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Oil Prices: Persistence and Breaks

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ABSTRACT

The rise of oil prices is a main issue in contemporary economics. This study examines the monthly, weekly and daily structure in several oil prices series using a modeling approach based on fractional integration and long range dependence. The results indicate that oil prices series are highly persistent, with orders of integration equal to or higher than 1. Breaks in the series do not alter the main conclusions of this study. That means that shocks have a permanent nature and strong policy measures must be implemented to return the series to their original long term projections.

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Keywords: Oil prices, fractional integration, persistence and breaks.
1. Introduction

Oil prices are a main issue in contemporary economics, since they are the trigger of all contemporary economic crises. Although crises can start elsewhere in economies, such as, for example, in the construction sector, they only generalize to all sectors when oil prices increase. Given the importance of oil prices, this study examines the degree of persistence in several monthly, weekly and daily oil prices using a fractional integration modeling framework (Lien and Root, 1999; Kang, Kang and Yoon, 2009) and allowing for breaks in the data. Modeling the degree of persistence is important in that it can reflect the stability in price over time. Thus, in the event of an exogenous shock, different policy measures should be adopted depending on their degree of persistence. First, if oil prices are stationary, shocks to oil prices will be transitory; however, if oil prices contain a unit root, shocks will have permanent effects. Second, given the importance of oil prices to other sectors of the economy if shocks to oil prices are indeed persistent, then one can expect that other sectors of the economy and macroeconomic aggregates will inherit this persistence, which in itself renders questions about the effectiveness of government intervention or stabilization policies.¹ By a shock, we mean an event which takes place at a particular point in the series, and which is not confined to the point at which it occurs. A shock is known to have a temporary or short term effect, if, after a number of periods, the series returns back to its original performance level (for example, oil prices might increase due to an economic boom, but drop back after the boom stimulus is withdrawn). On the other hand, a shock is known to have a persistent or long term impact if its short run impact is carried over forward to set a new trend in performance (for example, a persistence drop in oil prices that might result from an economic downturn, inflation). Breaks are another important feature that may be present in oil prices data. These reflect

¹ See, for example, Lean and Smyth (2009) for the relevance of testing for unit roots.
shocks in oil prices due to fluctuations in oil production, changes in the world geopolitical climate, and country-specific socio-economic events, among others.

Despite the importance of oil as an energy source and the previous research on the oil industry, there are no studies that specifically analyze the persistence, and breaks associated with oil prices. While studies consider, for example, oil consumption (Mohn and Osmundsen, 2008; Lean and Smyth, 2009), returns on investment in oil (Boone, 2001) and oil exhaustion (Tsoskounoglou et al. 2008; Höök and Aleklett, 2008; Karbassi et al. 2007), this study contemporarily explores the long memory property in oil prices, along the lines of Serletis (1992) and Elder and Serletis (2008) but allowing for potential breaks in the data.

The remainder of this study is organised as follows. Section 2 presents a review of the previous literature. Section 3 details the methodology. Section 4 discusses the data and the results, while Section 5 contains some concluding remarks.

2. Brief Overview of the Literature

Although there are some papers which investigate the presence of unit roots in energy consumption (Chen and Lee, 2007; Narayan and Smyth, 2007; Hsu et al, 2008; Mishra et al, 2009; Lean and Smyth, 2009; Rao and Rao, 2009), only a few studies examine oil prices. In a recent paper, Li and Thompson (2010) analyse monthly real prices of oil between 1990 and 2008 using a generalized autoregressive conditional heteroskedasticity GARCH model. Kilian (2010) analyses the relationship between demand and supply shocks between the price of gasoline in the U.S. and the price of crude oil in global markets, with a structural VAR model. Berument, Ceylan and Dogan (2010) analyse how oil price shocks affect the output growth of selected countries that are considered either net exporters or net importers of oil and which are too small to affect oil prices.
Vassilopoulos (2010) analyzes the price signals of the French wholesale electricity market building an operational research model and simulating it. Fattouh (2010) analyzes crude oil price differentials as a two-regime threshold autoregressive (TAR) model, concluding that standard unit root tests suggest that the prices of crude oil of different varieties move closely together and therefore the price was stationary. More in line with the present research, Serletis (1992) analyses the random walk type behavior in energy futures prices with unit root tests. Furthermore, Elder and Serletis (2008) analyze long memory behavior in energy futures prices with fractional integrating dynamics and a semi-parametric wavelet-based estimator, finding evidence of anti-persistence in its behaviour. The present research innovates in this context analysing monthly, weekly and daily oil prices with a fractionally integrated approach, and concluding that there is persistence in oil prices.

3. **Methodology**

One characteristic of many economic and financial time series are their nonstationary nature. There exists a variety of models to describe such nonstationarity. Until the 1980s a standard approach was to impose a deterministic (linear or quadratic) function of time, thus assuming that the residuals from the regression model were stationary. Later on, and especially after the seminal work of Nelson and Plosser (1982), there was a general agreement that the nonstationary component of most series was stochastic, and unit roots (or first differences) were commonly adopted. However, the unit root is merely one particular model to describe such behaviour. In fact, the number of differences required to render a series stationary I(0)\(^2\) may not necessarily be an integer value (usually 1) but any point in the real line. In such a case, the process is said to be fractionally integrated or I(d).

\[^2\] An I(0) process is defined as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. This includes the standard cases of white noise, stationary AR, MA, stationary ARMA, etc.
The I(d) models (with d > 0) belong to a wider class of processes called long memory. We can define long memory in the time domain or in the frequency domain.

Let us consider a zero-mean covariance stationary process \{x_t, t = 0, \pm 1, \ldots\} with autocovariance function \(\gamma_u = E(x_t x_{t+u})\). The time domain definition of long memory states that \(\sum_{u=-\infty}^{\infty} |\gamma_u| = \infty\). Now, assuming that \(x_t\) has an absolutely continuous spectral distribution, so that it has spectral density function

\[
f(\lambda) = \frac{1}{2\pi}\left(\gamma_0 + 2 \sum_{u=1}^{\infty} \gamma_u \cos(\lambda u)\right),
\]

the frequency domain definition of long memory states that the spectral density function is unbounded at some frequency \(\lambda\) in the interval \([0, \pi]\). Most of the empirical literature has concentrated on the case where the singularity or pole in the spectrum takes place at the 0-frequency. This is the standard case of I(d) models of the form:

\[
(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \ldots,
\]

where \(L\) is the lag-operator \((Lx_t = x_{t-1})\) and \(u_t\) is I(0). These processes were introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981) and since then have been widely employed to describe the behaviour of many economic time series (Diebold and Rudebusch, 1989; Sowell, 1992; Gil-Alana and Robinson, 1997; etc.). As earlier mentioned these processes are characterized by the spectral density function being unbounded at the zero frequency. The origin of these processes is in the 1960s, when Granger (1966) and Adelman (1965) pointed out that most aggregate economic time series have a typical shape where the spectral density increases dramatically as the frequency
approaches zero. However, differencing the data frequently leads to overdifferencing at the zero frequency.\(^3\)

In this study, we estimate \(d\) using a Whittle function in the frequency domain (Dahlhaus, 1989) along with a testing procedure developed by Robinson (1994) that permits us to test the null hypothesis \(H_0: d = d_o\) in (2) for any real value \(d_o\), where \(x_t\) can be the errors in a regression model of form:

\[
y_t = \beta^T z_t + x_t, \quad t = 1, 2, ..., \tag{3}
\]

where \(y_t\) is the time series we observe, \(\beta\) is a \((k \times 1)\) vector of unknown coefficients and \(z_t\) is a set of deterministic terms that might include an intercept (i.e., \(z_t = 1\)), an intercept with a linear time trend \((z_t = (1, t)^T)\), or any other type of deterministic processes such as dummy variables to examine the potential presence of breaks. This method is briefly described in Appendix 1. The main advantages of Robinson (1994) compared with other methods are the following: first, it permits us to test any real value \(d\), encompassing then stationary and nonstationary hypotheses; second, the limit distribution is standard normal and this limiting behavior holds independently of the type of deterministic terms included in the model and of the way of modeling the I(0) error term \(u_t\); finally, this method is the most efficient one in the Pitman sense against local departures from the null.

In the final part of this article we also consider the possibility of structural breaks. We suppose a single break and first assume that the break date is known and include dummy variables in the regression model (3). Thus, in this case, we suppose that there is a change in the deterministic part of the process, keeping constant the degree of integration of the series before and after the break. Finally, we also implement another approach (Gil-Alana, 2008; Appendix 2) that estimates endogenously the break date and that permits different orders of integration at each subsample. This method is based on the following model,

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\(^3\) Note, however, that fractional integration may also occur at some other frequencies away from 0, as in the case of seasonal/cyclical (fractional) models. (See, Arteche and Robinson, 1999; Arteche, 2002)
\[
y_i = \beta_1^T z_i + x_i; \quad (1 - L)^{d_1} x_i = u_i, \quad t = 1, \ldots, T_b, \quad (4)
\]

and
\[
y_i = \beta_2^T z_i + x_i; \quad (1 - L)^{d_2} x_i = u_i, \quad t = T_b + 1, \ldots, T, \quad (5)
\]

where the \(\beta\)'s are the coefficients corresponding to the deterministic terms; \(d_1\) and \(d_2\) may be real values; \(u_i\) is I(0); and \(T_b\) is the time of a break that is supposed to be unknown.\(^4\)

Note that given the difficulties in distinguishing between models with fractional orders of integration and those with broken deterministic trends (i.e., Teferovsky and Taqqu, 1997; Diebold and Inoue, 2001; Granger and Hyung, 2004; etc.), the use of a model that incorporates both features simultaneously seems clearly overdue.

### 4. Data and empirical results

The data examined correspond to the following time series: Spot oil price (West Texas Intermediate), which is a monthly series obtained from Dow Jones & Company; a weekly OPEC countries spot FOB weighted by estimated export volume (Dollars per barrel), and finally, the European Brent spot price FOB (Dollars per barrel) at a daily frequency. That means that we have oil spot prices at three different frequencies: monthly, weekly and daily. For the monthly data the series run from January 1997 to August 2009. For the weekly data, the starting date is January 3, 1997, ending on December 25, 2009. For the daily data, it is January 2, 1997 – January 5, 2010.

![Insert Figure 1 about here](image)

Figure 1 displays the plots of the three series. It can be observed that the three of them display a similar pattern with values increasing across time and a decrease that starts at July 2008 at the time after the housing crisis. Figure 2 displays the growth rate series

\(^4\) For the purposes of simplicity we have only considered here the case of a single break, though multiple breaks are also feasible with this procedure.
obtained at the first differences of the log-transformed data, while Figure 3 displays the first 50 sample autocorrelation values of each series.

[Insert Figures 2 and 3 about here]

The growth rate series have an appearance of stationarity, which is corroborated by the correlograms in Figure 3, however, we also observe in this Figure significant values at some lags, even away from zero, which may suggest some degree of long memory behaviour.

First, we employ the Whittle parametric approach. We report the estimated values of $d$ in the model given by

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t,$$

where $y_t$ is the observed logged time series; $\alpha$ and $\beta$ are the coefficients corresponding respectively to the intercept and a linear trend, and $x_t$ is supposed to be an I($d$) process. Thus, $u_t$ is I(0) and given the parametric nature of this method we must specify its functional form. We will first assume that $u_t$ is white noise and the results are reported in Table 1. We examine the three standard cases examined in the literature, i.e.: i) no regressors (i.e., $\alpha = \beta = 0$ a priori in (6)), an intercept ($\alpha$ unknown and $\beta = 0$ a priori), and an intercept with a linear time trend (i.e., $\alpha$ and $\beta$ unknown).

Table 1 reports the Whittle estimates of $d$ under the assumption that the error term is white noise. We also report from each case and each series the 95% confidence band of non-rejection values of $d$ using Robinson’s (1994) parametric approach.\(^5\) We observe that if we do not include regressors the unit root null cannot be rejected in any of the three series. However, including an intercept and an intercept with a linear trend, the unit root is rejected in favour of $d > 1$ in the cases of monthly and weekly data, but this hypothesis

\(^5\) Here we employ a grid of 0.001 values for $d$. That is, we test $H_0$: \(d = d_0\) in (6) with $d_0 = 0, 0.001, \ldots, 2$.  

7
cannot be rejected for the daily data. In general we observe a reduction in the degree of integration as we increase the frequency from monthly to weekly and daily data.

[Insert Tables 1 and 2 about here]

Table 2 displays the estimates of the selected models according to the specification of the deterministic terms. It is observed that the time trend is insignificant in the three series with only an intercept being required. The estimated values of d are 1.206 for the monthly series; 1.171 for the weekly data, and 1.010 for the daily data.

Next we consider the case of autocorrelated errors, supposing first that \( u_t \) is AR(1). Starting with the monthly data, we observe that the estimates are smaller than 1 in all cases. It is 0.549 with no regressors, 0.601 with an intercept, and 0.617 with a linear time trend. However, for the weekly and daily data, the results are a bit unclear. In fact, when using Robinson’s (1994) tests for a grid of values of d, we observe a lack of monotonicity in the value of the test statistic across d. Such a monotonicity should be a reasonable feature of the test statistic given correct specification and adequate sample size. Thus, for example, if the test rejects the null of \( d = d_o = 0.5 \) in favour of the alternative \( d > 0.5 \), (with a significantly large statistic), a more significant result in this direction (i.e. a larger magnitude in the test statistic) should be expected if \( d_o = 0.1 \) is tested. In the same way, if \( d = d_o = 2 \) is rejected in favour of \( d < 2 \) (with a significantly negative value in the test statistic) a similar result with a higher magnitude (in absolute value) should be expected with \( d_o = 2.5 \).

[Insert Figure 4 about here]

Figure 4 displays Robinson’s (1994) statistics in the context of AR(1) disturbances for the three series, for a range of values of \( d_o \) from -1 to 2. We observe a lack of monotonicity in the three cases, and for the weekly and daily data, there are two cases where the statistics cross the axe from positive values to negative ones. For the weekly
data, they correspond to the estimates 0.222 and 1.179 in case of an intercept, (and 0.245 and 1.180 with a linear trend, not reported). For the daily data the estimates are 0.020 and 1.011 with an intercept (and 0.035 and 1.013 with a linear trend, not reported). This apparent contradiction could be explained in terms of the competition between the AR parameters and the fractional differencing in describing the time dependence.\(^6\)

Due partly to this inconsistency we also implement another approach for the I(0) model that is based on the exponential spectral model of Bloomfield (1973). This is a non-parametric approach for modeling \(u_t\) that produces autocorrelations decaying exponentially as in the AR(MA) case. The main advantage of this model is that it mimics the behavior of ARMA structures with a small number of parameters. Moreover, it is stationary independent of the values of its coefficients unlike what happens in the AR case. The results based on this model are displayed in Table 4.

[Insert Table 4 about here]

We observe in this table that most of the estimates of \(d\) are above 1. For the monthly data, the unit root null cannot be rejected. It is rejected in favour of higher degrees of integration in case of weekly data, and the I(1) hypothesis cannot be rejected with daily observations.

Finally, we also examine the possibility of a mean shift around the time of the crisis. We take the break at July 2008 (monthly) and July 18\(^{th}\) 2008 (weekly and daily data) and examine the two cases of white noise and AR(1) disturbances. In particular, we examine the following model,

\[
y_t = \alpha + \alpha^* I(t > T^*) + x_t; \quad (1 - L)^d x_t = u_t, \tag{7}
\]

where \(T^*\) is the break date. The estimates based on white noise disturbances are displayed in Table 5, while Table 6 refers to the autocorrelated case.

\(^6\) In fact, when the estimates of \(d\) are close to 0 (as in the case of the daily data), the AR parameters are then found to be extremely close to 1.
Starting with the white noise case we observe that the unit root is rejected in the case of monthly and weekly data, while this hypothesis cannot be rejected for the daily data. The dummy variable for the mean shift is statistically significant in the three series. Allowing for autocorrelated disturbances, the unit root cannot be rejected for the weekly and daily data, the intercept is significant in the two cases and the AR coefficient is very close to 0, suggesting that the AR structure is almost insignificant.\textsuperscript{7}

As a final performance we implement the method of Gil-Alana (2008) that permits us to endogenously determine the break date. Using this method the break date was found at exactly the same dates as with the deterministic approaches, i.e, July 2008 in the case of the monthly data, and July 18\textsuperscript{th}, 2008 with weekly and daily observations. However, a problem with this approach occurs with the fact that the second subsamples are then formed by very few observations invalidating the analysis of fractional integration in the second subsamples. Therefore, we only report the results for the first subsamples across Tables 7, 8 and 9 (that is, with data ending at July 2008) in the three series using respectively white noise, AR(1) and Bloomfield-type errors.

Starting with the results based on white noise disturbances (Table 7) we see that the unit root is rejected in favour of higher degrees of integration in the majority of cases. In fact, the only two exceptions are the cases of one intercept and one intercept with a linear trend with daily data. In these two cases the estimate is slightly above 1 (1.007) but the unit root null cannot be rejected at the 95\% level. Allowing autocorrelated errors, either through the AR(1) or the Bloomfield (1973) model, the I(1) is almost never rejected and a small degree of mean reversion is observed for the daily data with deterministic

\textsuperscript{7} Convergence was not achieved in the case of the monthly data.
terms and Bloomfield-type disturbances. We can therefore conclude by saying that the degree of persistence is high in these series and it has been even reinforced after the crisis in 2007/08.

5. **Concluding comments**

This paper deals with the analysis of several oil prices series at different data frequencies from a fractionally integrated viewpoint. It is observed that independently of the data frequency and the way of modeling the I(0) error term, the series are highly persistent with orders of integration equal to or higher than 1 in the majority of the cases. Thus, there is no evidence of mean reversion in the data. Allowing for a mean shift around 2008, the evidence of unit roots is reinforced in the monthly, weekly and daily data, and using the procedure developed by Gil-Alana (2008) the results seem to indicate that persistence in oil prices has been reinforced in recent years.

In summary, it is clear that taking first differences in the oil price under the assumption of a unit root, may lead in some cases to series that still present a component of long memory behavior. Second, persistence behavior is another characteristic of these data signifying that the effects are persistent and do not disappear without policy action. Third, the existence of a potential break does not alter the main conclusions of this study.

The policy implication of this research is that oil prices tend to be persistent and the long term impact if its short run impact is carried over forward to set a new trend in performance (for example, a persistence drop in oil prices that is the result of an economic downturn, a persistent increase in oil prices that is related to economic growth). Therefore, in order to curb oil price increases any country it is necessary to control the oil price, either with price controls or perhaps promoting alternative sustainable energy that following the line of the Kyoto protocol will tend to substitute oil in the economic activity.
This substitution will affect the oil price increase, decreasing its demand. If the price is not managed, it will increase persistently over time. More research is needed to confirm the present conclusions.
Appendix 1: Robinson’s (1994) parametric approach

The LM test of Robinson (1994) for testing $H_0: d = d_o$ in (2) and (3) is

$$\hat{r} = \frac{T^{1/2}}{\sigma^2} \hat{A}^{-1/2} \hat{a},$$

where $T$ is the sample size and:

$$\hat{a} = -\frac{2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \hat{\sigma}^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{\lambda} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\epsilon}(\lambda_j)^{'} \times \left( \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j) \hat{\epsilon}(\lambda_j)^{'} \right)^{-1} \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|, \quad \hat{\epsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \lambda_j = \frac{2\pi j}{T}; \quad \hat{\tau} = \arg \min \sigma^2(\tau).$$

$\hat{a}$ and $\hat{\lambda}$ in the above expressions are obtained through the first and second derivatives of the log-likelihood function with respect to $d$ (see Robinson, 1994, page 1422, for further details). $I(\lambda_j)$ is the periodogram of $u_t$ evaluated under the null, i.e.:

$$\hat{u}_t = (1 - L)^{d_o} y_t - \hat{\beta}^{'} w_t;$$

$$\hat{\beta} = \left( \sum_{t=1}^{T} w_t w_t^{'} \right)^{-1} \sum_{t=1}^{T} w_t (1 - L)^{d_o} y_t; \quad w_t = (1 - L)^{d_o} z_t,$$

$z_t = (1, t)^T$, and $g$ is a known function related to the spectral density function of $u_t$:

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \quad -\pi < \lambda \leq \pi.$$
Appendix 2: Gil-Alana’s (2008) method for fractional integration with breaks

The model presented in (4) and (5) can also be written as:

\[(1 - L)^{d_1} y_t = \beta_1 \tilde{z}_i(d_1) + u_t, \quad t = 1, ..., T_b,\]

\[(1 - L)^{d_2} y_t = \beta_2 \tilde{z}_i(d_2) + u_t, \quad t = T_b + 1, ..., T,\]

where \(\tilde{z}_i(d_I) = (1 - L)^{d_I} z_i, \ i = 1, 2\). The procedure is based on the least square principle.

First we choose a grid for the values of the fractionally differencing parameters \(d_1\) and \(d_2\), for example, \(d_{10} = 0, 0.01, 0.02, \ldots, 1, i = 1, 2\). Then, for a given partition \(\{T_b\}\) and given initial \(d_1\), \(d_2\)-values, \((d_{10}^{(i)}, d_{20}^{(i)})\), we estimate the \(\alpha\)'s and the \(\beta\)'s by minimizing the sum of squared residuals,

\[
\min \sum_{i=1}^{T} \left[ (1 - L)^{d_{10}^{(i)}} y_t - \beta_1 \tilde{z}_i(d_{10}^{(i)}) \right]^2 + \sum_{i=2}^{T} \left[ (1 - L)^{d_{20}^{(i)}} y_t - \beta_2 \tilde{z}_i(d_{20}^{(i)}) \right]^2.
\]

Let \(\hat{\beta}(T_b; d_{10}^{(i)}, d_{20}^{(i)})\) denote the resulting estimates for partition \(\{T_b\}\) and initial values \(d_{10}^{(i)}\) and \(d_{20}^{(i)}\). Substituting these estimated values on the objective function, we have \(\text{RSS}(T_b; d_{10}^{(i)}, d_{20}^{(i)})\), and minimizing this expression across all values of \(d_{10}\) and \(d_{20}\) in the grid we obtain \(\text{RSS}(T_b) = \arg \min_{\{i,j\}} \text{RSS}(T_b; d_{10}^{(i)}, d_{20}^{(i)})\). Next, the estimated break date, \(\hat{T}_k\), is such that \(\hat{T}_k = \arg \min_{i=1,\ldots,m} \text{RSS}(T_i)\), where the minimization is taken over all partitions \(T_1, T_2, \ldots, T_m\), such that \(T_i - T_{i-1} \geq \epsilon T\). Then, the regression parameter estimates are the associated least-squares estimates of the estimated k-partition, i.e., \(\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})\), and their corresponding differencing parameters, \(\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})\), for \(i = 1\) and 2.
References


Figure 1: Time series data

i) Monthly frequency

ii) Weekly frequency

iii) Daily frequency
Figure 2: Growth rates

i) Monthly frequency

![Monthly frequency chart]

ii) Weekly frequency

![Weekly frequency chart]

iii) Daily frequency

![Daily frequency chart]
Figure 3: Correlograms of the growth rates

i) Monthly frequency

ii) Weekly frequency

iii) Daily frequency

The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.
Table 1: Estimates of $d$ and 95% confidence interval (White noise disturbances)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>0.973</td>
<td>1.206</td>
<td>1.207</td>
</tr>
<tr>
<td></td>
<td>(0.861, 1.120)</td>
<td>(1.079, 1.371)</td>
<td>(1.079, 1.371)</td>
</tr>
<tr>
<td>Weekly</td>
<td>1.006</td>
<td>1.171</td>
<td>1.171</td>
</tr>
<tr>
<td></td>
<td>(0.955, 1.066)</td>
<td>(1.116, 1.237)</td>
<td>(1.116, 1.237)</td>
</tr>
<tr>
<td>Daily</td>
<td>1.001</td>
<td>1.010</td>
<td>1.010</td>
</tr>
<tr>
<td></td>
<td>(0.978, 1.024)</td>
<td>(0.989, 1.033)</td>
<td>(0.989, 1.033)</td>
</tr>
</tbody>
</table>

The values in parenthesis refer to the 95% confidence band of the non-rejection values of $d$ using Robinson (1994) parametric approach.

Table 2: Estimates of $d$ and 95% confidence interval

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Estimates of $d$</th>
<th>Intercept</th>
<th>A time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>1.206</td>
<td>3.25262</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>(1.079, 1.371)</td>
<td>(37.43)</td>
<td></td>
</tr>
<tr>
<td>Weekly</td>
<td>1.171</td>
<td>3.13992</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>(1.116, 1.237)</td>
<td>(75.68)</td>
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</tr>
<tr>
<td>Daily</td>
<td>1.010</td>
<td>3.19678</td>
<td>------</td>
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<tr>
<td></td>
<td>(0.989, 1.033)</td>
<td>(127.50)</td>
<td></td>
</tr>
</tbody>
</table>

In parenthesis in column 3, $t$-values.

Table 3: Estimates of $d$ and 95% confidence interval (AR(1) disturbances)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>0.549</td>
<td>0.601</td>
<td>0.617</td>
</tr>
<tr>
<td></td>
<td>(0.457, 0.633)</td>
<td>(0.380, 0.700)</td>
<td>(0.507, 0.708)</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.259</td>
<td>0.222</td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>(0.215, 0.310)</td>
<td>(0.162, 0.291)</td>
<td>(0.187, 0.313)</td>
</tr>
<tr>
<td></td>
<td>1.125</td>
<td>1.179</td>
<td>1.180</td>
</tr>
<tr>
<td></td>
<td>(0.962, 1.250)</td>
<td>(1.084, 1.275)</td>
<td>(1.085, 1.275)</td>
</tr>
<tr>
<td>Daily</td>
<td>0.033</td>
<td>0.020</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(-0.042, 0.062)</td>
<td>(0.002, 0.020)</td>
<td>(0.011, 0.053)</td>
</tr>
<tr>
<td></td>
<td>1.02</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>(0.993, 1.063)</td>
<td>(0.983, 1.055)</td>
<td>(0.982, 1.054)</td>
</tr>
</tbody>
</table>

The values in parenthesis refer to the 95% confidence band of the non-rejection values of $d$ using Robinson (1994) parametric approach.
Figure 4: Robinson’s (1994) statistics for a range of values of d

i) Monthly frequency

ii) Weekly frequency

iii) Daily frequency

The thick lines refer to the standard N(0, 1) critical values.
Table 4: Estimates of $d$ and 95% confidence interval (Bloomfield disturbances)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>0.985 (0.651, 1.524)</td>
<td>1.154 (0.801, 1.743)</td>
<td>1.157 (0.793, 1.771)</td>
</tr>
<tr>
<td>Weekly</td>
<td>1.121 (1.030, 1.213)</td>
<td>1.171 (1.086, 1.275)</td>
<td>1.170 (1.087, 1.274)</td>
</tr>
<tr>
<td>Daily</td>
<td>1.023 (0.994, 1.061)</td>
<td>1.016 (0.981, 1.043)</td>
<td>1.015 (0.982, 1.041)</td>
</tr>
</tbody>
</table>

The values in parenthesis refer to the 95% confidence band of the non-rejection values of $d$ using Robinson (1994) parametric approach.
Table 5: Estimates of d and mean shift parameters with white noise disturbances

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Estimates of d</th>
<th>Intercept</th>
<th>A time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>1.196 (1.062, 1.366)</td>
<td>3.25160 (37.47)</td>
<td>-0.09836 (-2.13)</td>
</tr>
<tr>
<td>Weekly</td>
<td>1.170 (1.115, 1.236)</td>
<td>3.13994 (75.84)</td>
<td>-0.07472 (-1.80)</td>
</tr>
<tr>
<td>Daily</td>
<td>1.010 (0.988, 1.035)</td>
<td>3.19678 (127.49)</td>
<td>-0.00854 (-2.34)</td>
</tr>
</tbody>
</table>

The values in parenthesis refer to the 95% confidence band of the non-rejection values of d using Robinson (1994) parametric approach.

Table 6: Estimates of d and mean shift parameters with AR(1) disturbances

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Estimates of d</th>
<th>Intercept</th>
<th>Time trend</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly</td>
<td>1.043 (0.928, 1.157)</td>
<td>3.05125 (50.57)</td>
<td>0.08492 (1.99)</td>
<td>0.173</td>
</tr>
<tr>
<td>Daily</td>
<td>1.002 (0.968, 1.040)</td>
<td>3.18545 (89.82)</td>
<td>0.00995 (2.38)</td>
<td>0.013</td>
</tr>
</tbody>
</table>

In parenthesis in columns 3 and 4, t-values.
Table 7: Estimates of d and 95% confidence interval (White noise disturbances)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>1.111 (1.031, 1.214)</td>
<td>1.141 (1.049, 1.270)</td>
<td>1.148 (1.053, 1.277)</td>
</tr>
<tr>
<td>Weekly</td>
<td>1.122 (1.075, 1.181)</td>
<td>1.154 (1.101, 1.220)</td>
<td>1.157 (1.104, 1.222)</td>
</tr>
<tr>
<td>Daily</td>
<td>1.045 (1.019, 1.072)</td>
<td>1.007 (0.984, 1.033)</td>
<td>1.007 (0.984, 1.034)</td>
</tr>
</tbody>
</table>

The values in parenthesis refer to the 95% confidence band of the non-rejection values of d using Robinson (1994) parametric approach.

Table 8: Estimates of d and 95% confidence interval (AR(1) disturbances)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>1.143 (0.893, 1.312)</td>
<td>1.048 (0.917, 1.266)</td>
<td>1.056 (0.851, 1.277)</td>
</tr>
<tr>
<td>Weekly</td>
<td>1.028 (0.918, 1.138)</td>
<td>1.058 (0.975, 1.157)</td>
<td>1.061 (0.974, 1.161)</td>
</tr>
<tr>
<td>Daily</td>
<td>1.073 (0.1026, 1.114)</td>
<td>0.964 (0.933, 1.000)</td>
<td>0.963 (0.931, 1.000)</td>
</tr>
</tbody>
</table>

The values in parenthesis refer to the 95% confidence band of the non-rejection values of d using Robinson (1994) parametric approach.

Table 9: Estimates of d and 95% confidence interval (Bloomfield disturbances)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>No regressors</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td>1.229 (1.058, 1.441)</td>
<td>1.057 (0.896, 1.262)</td>
<td>1.061 (0.901, 1.269)</td>
</tr>
<tr>
<td>Weekly</td>
<td>1.074 (1.007, 1.062)</td>
<td>1.068 (0.996, 1.157)</td>
<td>1.063 (0.991, 1.160)</td>
</tr>
<tr>
<td>Daily</td>
<td>0.973 (0.942, 1.004)</td>
<td>0.960 (0.931, 0.998)</td>
<td>0.959 (0.930, 0.998)</td>
</tr>
</tbody>
</table>

The values in parenthesis refer to the 95% confidence band of the non-rejection values of d using Robinson (1994) parametric approach.