\lambda d \leq k - m \leq (3/2)\lambda d$. At a *kinked* equilibrium, the marginal relocation of a firm will again result in a kink configuration. This is an endless process that doesn't allow us to evaluate the long-run profit function and thus the existence of a locational *kinked* equilibrium.

Again, the problem of non-existence of equilibrium in locations in the model with linear transportation costs was solved when the costs of transport were assumed to be quadratic.

S. Brenner (2005) finds a perfect price equilibrium in both locations and prices in the unit-line market with quadratic transportation costs and with the reservation price set sufficiently high so as to ensure that all consumers buy one unit of the good.[22] He shows that a unique equilibrium in prices exists for every given pattern of locations for $n \geq 3$ firms. The price equilibrium is characterised by a U-shape distribution of prices, with

[21] We call it *competitive* following S. Salop (1979), who defines competitive equilibrium as the one in which the marginal consumer between two firms is strictly better off by consuming the good from either firm than by not consuming the good at all.

[22] This corresponds to the *competitive* equilibrium in S. Salop (1979) and N. Economides (1993).

highest prices at the edges and lowest prices at the centre. For $n = \{3,4\}$, profits are highest for the central firm(s). For $n \geq 5$, profits exhibit a U-shape pattern. The reason is that, as n increases, there are more firms sharing the centre of the market, thus decreasing the market power of each of them.[23] As the firms closest to the edges have a monopoly power over the consumers located between them and the ends, the higher the number of firms, the stronger the relative position of the extreme firms with respect to the central firms. The location equilibrium exhibits equidistant locations and no firms located at the edges. Again, the use of quadratic transportation cost contributes to the existence of location equilibria and also eliminates the incentives of firms to move towards the centre.

4. Concluding comments

Once completed the review of the different models of horizontal product differentiation, I will briefly summarise the main ideas: The problem of non-existence of perfect equilibrium in the original model of H. Hotelling (1929) and the principle of minimum differentiation he suggested have been tackled on different grounds. The assumption of quadratic transportation costs benefits the existence of a perfect equilibrium, yielding maximum product differentiation. Relaxing other assumptions, such as the perfect inelasticity of demand, the one-dimensional differentiation and the duopolistic market structure, yield perfect equilibria where the degree of differentiation is intermediate between maximum and minimum differentiation.

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[23] We will see in Section 5 the implication of this change in the shape of the distribution of profits for market definition with supply-side substitution.

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