



Universidad  
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Miguel Casares  
Universidad Pública de Navarra

Antonio Moreno  
Universidad de Navarra

Jesús Vázquez  
Universidad del País Vasco

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## ABSTRACT

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Miguel Casares

Departamento de Economía, Universidad Pública de Navarra,  
31006, Pamplona, Spain

[mcasares@unavarra.es](mailto:mcasares@unavarra.es)

Antonio Moreno

Departamento de Economía, Universidad de Navarra

[antmoreno@unav.es](mailto:antmoreno@unav.es)

Jesús Vázquez

Departamento FAE II, Universidad del País Vasco

[jesus.vazquez@ehu.es](mailto:jesus.vazquez@ehu.es)

# An Estimated New-Keynesian Model with Unemployment as Excess Supply of Labor\*

Miguel Casares<sup>†</sup>, Antonio Moreno<sup>‡</sup> and Jesús Vázquez<sup>§</sup>

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## Abstract

Wage stickiness is incorporated to a New-Keynesian model with variable capital in a way that generates endogenous unemployment fluctuations as the log difference between aggregate labor supply and aggregate labor demand. After estimation with U.S. data, the implied second-moment statistics of the unemployment rate provide a reasonable match with those observed in the data. Our results also show that wage-push shocks, demand shifts and monetary policy shocks are the three major determinants of unemployment fluctuations. Compared to an estimated canonical DSGE model without unemployment: wage stickiness is higher, labor supply elasticity is lower, the slope of the New-Keynesian Phillips curve is flatter, and the importance of technology innovations on output growth variability increases.

Key words: sticky wages, unemployment, business cycles, New-Keynesian models.

JEL codes: C32, E30.

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<sup>†</sup>Corresponding author. Postal address: Departamento de Economía, Universidad Pública de Navarra, 31006, Pamplona, Spain. E-mail: mcasares@unavarra.es

<sup>‡</sup>Departamento de Economía, Universidad de Navarra. E-mail: antmoreno@unav.es

<sup>§</sup>Departamento FAE II, Universidad del País Vasco. E-mail: jesus.vazquez@ehu.es

# 1 Introduction

The New-Keynesian macro model has been extended in recent years to incorporate the endogenous determination of unemployment fluctuations in the labor market.<sup>1</sup> Taking the search frictions approach, Walsh (2005) and Trigari (2009) introduced unemployment as the gap between job creation and destruction that results in a labor market with real rigidities *à la* Mortensen and Pissarides (1994). Alternatively, Casares (2007, 2010) and Galí (2011) assume nominal rigidities on wage setting to produce mismatches between labor supply and labor demand that delivers unemployment fluctuations. In a recent paper, Michaillat (2012) explores the interactions between search frictions, job rationing and wage rigidity and finds asymmetric patterns in business cycle fluctuations of unemployment.

This paper presents novel theoretical and empirical contributions. On the theoretical side, our model simultaneously accommodates unemployment fluctuations due to sticky wages, and variable capital, thus affecting labor-capital reallocations at the firm level. In contrast to the vast majority of the related literature, our model generates unemployment fluctuations without resorting search frictions in the labor market. Hence, the model combines most of the nominal and real rigidities of full-fledged New-Keynesian models – Calvo-type price stickiness, consumption habits, investment adjustment costs, variable capital utilization, etc. –, with a labor market which generalizes that of Casares (2010) to include a rental market for capital. In that regard, we replace the standard way of introducing wage rigidities on labor contracts set by households (which follows the seminal paper by Erceg, Henderson and Levin, 2000) for a labor market structure in which excess-labor-supply unemployment stems from sticky wages. As a result, wage dynamics depend inversely upon fluctuations of the rate of unemployment. We also discuss the implications on inflation dynamics: the New-Keynesian Phillips curve turns flatter because of the negative effect of relative prices over relative nominal wages at the firm level.

On the empirical front, this paper includes unemployment -due to sticky nominal wages- in estimation and provides a comparison between our proposed model and the Smets and Wouters (2007) New-Keynesian model, which is a well-known reference model in the DSGE literature.

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<sup>1</sup>Referential New-Keynesian models without unemployment are Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2007) and all the model variants collected in Woodford (2003). They belong to the family of dynamic stochastic general equilibrium (DSGE) models.

We follow a Bayesian econometric strategy to estimate the two models using U.S. quarterly data during the period 1984:1-2009:3. The estimation results provide a good fit to the data since both models capture most of the business cycle statistics. In the comparison across models, we find similar estimates of most structural model parameters, with three main differences. First, wage stickiness is significantly higher in the model with unemployment while price stickiness is nearly the same across models. As a consequence, the introduction of unemployment as excess supply of labor raises the average length of labor contracts (5.6 quarters with unemployment and 2.3 quarters without unemployment). Second, the labor supply curve is significantly more inelastic in the model with unemployment. Finally, the elasticity of capital adjustment costs is lower in the model with unemployment.

Our estimated New-Keynesian model reproduces the U.S. business cycle features at least as well as DSGE models without unemployment and, crucially, it provides a good characterization of U.S. unemployment fluctuations. In particular, the model captures the volatility, countercyclicality and persistence of the quarterly U.S. unemployment rate. Furthermore, the impulse-response functions provide reasonable reactions of unemployment to technology innovations, demand shocks, monetary shocks and cost-push shocks. In the variance-decomposition analysis, model results indicate that the driving forces of unemployment fluctuations are wage inflation shocks, risk premium (demand-side) shocks, and monetary shocks, with little influence from technology shocks. Besides, the model provides a good matching of the lead-lag comovement between the unemployment rate and output growth.

Our paper is most closely related to Galí, Smets and Wouters (2011), though developed independently. There are four important differences across models. First, and most importantly, in Galí, Smets and Wouters (2011) -following Galí (2011) - unemployment is perfectly correlated with the average wage markup and therefore moves in tandem with workers' market power. In contrast to assuming market power of households, this paper includes an intertemporal equation to set wages that match labor supply and labor demand. In turn, unemployment is introduced at a decentralized level that interacts with the price setting behavior. Second, both models feature a similar number of shocks but Galí, Smets and Wouters (2011) resort to wage and price markup shocks, whereas we introduce wage and price push shocks. Third, our model provides a reasonable characterization of the joint dynamics of the labor force, consumption and the wage

over the business cycle without having to assume away short-run wealth effects on labor as in Galí, Smets and Wouters (2011). Finally, our estimated model attributes more than 60% of long-run fluctuations in unemployment to demand shocks, while in Galí, Smets and Wouters (2011) wage markup shocks explain 80% of the fluctuations.

The remainder of the paper proceeds as follows. Section 2 describes the model with sticky wages, unemployment as excess supply of labor and variable capital. Section 3 introduces the estimation procedure and discusses the estimation results. Section 4 presents the empirical fit of the two models along three important dimensions (second-moment statistics, variance decomposition and impulse-response functions) and also compares some of the model-implied dynamic cross-correlations with those in the data. Section 5 concludes with a summary of the main results.

## 2 A model with unemployment as excess supply of labor and variable capital

This section introduces unemployment in a New-Keynesian model with endogenous capital accumulation. Thus, we borrow most of the elements of the New-Keynesian model described in Smets and Wouters (2007) except for the labor market and wage setting behavior. On that dimension, we extend Casares (2010), with the addition of variable capital accumulation, to incorporate unemployment as excess supply of labor. In contrast to Smets and Wouters (2007), employment variability is determined only by the extensive margin of labor (number of employees), assuming that the number of hours per worker is inelastically supplied as in Hansen (1985).<sup>2</sup> Hence, there is a representative household that supplies a variable number of workers for all differentiated types of labor while each firm demands one specific kind.<sup>3</sup> Let us denote  $L_t^d(i)$  as the labor demand for jobs in type- $i$  firm and  $L_t^s(i)$  as the labor supply of workers in

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<sup>2</sup>This assumption relies on the generally accepted view that most variability of total hours worked in modern economies is explained by changes in the number of employed people whereas fluctuations of the number of hours at work have significantly less influence (Cho and Cooley, 1994; Mulligan, 2001).

<sup>3</sup>Woodford (2003, chapter 3) uses this labor market scenario for fluctuations of the intensive margin of labor (hours), claiming that the existence of heterogeneous labor services is more adequate for sticky-price models than the common assumption of an homogeneous labor market.

period  $t$  for that  $i$  firm, so that the rate of unemployment at the  $i$  firm in period  $t$  is

$$u_t(i) = 1 - \frac{L_t^d(i)}{L_t^s(i)}. \quad (1)$$

By assumption, wage rigidity causes unemployment fluctuations in the model around the (constant) natural rate,  $u^n$ . Thus, if wages were flexible they would adjust to make the current rate of unemployment equal to  $u^n$ . Following Bénassy (1995), and, more recently, Casares (2007, 2010), labor contracts are revised in that way only if firms and households can get down together to agree on the natural-rate wage at the firm level. If labor contracts cannot be revised on a given period, the nominal wage will be automatically adjusted by applying an *ad-hoc* indexation rule. Introducing wage stickiness *à la* Calvo (1983), the nominal wage for labor contracts in firm  $i$  is set at the value that results from the intertemporal equilibrium condition:

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j u_{t+j}(i) = \frac{u^n}{1 - \bar{\beta} \xi_w}, \quad (2)$$

where  $\bar{\beta}$  is the discount rate that incorporates detrending from long-run growth,  $\xi_w$  is the Calvo (1983)-type constant probability of not experiencing a labor contract revision, and  $E_t^{\xi_w}$  is the rational expectations operator conditional on the lack of revisions in the future. With fully-flexible wages ( $\xi_w = 0.0$ ), (2) yields  $u_t(i) = u^n$ .

Plugging (1) and the corresponding expressions for future periods in (2) and taking a loglinear approximation give the following expression

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j \left( l_{t+j}^s(i) - l_{t+j}^d(i) \right) = 0, \quad (3)$$

where  $l_{t+j}^s(i)$  and  $l_{t+j}^d(i)$  represent the log deviations, in any  $t + j$  period, from their respective steady-state levels of the labor supply of workers and the labor demand for jobs of type- $i$  labor.<sup>4</sup> In the absence of wage stickiness ( $\xi_w = 0.0$ ), the wage setting condition (3) would bring a perfect matching between fluctuations of labor supply and labor demand at firm level,  $l_t^s(i) = l_t^d(i)$ . Put differently, wage rigidities bring about gaps between the amounts of supply of labor (workers provided by the household) and the demand for labor (jobs demanded by the firm) that make the effective rate of unemployment deviate from a constant natural rate of unemployment.

Hence, the value of the wage agreed upon labor contract revision depends on how labor supply and labor demand enter (3). Adapting the household optimizing program of Smets

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<sup>4</sup>Throughout the paper, lower-case variables denote log deviations with respect to steady-state levels.

and Wouters (2007), the first order condition on type- $i$  labor supply implies drives a positive reaction to both the firm-specific wage and relative unemployment rate:<sup>5</sup>

$$l_t^s(i) = \frac{1}{\sigma_l} \left( \log W_t(i) - p_t - \frac{1}{1-u^n} (u_t(i) - u^n) - \frac{1}{(1-\lambda/\gamma)} (c_t - (\lambda/\gamma) c_{t-1}) \right).$$

Loglinearizing and aggregating across all types of labor services yields the relative labor supply

$$l_t^s(i) - l_t^s = \frac{1}{\sigma_l} \left( \widetilde{W}_t(i) - \frac{1}{1-u^n} (u_t(i) - u_t) \right), \quad (4)$$

where  $l_t^s = \int_0^1 l_t^s(i) di$  is the log deviation of aggregate labor supply from the steady-state level,  $\widetilde{W}_t(i) = \log W_t(i) - \log W_t$  is the relative nominal wage, and  $u_t$  is the aggregate rate of unemployment defined as 1.0 minus the ratio between aggregate labor demand (effective employment,  $L_t$ )<sup>6</sup> and aggregate labor supply (labor force,  $L_t^s$ )

$$u_t = 1 - \frac{L_t}{L_t^s}.$$

In semi-loglinear terms, firm-level and aggregate rates of unemployment become

$$u_t(i) = u^n + (1 - u^n) \left( l_t^s(i) - l_t^d(i) \right). \quad (5a)$$

$$u_t = u^n + (1 - u^n) (l_t^s - l_t). \quad (5b)$$

Substituting both (5a) and (5b) in (4) yields

$$l_t^s(i) - l_t^s = \frac{1}{\sigma_l} \left( \widetilde{W}_t(i) - \left( l_t^s(i) - l_t^d(i) - l_t^s + l_t \right) \right). \quad (6)$$

Regarding firm-level labor demand, we also borrow the production technology and factor markets used in Smets and Wouters (2007) to derive the optimality condition that makes the ratio of marginal products (capital and labor) equal to the ratio of factor prices (real wage and rental rate)

$$\frac{1 - \alpha}{\alpha} \frac{K_t^d(i)}{L_t^d(i)} = \frac{W_t(i)}{r_t^k}, \quad (7)$$

where  $\alpha$  is the capital share in the Cobb-Douglas production technology,  $K_t^d(i)$  and  $L_t^d(i)$  are the levels of capital and labor demanded by firm  $i$ , the aggregate price level is  $P_t$  and  $r_t^k$  is the real

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<sup>5</sup>See the technical appendix -section 1- for the optimizing program of the representative household and the derivation of the first order conditions.

<sup>6</sup>The effective amount of employment is demand determined, as usually assumed in Keynesian models and many macroeconomics textbooks (e.g., Abel, Bernanke, and Croushore, 2011; Blanchard, 2011).

rental rate on capital goods. Also as in Smets and Wouters (2007), the loglinearized production function, with AR(1) technology shocks  $\varepsilon_t^a$ , is

$$y_t(i) = (1 - \alpha) l_t^d(i) + \alpha k_t^d(i) + \varepsilon_t^a. \quad (8)$$

Taking away  $k_t^d(i)$  using (8), and inserting the log-linear relationship between demand-determined relative output and the relative price,  $\tilde{y}_t(i) = -\theta \tilde{P}_t(i)$  results in the relative labor demand equation<sup>7</sup>

$$l_t^d(i) - l_t = -\theta \tilde{P}_t(i) - \alpha \tilde{W}_t(i), \quad (9)$$

which introduces the relative price  $\tilde{P}_t(i) = \log P_t(i) - \log P_t$  and  $l_t$  as the log deviation from steady state of demand-determined employment obtained from the aggregation of log deviations on firm-specific labor demand

$$l_t = \int_0^1 l_t^d(i) di. \quad (10)$$

Equation (3) governs wage setting with the intertemporal targeting of  $l_{t+j}^s(i) - l_{t+j}^d(i)$ . Recalling labor supply and labor demand schedules (equations 6 and 9, respectively) for any  $t + j$  period,

$$l_{t+j}^s(i) - l_{t+j}^d(i) = \frac{1}{\sigma_l} \left( \tilde{W}_{t+j}(i) - \left( l_{t+j}^s(i) - l_{t+j}^d(i) - l_{t+j}^s + l_{t+j} \right) \right) + l_{t+j}^s - l_{t+j} + \theta \tilde{P}_{t+j}(i) + \alpha \tilde{W}_{t+j}(i),$$

which can be simplified as follows

$$l_{t+j}^s(i) - l_{t+j}^d(i) = \frac{\sigma_l^{-1} + \alpha}{1 + \sigma_l^{-1}} \tilde{W}_{t+j}(i) + \frac{\theta}{1 + \sigma_l^{-1}} \tilde{P}_{t+j}(i) + (l_{t+j}^s - l_{t+j}).$$

Using the last expression and (5b) in (3) yields

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \beta^j \xi_w^j \left[ \frac{\sigma_l^{-1} + \alpha}{(1 + 1/\sigma_l)} \tilde{W}_{t+j}(i) + (1 - u^n)^{-1} (u_{t+j} - u^n) + \frac{\theta}{1 + 1/\sigma_l} \tilde{P}_{t+j}(i) \right] = 0. \quad (11)$$

For non-revised labor contracts, the nominal wage is automatically adjusted by applying an indexation rule that combines a weight  $0 < \iota_w < 1$  for lagged inflation and the complementary weight  $1 - \iota_w$  for the steady-state inflation rate plus a stochastic wage-push shock  $\varepsilon_t^w$

$$W_t(j) = W_{t-1}(j) \left[ (1 + \pi_{t-1})^{\iota_w} (1 + \pi + \varepsilon_t^w)^{1 - \iota_w} \right]. \quad (12)$$

This indexation rule is very similar to the one assumed in Smets and Wouters (2007), with the only difference that we include the wage indexation shock  $\varepsilon_t^w$  to replace wage mark-up shocks of their model.

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<sup>7</sup>See technical appendix 1 -section 1- for further details.

Let  $\widetilde{W}_t^*(i) = \log W_t^*(i) - \log W_t$  be the log difference between the wage set in the firm  $i$ ,  $W_t^*(i)$ , if receiving a Calvo-type signal allowing to apply (3) and the aggregate wage,  $W_t$ . Then, defining wage inflation as  $\pi_t^w = \log W_t - \log W_{t-1}$ , the relative wage consistent with (11) and (12) is<sup>8</sup>

$$\begin{aligned} \widetilde{W}_t^*(i) &= -\frac{(1-\bar{\beta}\xi_w)}{\sigma_l^{-1}+\alpha} E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j \left( \frac{1+1/\sigma_l}{1-u^n} (u_{t+j} - u^n) + \theta \widetilde{P}_{t+j}(i) \right) \\ &\quad - E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_w^j (\iota_w \pi_{t+j-1} + (1-\iota_w) \varepsilon_{t+k}^w - \pi_{t+j}^w), \end{aligned} \quad (13)$$

Equation (13) shows that the value of the nominal wage newly set at the firm depends negatively on the stream of the economy-wide rate of unemployment and also negatively on the stream of relative prices. As in Casares (2010), let us introduce the following guess: relative optimal pricing and relative wage setting are related as follows

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1 \widetilde{W}_{t-1}(i), \quad (14a)$$

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \widetilde{P}_t(i), \quad (14b)$$

where  $\widetilde{P}_t^*(i) = \log P_t^*(i) - \log P_t$  is the firm-specific relative optimal price,  $\widetilde{P}_t^* = \int_0^1 \log P_t^*(i) di - \log P_t$  is the aggregate relative optimal price,  $\widetilde{W}_t^* = \int_0^1 \log W_t^*(i) di - \log W_t$  is the aggregate relative labor-clearing wage, and  $\tau_1$  and  $\tau_2$  are coefficients to be determined by equilibrium conditions. Using firm-specific relationships (14a)-(14b), and going through some algebra, equation (13) can be rewritten in the following way<sup>9</sup>

$$\begin{aligned} (1+\Lambda) \widetilde{W}_t^*(i) &= -\frac{\theta(1-\bar{\beta}\xi_w)}{(\sigma_l^{-1}+\alpha)(1-\bar{\beta}\xi_w\xi_p)} \widetilde{P}_t(i) - \frac{(1-\bar{\beta}\xi_w)(1+1/\sigma_l)}{(\sigma_l^{-1}+\alpha)(1-u^n)} E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j (u_{t+j} - u^n) \\ &\quad + (1+\Lambda) E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_w^j (\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1-\iota_w) \varepsilon_{t+j}^w), \end{aligned} \quad (15)$$

with  $\Lambda = \frac{\tau_1 \bar{\beta} \xi_w \theta}{\sigma_l^{-1} + \alpha} \left( 1 - \frac{\xi_p (1 - \bar{\beta} \xi_w)}{1 - \bar{\beta} \xi_w \xi_p} \right)$ . Equation (15) proves right the proposed linear relation (14b), with the following solution for  $\tau_2$

$$\tau_2 = \frac{\theta(1-\bar{\beta}\xi_w)}{(\sigma_l^{-1}+\alpha)(1-\bar{\beta}\xi_w\xi_p)(1+\Lambda)},$$

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<sup>8</sup>It should be noticed that

$$\begin{aligned} E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j \widetilde{W}_{t+j}(i) &= \\ E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j \left( \widetilde{W}_t^*(i) + \sum_{k=1}^j (\iota_w \pi_{t+k-1} + (1-\iota_w) \varepsilon_{t+k}^w - \pi_{t+k}^w) \right). \end{aligned}$$

<sup>9</sup>The proof is shown in the technical appendix -section 2-.

and the following expression for the aggregate relative wage set in period  $t$

$$\widetilde{W}_t^* = -\frac{(1-\bar{\beta}\xi_w)(1+1/\sigma_l)}{(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j (u_{t+j} - u^n) + E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_w^j (\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1-\iota_w)\varepsilon_{t+j}^w). \quad (16)$$

Calvo-type wage stickiness and the wage indexation rule (10) imply a proportional relationship between relative wages and the rate of wage inflation adjusted by the indexation factors

$$\widetilde{W}_t^* = \frac{\xi_w}{1-\xi_w} (\pi_t^w - \iota_w \pi_{t-1} - (1-\iota_w)\varepsilon_t^w). \quad (17)$$

Combining (16) and (17) yields

$$\begin{aligned} \pi_t^w &= \iota_w \pi_{t-1} + (1-\iota_w)\varepsilon_t^w - \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)(1+1/\sigma_l)}{\xi_w(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j (u_{t+j} - u^n) \\ &\quad + \frac{1-\xi_w}{\xi_w}E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_w^j (\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1-\iota_w)\varepsilon_{t+j}^w). \end{aligned}$$

Thus,  $\pi_t^w - \bar{\beta}\xi_w E_t \pi_{t+1}^w$  can be expressed as

$$\begin{aligned} \pi_t^w - \bar{\beta}\xi_w E_t \pi_{t+1}^w &= \iota_w \pi_{t-1} - \bar{\beta}\xi_w \iota_w \pi_t + (1-\iota_w)\varepsilon_t^w - \bar{\beta}\xi_w(1-\iota_w)E_t \varepsilon_{t+1}^w \\ &\quad - \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)(1+1/\sigma_l)}{\xi_w(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}(u_t - u^n) + \frac{1-\xi_w}{\xi_w}\bar{\beta}\xi_w E_t (\pi_{t+1}^w - \iota_w \pi_t - (1-\iota_w)\varepsilon_{t+1}^w), \end{aligned}$$

which collapses to the wage inflation equation

$$\pi_t^w = \bar{\beta}E_t \pi_{t+1}^w + \iota_w \pi_{t-1} - \bar{\beta}\iota_w \pi_t - \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)(1+1/\sigma_l)}{\xi_w(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}(u_t - u^n) + (1-\iota_w)(\varepsilon_t^w - \bar{\beta}E_t \varepsilon_{t+1}^w), \quad (18)$$

Thus, wage inflation dynamics are inversely related to the rate of unemployment.<sup>10</sup> For the real wage equation, we can take the log difference to its definition,  $w_t = \log\left(\frac{W_t}{P_t}\right)$ , to obtain

$$w_t - w_{t-1} = \pi_t^w - \pi_t. \quad (19)$$

Using (18) in (19) and solving out for the log of the real wage leads to

$$w_t = w_1 w_{t-1} + (1-w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 (u_t - u^n) + w_5 (\varepsilon_t^w - \bar{\beta}E_t \varepsilon_{t+1}^w), \quad (20)$$

where  $w_1 = \frac{1}{1+\bar{\beta}}$ ,  $w_2 = \frac{1+\bar{\beta}\iota_w}{1+\bar{\beta}}$ ,  $w_3 = \frac{\iota_w}{1+\bar{\beta}}$ ,  $w_4 = \frac{1}{1+\bar{\beta}} \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)(1+1/\sigma_l)}{\xi_w(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}$ , and  $w_5 = \frac{1-\iota_w}{1+\bar{\beta}}$ .

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<sup>10</sup>The slope coefficient in the wage inflation equation (18) is different from the one found in Casares (2010) due to the presence of variable capital and a competitive rental market together with a constant natural rate of unemployment.

Let us turn now to derive the New-Keynesian Phillips curve. The presence of unemployment as an excess supply of labor is going to influence inflation dynamics through the effect of firm-specific wage setting on firm-specific real marginal costs. For its derivation, we start from the loglinearized equation for the optimal price in Smets and Wouters (2007):<sup>11</sup>

$$p_t^*(i) = (1 - \bar{\beta}\xi_p) E_t^{\xi_p} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j \left( A \left( mc_{t+j}(i) + \lambda_{t+j}^p \right) + p_{t+j} - \iota_p \sum_{k=1}^j \pi_{t+k-1} \right), \quad (21)$$

where  $p_t^*(i)$  is the log of the optimal price set by firm  $i$ ,  $A > 0$  is a constant parameter that depends upon the Kimball (1995) goods market aggregator and the steady-state price mark-up,<sup>12</sup> and  $E_t^{\xi_p}$  is the rational expectations operator conditional on the lack of optimal pricing after period  $t$ . The log of the optimal price depends on the expectation of three factors: the log of the real marginal costs,  $mc_{t+j}(i)$ , exogenous price mark-up variations,  $\lambda_{t+j}^p$ , and the log of the aggregate price level adjusted by the indexation rule,  $p_{t+j} - \iota_p \sum_{k=1}^j \pi_{t+k-1}$ . Since  $p_{t+j} = p_t + \sum_{k=1}^j \pi_{t+k}^p$ , the following optimal relative price ( $\tilde{P}_t^*(i) = p_t^*(i) - p_t$ ) obtains:

$$\tilde{P}_t^*(i) = A (1 - \bar{\beta}\xi_p) E_t^{\xi_p} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j \left( mc_{t+j}(i) + \lambda_{t+j}^p \right) + E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j (\pi_{t+j} - \iota_p \pi_{t+j-1}). \quad (22)$$

Unlike Smets and Wouters (2007), the real marginal cost is firm-specific in our model as a consequence of firm-specific nominal wages. Taking logs in the definition of the firm-specific real marginal cost gives<sup>13</sup>

$$mc_t(i) = (1 - \alpha) (\log W_t(i) - p_t) + \alpha \log r_t^k - z_t, \quad (23)$$

where  $z_t$  is the log of capital utilization. Summing up across all firms and subtracting the result from (23) leads to

$$mc_t(i) = mc_t + (1 - \alpha) \tilde{W}_t(i). \quad (24)$$

Generalizing (24) for  $t + j$  periods and inserting the resulting expressions in (22) yields

$$\tilde{P}_t^*(i) = A (1 - \bar{\beta}\xi_p) E_t^{\xi_p} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j \left( mc_{t+j} + (1 - \alpha) \tilde{W}_{t+j}(i) + \lambda_{t+j}^p \right) + A E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j \left( \pi_{t+j} - \iota_p \pi_{t+j-1}^p \right). \quad (25)$$

<sup>11</sup>This result is provided in the technical appendix of Smets and Wouters (2007), available at [http://www.aeaweb.org/aer/data/june07/20041254\\_app.pdf](http://www.aeaweb.org/aer/data/june07/20041254_app.pdf).

<sup>12</sup>Concretely,  $A = 1/((\Phi - 1)\varepsilon_p + 1)$  where  $\varepsilon_p$  is the curvature of the Kimball aggregator and  $\Phi$  is the steady-state price mark-up.

<sup>13</sup>The real marginal cost of firm  $i$  in period  $t$  is  $MC_t(i) = \frac{(W_t(i))^{1-\alpha} (R_t^k)^\alpha}{Z_t \alpha^\alpha (1-\alpha)^{1-\alpha}}$ .

Recalling the firm-specific relationships (14a) and (14b), and doing some algebra, equation (25) can be written as follows<sup>14</sup>

$$(1 + \Theta) \tilde{P}_t^*(i) = \frac{A(1-\alpha)(1-\bar{\beta}\xi_p)\xi_w}{1-\bar{\beta}\xi_w\xi_p} \tilde{W}_{t-1}(i) + A(1-\bar{\beta}\xi_p) E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j (mc_{t+j} + \lambda_{t+j}^p) \quad (26)$$

$$+ (1 + \Theta) E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j (\pi_{t+j} - \iota_p \pi_{t+j-1}),$$

where  $\Theta = \tau_2 A(1-\alpha) \left(1 - \frac{(1-\bar{\beta}\xi_p)\xi_w}{1-\bar{\beta}\xi_p\xi_w}\right)$ . Equation (26) validates (14a) with  $\tau_1$  given by

$$\tau_1 = \frac{A(1-\alpha)(1-\bar{\beta}\xi_p)\xi_w}{(1-\bar{\beta}\xi_p\xi_w)(1+\Theta)}, \quad (27)$$

and also determines the dynamics of the aggregate relative prices across all firms that were able to optimally set prices in period  $t$

$$\tilde{P}_t^* = \frac{A(1-\bar{\beta}\xi_p)}{1+\Theta} E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j (mc_{t+j} + \lambda_{t+j}^p) + E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j (\pi_{t+j} - \iota_p \pi_{t+j-1}). \quad (28)$$

Calvo pricing combined with the same price indexation rule as in Smets and Wouters (2007) determine, after loglinearization, that relative optimal prices and the rate of inflation are related as follows

$$\tilde{P}_t^* = \frac{\xi_p}{1-\xi_p} (\pi_t - \iota_p \pi_{t-1}),$$

which can be substituted into the left-hand side of (28) to obtain

$$\pi_t = \iota_p \pi_{t-1} + \frac{A(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p(1+\Theta)} E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j (mc_{t+j} + \lambda_{t+j}^p) + \frac{1-\xi_p}{\xi_p} E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j (\pi_{t+j} - \iota_p \pi_{t+j-1}). \quad (29)$$

Rewriting (29) one period ahead to compute  $\bar{\beta}\xi_p E_t \pi_{t+1}$  and then subtracting it from (29) results in

$$\pi_t - \bar{\beta}\xi_p E_t \pi_{t+1} = \iota_p \pi_{t-1} - \bar{\beta}\xi_p \iota_p \pi_t^p + \frac{A(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p(1+\Theta)} (mc_t + \lambda_t^p) + \frac{1-\xi_p}{\xi_p} \bar{\beta}\xi_p E_t (\pi_{t+1} - \iota_p \pi_t).$$

Finally, we can put together terms on current and expected next period's inflation, re-scale the mark-up shock at  $\varepsilon_t^p = \pi_3 \lambda_t^p$  and -following the Smets and Wouters (2007) convention- introduce  $\mu_t^p$  as the log deviation of the price mark-up ( $mc_t = -\mu_t^p$ ), so that the inflation equation becomes

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p, \quad (30)$$

<sup>14</sup>The algebra involved is provided in the technical appendix -section 3-.

where  $\pi_1 = \frac{\iota_p}{1+\bar{\beta}\iota_p}$ ,  $\pi_2 = \frac{\bar{\beta}}{1+\bar{\beta}\iota_p}$ ,  $\pi_3 = \frac{1}{1+\bar{\beta}\iota_p} \frac{(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)(1+\Theta)}$ . Interestingly, the inflation equation (30) is a hybrid New-Keynesian Phillips curve where the slope coefficient is affected by the presence of nominal rigidities on both the goods and labor market. Thus, the slope of (30) depends on the value of the sticky-wage probability,  $\xi_w$ , that is contained in  $\Theta$ , reflecting the complementarities between pricing and wage setting assumed in (14a) and (14b) that are absent in standard DSGE models.<sup>15</sup> More precisely, the New-Keynesian Phillips curve (30) is flatter than the one derived in Smets and Wouters (2007) which had a slope coefficient  $\pi_3^{SW} = \frac{1}{1+\bar{\beta}\iota_p} \frac{(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)} = \pi_3(1+\Theta)$ . Intuitively, the response of inflation to an increase in the real marginal cost is weaker in our model because relative wages will move downwards due to higher relative prices as indicated by equation (14b). Therefore, a more moderate initial increase in prices set by firms will be enough to maximize intertemporal profit as they anticipate lower marginal costs when nominal wages are reset.

Hence, equations (20) and (30) collect the effects of unemployment as excess supply of labor in the dynamics of the real wage and inflation. The demand-side equations, the monetary policy rule and all the stochastic elements of the model (except for the wage indexation shock) are borrowed from Smets and Wouters (2007) since all these equations can be reached with no influence of the wage setting behavior and unemployment fluctuations. Thus, the shock processes are: the AR(1) technology shock  $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ , the AR(1) risk premium disturbance that shifts the demand for purchases of consumption and investment goods  $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ , the exogenous spending shock driven by an AR(1) process with an extra term capturing the potential influence of technology innovations on exogenous spending  $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$ , the AR(1) investment shock  $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ , the AR(1) monetary policy shock:  $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$ , the ARMA(1,1) price mark-up shock:  $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$ , and the ARMA(1,1) wage shock  $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$ . The technical appendix -sections 4 and 5- displays the complete set of equations of the model and discusses the equation-to-equation comparison across models. From now on, we will refer to the model with unemployment as the CMV model while the model of Smets and Wouters (2007) will be the SW model, taking the initial letters of the authors' last

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<sup>15</sup>The slope coefficient of the New-Keynesian Phillips curve (30) is also analytically different from the one obtained in the model of Casares (2010) which features unemployment as excess supply of labor and constant capital.

names.

### 3 Estimation

We estimate both models with U.S. data from the first quarter of 1984 to the third quarter of 2009. Except for some of the last quarters of the sample, corresponding to the 2007-08 financial crises, this period is characterized by mild fluctuations (the so-called Great Moderation) of aggregate variables (see Stock and Watson, 2002, among others). Thus, the estimation exercises do not suffer from some potential miss-specification sources, such as parameter instability in both the private sector -for instance, Calvo probabilities (Moreno, 2004)- and the monetary policy reactions to inflation or output. Indeed, some authors argue that a sound monetary policy implementation is the main factor behind the low business cycle volatility in this period (Clarida, Galí and Gertler, 1999).

Regarding the data set, we take as observable variables quarterly time series of the inflation rate, the Federal funds rate, civilian employment and the log differences of the real Gross Domestic Product (GDP), real consumption, real investment, and the real wage.<sup>16</sup> Thus, variables displaying a long-run trend enter the estimation procedure in log differences to extract their stationary business cycle component.<sup>17</sup> In the estimation of the CMV model, we add the quarterly unemployment rate as another observable variable and ignore the log of employment in order to consider the same (number of) shocks in the two models. The data were retrieved from the Federal Reserve of St. Louis (FRED2) database.

The estimation procedure also follows Smets and Wouters (2007). Thus, we consider a two-step Bayesian procedure. In the first step, the log posterior function is maximized in a way that combines the prior information of the parameters with the empirical likelihood of the data. In a second step, we perform the Metropolis-Hastings algorithm to compute the posterior distribution

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<sup>16</sup>The rate of inflation is obtained as the first difference of (the log of) the implicit GDP deflator, whereas the real wage is computed as the ratio between nominal compensation per hour and the GDP price deflator. Smets and Wouters (2007) estimate their model with (the log of) hours. As an alternative, and in order to facilitate a comparison with the estimation results of the CMV model, this paper estimates a version of the SW model where employment variability is determined only by the extensive margin of labor. Thus, we estimate the SW model using (the log of) civilian employment.

<sup>17</sup>In this way, we avoid the well-known measurement error implied by standard filtering treatments.

of the parameter set.<sup>18</sup> It should be noted that in the estimation of the CMV model, the slope coefficients in the inflation and real wage equations were introduced as implicit functions of the undetermined coefficients  $\tau_1$  and  $\tau_2$ . These coefficients can be analytically solved through a non-linear two-equation system. We choose the positive values associated with these solutions, as implied by theory.

In terms of the priors, we select the same prior distributions as Smets and Wouters (2007) for the estimation of the two models (see the first three columns in Tables 1A and 1B), and we also borrow their notation for the structural parameters. In the CMV model we have two additional parameters: the Dixit-Stiglitz elasticity of substitution across goods,  $\theta$ , and the steady-state unemployment rate,  $u$ . Prior mean of these two parameters is set at 6.0, in line with previous studies.

Tables 1A and 1B show the estimation results of both the SW and CMV models and report the posterior mean estimates together with the 5% and 95% quantiles of the posterior distribution.

[Insert Table 1A and Table 1B here]

As the last three columns of Tables 1A and 1B show, our version of the SW model -with a five-year longer sample period- confirms their estimates of the structural parameters. Across models, there are three main differences. First, the labor supply is less elastic in the CMV model, where  $\sigma_l$  is 4.79, while it is 2.43 in the SW model. Second, the Calvo probability of wage stickiness is higher in the CMV model, where  $\xi_w$  is 0.82, while it is 0.57 in the SW model. Therefore, the introduction of unemployment as excess supply of labor increases the estimated average length of labor contracts from  $(1 - 0.56)^{-1} = 2.27$  quarters to  $(1 - 0.82)^{-1} = 5.56$  quarters. Third, the elasticity of capital adjustment costs is higher in the SW model, where  $\varphi$  is 4.51, while it is 3.35 in the CMV model.

In both models, the inverse of the elasticity of intertemporal substitution ( $\sigma_c$ ) is quite low, slightly below 1.0, and the presence of habit persistence is significant and moderate, as  $h$  is in the vicinity of 0.5. The Calvo probability of price stickiness ( $\xi_p$ ) is high, around 0.76, and also

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<sup>18</sup>All estimation exercises are performed with DYNARE free routine software, which can be downloaded from <http://www.dynare.org>. A sample of 250,000 draws was used (ignoring the first 20% of draws). A step size of 0.3 resulted in an average acceptance rate of roughly 30% across the five Metropolis-Hastings blocks used.

very similar across models. Both wage and price indexation parameters ( $\iota_w$  and  $\iota_p$ , respectively) are slightly higher in the SW model. Even though price rigidities and indexation parameters are similar in both models, the New-Keynesian Phillips curve is flatter in the CMV model (see discussion in Section 2). Concretely, the estimated slopes are  $\pi_3 = 0.0055$  in the CMV model, lower than  $\pi_3^{SW} = 0.0083$  in the SW model as a consequence of an estimated  $\Theta = 0.4037$ .

Meanwhile, the elasticity of capital utilization adjustment cost  $\psi$  is 0.82 in the CMV model and somewhat lower at 0.68 in the SW model. Monetary policy parameters are similar across models, with a stabilizing interest reaction of inflation,  $r_\pi$ , between 1.75 and 1.93, a response to output gap growth,  $r_{\Delta y}$ , between 0.20 and 0.23, and a high policy rule persistence parameter,  $\rho$ , between 0.82 and 0.84. The only noticeable difference is that the response to the output gap,  $r_y$ , is not significantly different from zero in the SW model, whereas it is small, 0.14, but significant in the CMV model. The estimate of one plus the fixed-cost share,  $\Phi$ , is slightly higher in the CMV model. The estimates of the steady-state parameter that determines the long-run rate of growth,  $\gamma$ , the real interest rate,  $100(\beta^{-1} - 1)$ , and the rate of inflation,  $\pi$ , are similar in the two models, as well as the estimate of the capital share in the production function,  $\alpha$ . Finally, the elasticity of substitution across goods,  $\theta$ , and the steady state rate of unemployment,  $u$ , are only estimated in the CMV model, and are in line with the values chosen as priors.

Table 1B shows the standard deviations and autocorrelations of the seven structural shocks. The estimates of the standard deviations of the innovations look similar in both models. The only difference lies in the volatility of the wage-push innovation, which is significantly higher in the CMV model. As shown in the technical appendix, this is due to the fact that the wage inflation equation differs across models and the wage-push shock also has a different interpretation; it is a wage indexation shock in the CMV model while it is a wage mark-up shock in the SW model.<sup>19</sup> Again, the estimates of persistence and moving-average parameters are similar across models. The only difference lies in the persistence parameter associated with the wage-push shock, which is lower in the CMV model. Moreover, the monetary policy shock is the one exhibiting the lowest first-order autocorrelation -around 0.30- in both models. Technology, risk premium and

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<sup>19</sup>Moreover, the wage-push shock in the real wage equation of the CMV model appears multiplied by the coefficient  $w_5$ , which is estimated to be close to 0.32 while it is a unit coefficient in the real wage equation of the SW model. As a result, the effective size of the wage-push shock is similar across models.

exogenous spending innovations are highly persistent across models. The remaining shocks show less persistence in the SW model.

## 4 Empirical Fit

This section compares the performance of the SW and CMV models along three dimensions in the first three subsections. First, we analyze the ability of the two models to reproduce second-moment statistics found in U.S. quarterly data. Second, we study the contribution of each structural shock in explaining the total variance decomposition of macroeconomic variables. Third, we carry out an impulse-response analysis. Finally, the fourth subsection analyzes the ability of the CMV model to replicate U.S. lead-lag comovements between the unemployment rate and the output growth rate and between the rate of inflation and the output growth rate.

### 4.1 Second-moment statistics

Table 2 shows second-moment statistics obtained from actual data, and the ones found in the estimated CMV and SW models.

[Insert Table 2 here]

In general, the two models do a good job in reproducing the cyclical features of the data. Thus, both models match quite well the historical volatility of output growth, consumption growth, investment growth, the real wage growth, the log of civilian employment, price inflation, and the nominal interest rate. The CMV model matches all these volatilities better except for the nominal interest rate. Importantly, the introduction of unemployment as excess supply of labor in the estimated CMV model reproduces the unemployment rate volatility very accurately. Moreover, the CMV model by introducing unemployment is able to distinguish employment from the labor force (i.e. labor supply). Thus, the CMV model is able to reproduce rather well labor force volatility.

The contemporaneous correlations between each variable and the output growth rate are also reported in Table 2 as a measure of their procyclical or countercyclical behavior. Both models provide the sign found in the data for these correlations except for the ones of the real wage

growth and the nominal interest rate. In general, most of the model implied contemporaneous correlations are close to their data counterparts. Finally, the two models do a reasonable job in replicating the first-order autocorrelation of all variables, with the exception of an excessive inflation persistence in the SW model. Interestingly, our model does a good job in reproducing labor market dynamics without resorting to the device of considering an arbitrarily small short-term wealth effect introduced in Galí, Smets and Wouters (2011).

## 4.2 Variance decomposition

Table 3 shows the total variance decomposition analysis for the CMV and SW models. In the CMV model, technology innovations  $\eta^a$  explain nearly 40% of the fluctuations in labor force and output and consumption growth. Meanwhile, demand (risk-premium) shocks,  $\eta^b$ , drive around 75% of the variability of the nominal interest rate and more than one third of both employment and the rate of unemployment. The influence of exogenous spending (fiscal/net exports) shocks,  $\eta^g$ , is rather low as it determines 14% of output growth and even lower shares of the other variables.<sup>20</sup> Innovations in the investment adjustment costs,  $\eta^i$ , only have a substantial impact on investment fluctuations (73%) whereas monetary policy shocks,  $\eta^R$ , explain between 15% and 21% of fluctuations of output growth, consumption growth, employment and the rate of unemployment. Meanwhile, inflation (price-push) shocks,  $\eta^p$ , are the main determinant of inflation variability (56%) but account for less than 10% of the variance share of the other variables. The wage-push (indexation) shock,  $\eta^w$ , exerts a strong influence on the real wage growth (81%) and more moderate influence on the rate of unemployment (33%), labor force (21%) and inflation (14%), while having a weak impact on the rest of the variables.

[Insert Table 3 here]

Therefore, the introduction of unemployment as excess supply of labor in an estimated New-Keynesian model implies that the main driving forces behind U.S. unemployment fluctuations are wage-push shocks (33%), demand shifts driven by risk-premium variations (35%) and

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<sup>20</sup>As in Smets and Wouters (2007), the role of the exogenous spending shock is to bring demand-determined changes in output that are not collected by either private consumption or private investment, such as fiscal shocks or exports/imports variations.

also monetary policy shocks (21%). Innovations in technology barely explain 2% of unemployment variability. By contrast, the dynamics of output growth are substantially influenced by technology shocks (39%), as well as a mix of demand-side perturbations (risk-premium shocks, exogenous spending innovations and monetary policy shocks that jointly take 59% in its variance decomposition).

The SW model estimated here confirms that technology innovations,  $\eta^a$ , are less influential on business cycle fluctuations than in the CMV model, affecting 21% of output changes and lower percentages of the rest of the variables. The risk-premium shocks,  $\eta^b$ , account for approximately one fourth of the variability of output growth and inflation, similarly to the CMV model, while their influence on the movements of the nominal interest rate is still high at 71% of total variability. As in the CMV model, the influence of exogenous spending shocks,  $\eta^g$ , is not substantial with only 14% of fluctuations of output growth, while the investment shock,  $\eta^i$ , mainly affects private investment (72% of its variability) and has a minor influence on the remaining variables. Monetary policy shocks,  $\eta^R$ , account for 15% and 22% for changes in output and consumption, respectively; whereas inflation shocks,  $\eta^p$ , only explain significant fractions of variability of inflation (29%) and the changes in the real wage (18%). Wage-push shocks are more influential in the SW model than in the CMV model as they explain 63% of variations in the real wage growth, but in addition they also determine a high share of employment fluctuations (81%), 30% of inflation variability, 24% of consumption growth fluctuations, and 14% of variability on both nominal interest rate and output growth.<sup>21</sup>

It is worthwhile highlighting that the results for long-run variance decompositions obtained from the estimated CMV model present some relevant differences with respect to those in Galí, Smets and Wouters (2007). Most importantly, our model gives a relatively large importance to demand (risk premium, monetary policy and fiscal/net export) shocks in explaining most cyclical fluctuations, whereas in their case wage markup shocks emerge as a key driving force behind many variables. In particular, results in terms of unemployment dynamics are in stark contrast, as they attribute them almost entirely to wage markup shocks, whereas in our case demand shocks account for more than half of the variance. Despite these differences, both models confer

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<sup>21</sup>This result will be reflected in the relatively large reactions of these variables to the estimated wage-push shock displayed in the impulse-response analysis conducted below.

a large importance to productivity shocks in explaining output fluctuations.

### 4.3 Impulse-response functions

Figures 1, 2 and 3 plot the impulse response functions obtained in the SW and CMV estimated models to the seven -one standard deviation- structural shocks. Across figures, we observe that the responses are quite similar for both models in terms of sign and dynamics. However, the CMV model shows greater responses to technology shocks and weaker to wage-push and price-push shocks, than in the SW model. In particular, Figure 1 shows that the technology shock increases output, consumption and investment, with the effects being higher and more persistent in the CMV model. The risk premium shock results in similar declines of these three variables, whereas the investment shock increases output and investment at the expense of a drop in consumption due to the consequent monetary policy tightening -see Figure 3 below-. The fiscal-net exports (exogenous spending) shock increases output but crowds out investment and consumption in both models, whereas the interest rate shock has a negative impact on these three variables, as typical from sticky-price models. Models disagree substantially in the effects of the wage-push shocks. These different effects do not come as a surprise since the interpretation of this shock in the two models, as explained above, is different. Thus, the CMV model provides a slight decrease in output while in the SW model we find a much deeper and more persistent fall in output. Price mark-up shocks are also more contractionary on output, consumption and investment in the SW model, through the implied increase in interest rates in response to higher inflation.

[Insert Figure 1 here]

[Insert Figure 2 here]

[Insert Figure 3 here]

Figure 2 shows the responses of the labor market variables to the structural shocks. While both the SW and CMV models include log fluctuations of the real wage and labor (employment), the CMV model captures the response of the unemployment rate as well, in contrast to the SW model. The real wage rises after technology, investment, fiscal-net exports and wage-push shocks, whereas it decreases after risk premium, monetary policy and price-push shocks. Meanwhile,

employment falls countercyclically in both models when there is a positive technology shock. This is a characteristic response in New-Keynesian models with sticky prices, as discussed in Galí (1999). By contrast, procyclical reactions of employment are always reported after demand-side disturbances such as risk-premium shocks, investment shocks, and fiscal-net exports shocks. Both models imply declines of employment in reaction to price and wage cost-push shocks. However, the fall of employment after a wage-push shock is much deeper and persistent in the SW model than in the CMV model (see Figure 2), which is consistent with the variance decomposition analysis conducted above.<sup>22</sup>

The reactions of unemployment –only reported in the CMV model– are closely and inversely related to those of employment, as the influence of labor supply variability is small due to the low estimated labor supply elasticity.<sup>23</sup> A positive technology shock increases unemployment only during the quarter of the shock. Demand shocks (risk premium, investment, fiscal-net exports and monetary policy) bring procyclical reactions of unemployment, that display quite persistent dynamic patterns. The unemployment rate also raises after both wage and price cost-push shocks, since monetary policy reacts through higher interest rates to these supply-side disturbances.

Figure 3 shows the responses of the nominal interest rate and inflation. The plots are rather similar across models, although the reactions in the SW model show more amplitude after wage-push shocks. Technology innovations bring countercyclical cuts of inflation and the nominal interest rate whereas three of the demand shocks (risk premium, investment, and fiscal-net exports) result in procyclical responses of inflation and interest rates. The interest rate shock represents an unexpected monetary policy tightening that brings a realistic U-shaped decline in inflation (as observed in Romer and Romer, 2004). Finally, both wage and price push

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<sup>22</sup>This higher sensitivity to wage-push shocks in the SW model is the result of its particular labor market assumptions. Households must attend firm-specific relative labor demand as constraints in their optimizing programs. In turn, those households that apply the indexation rule with the positive wage shock will suffer from a significant employment cut. This implies contractionary effects on consumption due to the non-separability between labor and consumption in the utility function, which justifies the difference in the response of output growth across models (see Figure 1).

<sup>23</sup>In log fluctuations from steady-state, labor supply (labor force) can be obtained as the sum of the response of labor plus the response of the rate of unemployment.

shocks increase inflation and, as a result, trigger a gradual and persistent increase in the nominal interest rate.

#### 4.4 Dynamic Cross-Correlation Functions

This section studies the ability of the CMV model to reproduce two important comovement patterns observed in U.S. business cycles. First, we examine the dynamic correlations between the rate of unemployment and the output growth rate in a model-to-data comparison (Figure 4). Then, we assess the capacity of the model to replicate the dynamic cross correlations between inflation and output growth rates (Figure 5). These figures compare the lead-lag correlation functions in the data with those implied by the CMV model. They also show the  $\pm$  two-standard deviation confidence interval (CI) bands derived from simulated data obtained from 5,000 independent draws for the seven innovations of the CMV model.

[Insert Figure 4 here]

[Insert Figure 5 here]

Figure 4 shows the lead and lag correlation structure between output growth and the rate of unemployment observed in actual U.S. data (dashed line) and the corresponding comovement implied by the CMV model (solid line) within the model-implied statistical confidence interval. The estimated model reproduces the negative contemporaneous comovement between the U.S. rates of unemployment and output growth. In addition, the model replicates the positive correlation between lagged rates of unemployment and current output growth rate (i.e., cases with  $j < 0$  in Figure 4) and the negative correlation between future unemployment rates and current output growth rate (i.e., cases with  $j > 0$  in Figure 4).

Similarly, Figure 5 compares the dynamic comovement patterns between inflation and output growth found in US data (dashed line) with those implied by the CMV model (solid line). Once again the estimated model does a good job in reproducing the lead-lag pattern shape displayed by actual data.<sup>24</sup> Thus, the model reproduces two stylized facts. First, higher lagged inflation anticipates a lower current output growth rate (i.e. for  $j < 0$ ). Second, higher current output growth anticipates higher future inflation 1-4 quarters ahead.

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<sup>24</sup>Smetts and Wouters (2007) also report a good matching to dynamic cross correlations between inflation and Hodrick-Prescott filtered output in their model without unemployment.

## 5 Conclusions

This paper introduces a model with both sticky prices and sticky wages that combines elements of Smets and Wouters (2007) and Casares (2010) in a way that incorporates unemployment as excess supply of labor in a medium-scale New-Keynesian model. The alternative labor market assumptions have implications for the real wage equation (where the real wage is inversely related to the rate of unemployment) and also for the New Keynesian Phillips curve (where the slope coefficient is lower and it depends upon the level of wage stickiness).

The structural model parameters were estimated with Bayesian techniques and then compared to the estimates of the benchmark New-Keynesian model of Smets and Wouters (2007). Most parameter estimates are quite similar across models. The only substantial differences are that in the model with unemployment the labor supply curve is less elastic, wages are stickier with a longer average in the length of labor contracts, and the elasticity of capital adjustment costs is lower. The empirical comparison also shows that the two models do a similar job in reproducing many of the features characterizing the recent U.S. business cycles. Importantly, our model with unemployment is able to explain the most salient features -volatility, cyclical correlation and persistence- characterizing U.S. unemployment rate fluctuations. The impulse-response functions show that the rate of unemployment reacts in a countercyclical way to demand shocks and price-push shocks, whereas the response is initially procyclical and later countercyclical after productivity innovations and clearly procyclical after wage-push innovations.

Our results also indicate that fluctuations in the unemployment rate are mostly driven by wage-push shocks and by demand-side shocks such as risk-premium disturbances and monetary policy shocks, while technology shocks play a more secondary role. Regarding output growth variability, changes in output growth are evenly driven by technology innovations (nearly 50% of total variability) and demand shocks in the model with unemployment. The model without unemployment gives less influence to technology shocks and more to cost-push shocks. Finally, the estimated model with unemployment is able to provide a good match of the U.S. dynamic cross correlation between unemployment and output growth rates.

## References

- Abel, Andrew, Ben Bernanke, and Dean Croushore. 2011. *Macroeconomics*, 7th ed., Pearson Education.
- Bénassy, Jean P. 1995. “Money and Wage Contracts in an Optimizing Model of the Business Cycle.” *Journal of Monetary Economics*, 35: 303-315.
- Blanchard, Olivier, and Jordi Galí. 2010. “Labor Market Frictions and Monetary Policy: A New Keynesian Model with Unemployment.” *American Economic Journal: Macroeconomics*, 2(2): 1-30.
- Blanchard, Olivier. 2011. *Macroeconomics*, 5th ed., Pearson Education.
- Calvo, Guillermo A. 1983. “Staggered Pricing in a Utility-Maximizing Framework.” *Journal of Monetary Economics*, 12: 383-396.
- Casares, Miguel. 2007. “Firm-Specific or Household-Specific Sticky Wages in the New-Keynesian Model?” *International Journal of Central Banking*, 3: 181-240.
- Casares, Miguel. 2010. “Unemployment as Excess Supply of Labor.” *Journal of Monetary Economics*, 57: 233-243.
- Cho, Jang-Ok, and Thomas F. Cooley. 1994. “Employment and Hours over the Business Cycle.” *Journal of Economic Dynamics and Control*, 18: 411-432.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.” *Journal of Political Economy*, 113: 1-45.
- Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin. 2010. “Involuntary Unemployment and the Business Cycle.” NBER Working Paper 15801.
- Clarida, Richard, Jordi Galí, and Mark Gertler. 1999. “The Science of Monetary Policy: A New-Keynesian Perspective.” *Journal of Economic Literature*, 37: 1661-1707.
- Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. 2000. “Optimal Monetary Policy with Staggered Wage and Price Contracts.” *Journal of Monetary Economics*, 46: 281-313.
- Galí, Jordi. 1999. “Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?” *American Economic Review*, 89: 249-271.
- Galí, Jordi. 2011. “The Return of the Wage Phillips Curve.” *Journal of the European*

*Economic Association* 9: 436-461.

Galí, Jordi, Frank R. Smets, and Rafael Wouters. 2011. "Unemployment in an Estimated New Keynesian Model," NBER Working Paper 17084.

Gertler, Mark, Luca Sala, and Antonella Trigari. 2008. "An Estimated Monetary DSGE Model with Un-employment and Staggered Nominal Wage Bargaining." *Journal of Money, Credit, and Banking*, 40: 1713-1764.

Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*, 16: 309-327.

Kimball, Miles S. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model." *Journal of Money, Credit, and Banking*, 27: 1241-1277.

Michaillat, Pascal. 2012. "Do Matching Frictions Explain Unemployment? Not in Bad Times." *American Economic Review*, forthcoming.

Moreno, Antonio. 2004. "Reaching Inflation Stability." *Journal of Money, Credit, and Banking*, 36: 801-825.

Mortensen, Dale T. and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, 61: 397-415.

Mulligan, Casey. 2001. "Aggregate Implications of Indivisible Labor." *Advances in Macroeconomics, Berkeley Electronic Press*, 1 (1): article 4.

Romer, Christina D. and David H. Romer. 2004. "A New Measure of Monetary Shocks: Derivation and Implications." *American Economic Review*, 94: 1055-1084.

Smets, Frank R., and Rafael Wouters. 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association*, 1: 1123-1175.

Smets, Frank R., and Rafael Wouters. 2007. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97: 586-606.

Stock, James H., and Mark W. Watson. 2002. "Has the Business Cycle Changed and Why?" In *NBER Macroeconomics Annual 2002*, eds. Mark Gertler and Kenneth Rogoff, 159-218. Cambridge: MIT Press.

Trigari, Antonella. 2009. "Equilibrium Unemployment, Job Flows, and Inflation Dynamics." *Journal of Money, Credit, and Banking*, 41: 1-33.

Walsh, Carl E. 2005. "Labor Market Search, Sticky Prices, and Interest Rate Policies." *Review of Economic Dynamics*, 8: 829-849.

Woodford, Michael 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

Table 1A. Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
	Distr	Mean	Std D.	CMV model			SW model		
				Mean	5%	95%	Mean	5%	95%
$\varphi$	Normal	4.00	1.50	3.35	1.60	4.97	4.51	2.80	6.12
$h$	Beta	0.70	0.10	0.46	0.32	0.59	0.57	0.47	0.69
$\sigma_c$	Normal	1.50	0.37	0.97	0.67	1.30	0.82	0.56	1.05
$\sigma_l$	Normal	2.00	0.75	4.79	3.92	5.61	2.43	1.47	3.36
$\xi_p$	Beta	0.50	0.10	0.77	0.71	0.83	0.76	0.67	0.83
$\xi_w$	Beta	0.50	0.10	0.82	0.77	0.87	0.56	0.43	0.70
$\iota_w$	Beta	0.50	0.15	0.37	0.17	0.55	0.49	0.25	0.73
$\iota_p$	Beta	0.50	0.15	0.30	0.10	0.56	0.45	0.17	0.68
$\psi$	Beta	0.50	0.15	0.82	0.71	0.94	0.68	0.51	0.86
$\Phi$	Normal	1.25	0.12	1.61	1.44	1.76	1.55	1.42	1.69
$r_\pi$	Normal	1.50	0.25	1.75	1.39	2.10	1.93	1.62	2.23
$\rho$	Beta	0.75	0.10	0.84	0.80	0.88	0.82	0.78	0.86
$r_y$	Normal	0.12	0.05	0.14	0.07	0.21	0.02	-0.01	0.05
$r_{\Delta y}$	Normal	0.12	0.05	0.23	0.18	0.28	0.20	0.15	0.24
$\pi$	Gamma	0.62	0.10	0.65	0.54	0.77	0.66	0.52	0.79
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	0.23	0.10	0.36	0.23	0.11	0.34
$\bar{l}$	Normal	0.00	2.00	-	-	-	-0.43	-2.55	1.81
$\gamma$	Normal	0.40	0.10	0.40	0.32	0.47	0.39	0.36	0.43
$\alpha$	Normal	0.30	0.05	0.14	0.10	0.16	0.11	0.09	0.14
$\theta$	Normal	6.00	1.50	6.92	4.74	9.36	-	-	-
$u$	Normal	6.00	2.00	6.28	5.66	6.91	-	-	-

Table 1B. Priors and estimated posteriors of the shock processes

Priors				Posteriors					
				CMV model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
$\sigma_a$	Invgamma	0.10	2.00	0.46	0.37	0.54	0.35	0.30	0.39
$\sigma_b$	Invgamma	0.10	2.00	0.16	0.12	0.20	0.18	0.12	0.24
$\sigma_g$	Invgamma	0.10	2.00	0.40	0.35	0.45	0.35	0.31	0.39
$\sigma_i$	Invgamma	0.10	2.00	0.46	0.36	0.55	0.43	0.31	0.54
$\sigma_R$	Invgamma	0.10	2.00	0.13	0.12	0.15	0.13	0.11	0.15
$\sigma_p$	Invgamma	0.10	2.00	0.15	0.12	0.17	0.14	0.11	0.16
$\sigma_w$	Invgamma	0.10	2.00	1.47	0.88	1.95	0.32	0.25	0.38
$\rho_a$	Beta	0.50	0.20	0.98	0.97	0.99	0.94	0.91	0.97
$\rho_b$	Beta	0.50	0.20	0.92	0.88	0.96	0.90	0.85	0.95
$\rho_g$	Beta	0.50	0.20	0.96	0.94	0.99	0.97	0.95	0.99
$\rho_i$	Beta	0.50	0.20	0.69	0.56	0.82	0.67	0.51	0.81
$\rho_R$	Beta	0.50	0.20	0.28	0.15	0.41	0.31	0.19	0.44
$\rho_p$	Beta	0.50	0.20	0.54	0.16	0.82	0.66	0.41	0.94
$\rho_w$	Beta	0.50	0.20	0.73	0.52	0.92	0.98	0.96	0.99
$\mu_p$	Beta	0.50	0.20	0.50	0.23	0.77	0.57	0.33	0.83
$\mu_w$	Beta	0.50	0.20	0.42	0.21	0.63	0.71	0.52	0.92
$\rho_{ga}$	Beta	0.50	0.20	0.29	0.14	0.45	0.63	0.47	0.77

Table 2. Second-moment statistics

	$\Delta y$	$\Delta c$	$\Delta i$	$\Delta w$	$l$	$u$	$lforce$	$R$	$\pi$
<i>U.S. data (1984:1-2009:3):</i>									
Standard deviation (%)	0.63	0.61	2.26	0.69	2.08	1.17	1.10	0.63	0.26
Correlation with output growth	1.0	0.67	0.70	-0.03	0.01	-0.07	-0.05	0.27	-0.05
Autocorrelation	0.35	0.30	0.61	0.05	0.98	0.97	0.98	0.98	0.51
<i>Estimated CMV model:</i>									
Standard deviation (%)	0.73	0.69	2.41	0.77	1.42	1.14	0.93	0.48	0.29
Correlation with output growth	1.0	0.68	0.55	0.23	0.08	-0.22	-0.16	-0.06	-0.11
Autocorrelation	0.30	0.36	0.62	0.24	0.94	0.94	0.95	0.96	0.62
<i>Estimated SW model:</i>									
Standard deviation (%)	0.77	0.70	2.52	0.79	3.30	-	3.30	0.52	0.46
Correlation with output growth	1.0	0.74	0.57	0.28	0.08	-	0.08	-0.10	-0.28
Autocorrelation	0.38	0.49	0.64	0.29	0.99	-	0.99	0.96	0.82

Table 3. Long-run variance decomposition<sup>25</sup>

<i>Estimated CMV model:</i>									
Innovations	$\Delta y$	$\Delta c$	$\Delta i$	$\Delta w$	$l$	$u$	$lforce$	$R$	$\pi$
Technology, $\eta^a$	0.39	0.39	0.03	0.02	0.15	0.02	0.41	0.06	0.05
Risk premium, $\eta^b$	0.21	0.26	0.13	0.05	0.36	0.35	0.09	0.75	0.21
Fiscal/Net exports, $\eta^g$	0.15	0.10	0.00	0.00	0.09	0.01	0.18	0.02	0.01
Investment, $\eta^i$	0.08	0.04	0.73	0.01	0.10	0.06	0.06	0.06	0.01
Interest-rate, $\eta^R$	0.15	0.19	0.08	0.03	0.19	0.21	0.03	0.06	0.03
Wage-push, $\eta^w$	0.00	0.01	0.01	0.81	0.06	0.33	0.21	0.04	0.14
Price-push, $\eta^p$	0.02	0.02	0.02	0.09	0.05	0.02	0.03	0.02	0.56
<i>Estimated SW model:</i>									
Innovations	$\Delta y$	$\Delta c$	$\Delta i$	$\Delta w$	$l$	$u$	$lforce$	$R$	$\pi$
Technology, $\eta^a$	0.21	0.06	0.02	0.01	0.01	–	0.01	0.03	0.01
Risk premium, $\eta^b$	0.23	0.34	0.09	0.11	0.06	–	0.06	0.71	0.30
Fiscal/Net exports, $\eta^g$	0.14	0.04	0.00	0.00	0.04	–	0.04	0.01	0.00
Investment, $\eta^i$	0.06	0.02	0.72	0.01	0.02	–	0.02	0.03	0.01
Interest-rate, $\eta^R$	0.15	0.22	0.05	0.06	0.03	–	0.03	0.05	0.08
Wage-push, $\eta^w$	0.14	0.24	0.07	0.63	0.81	–	0.81	0.14	0.30
Price-push, $\eta^p$	0.08	0.08	0.06	0.18	0.04	–	0.04	0.04	0.29

<sup>25</sup>For a 100-period ahead forecast.

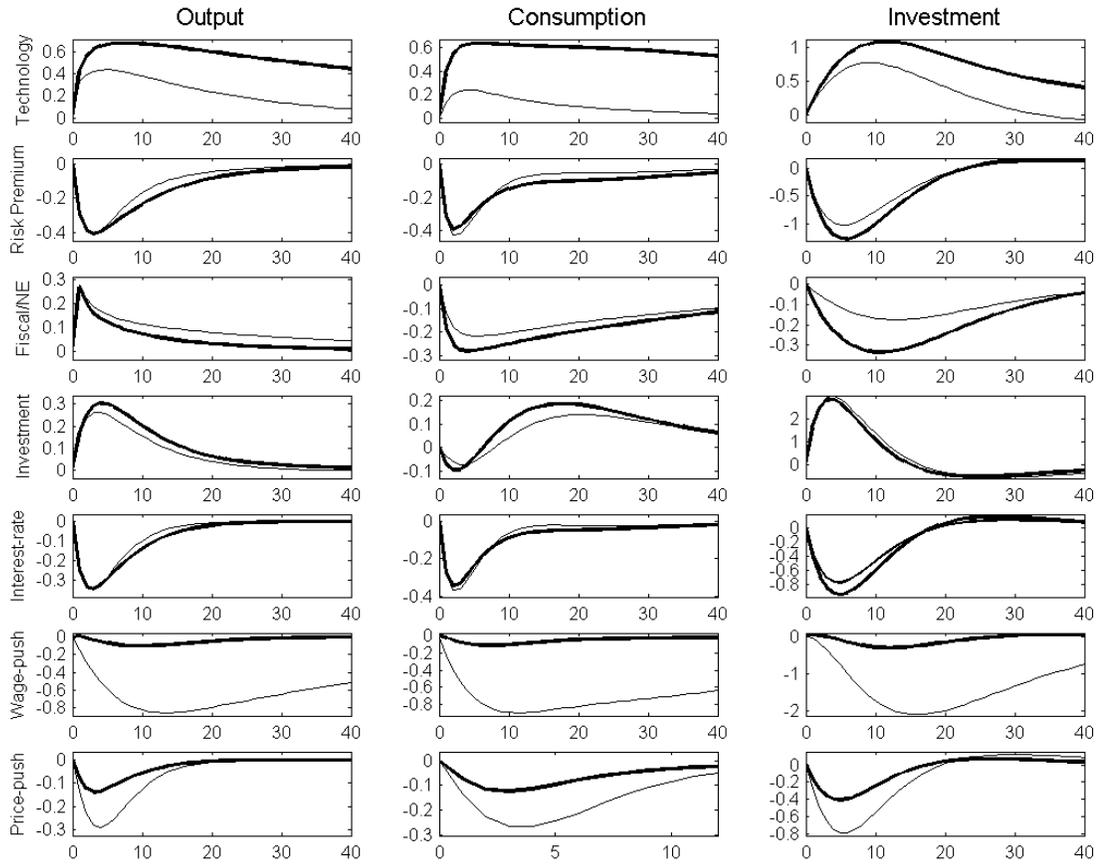


Figure 1: Impulse response functions of output, consumption, and investment. CMV model (thick line) and SW model (thin line).

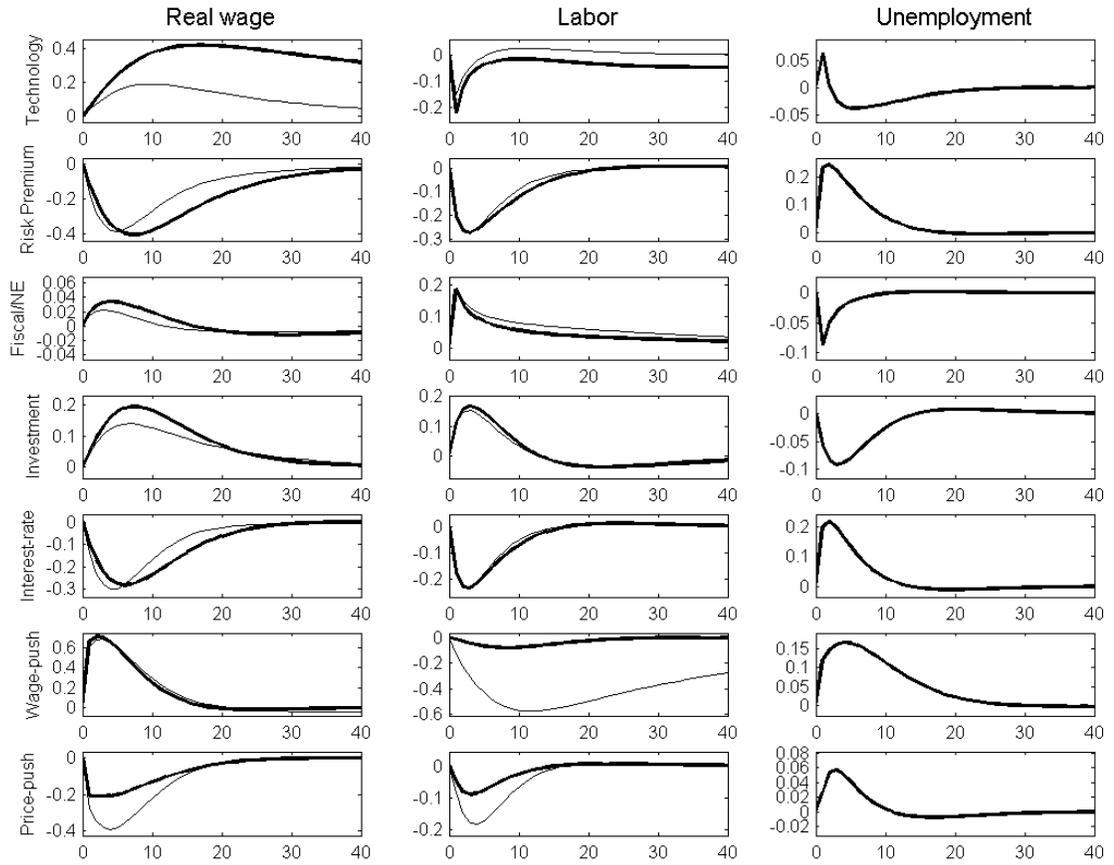


Figure 2: Impulse responses of the real wage, labor, and the rate of unemployment. CMV model (thick line) and SW model (thin line).

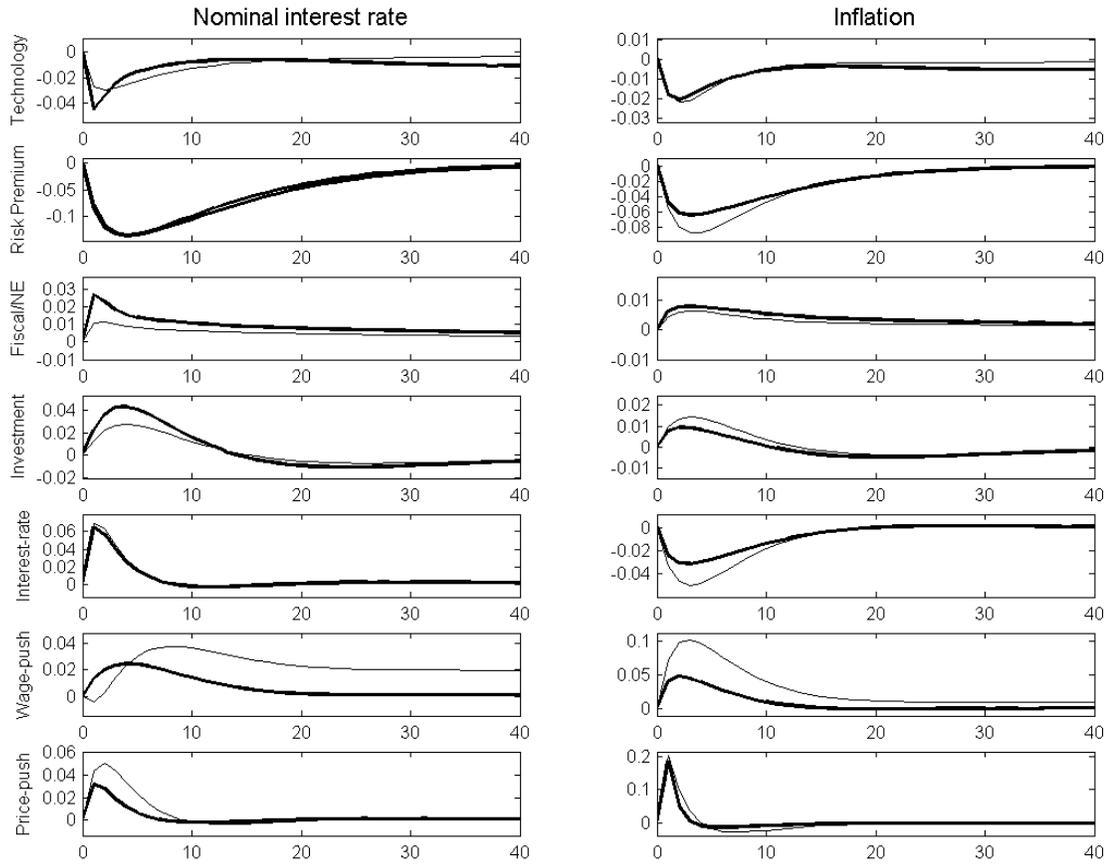


Figure 3: Impulse response functions of the nominal interest rate and the rate of inflation. CMV model (thick line) and SW model (thin line).

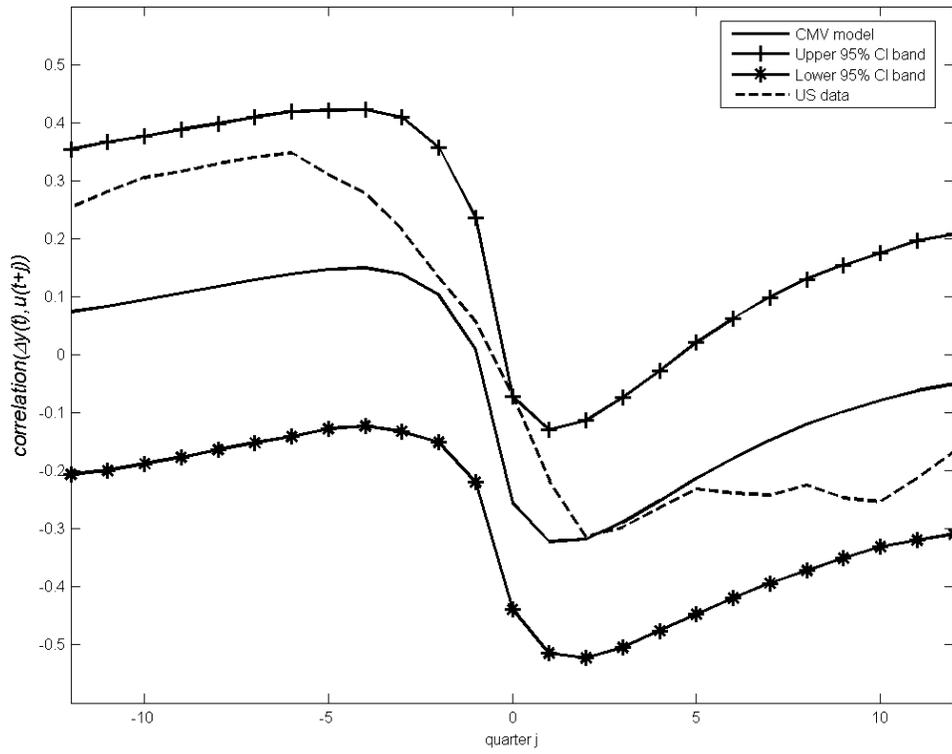


Figure 4: Dynamic cross-correlation between output growth,  $\Delta y_t$ , and the unemployment rate,  $u_{t+j}$ .

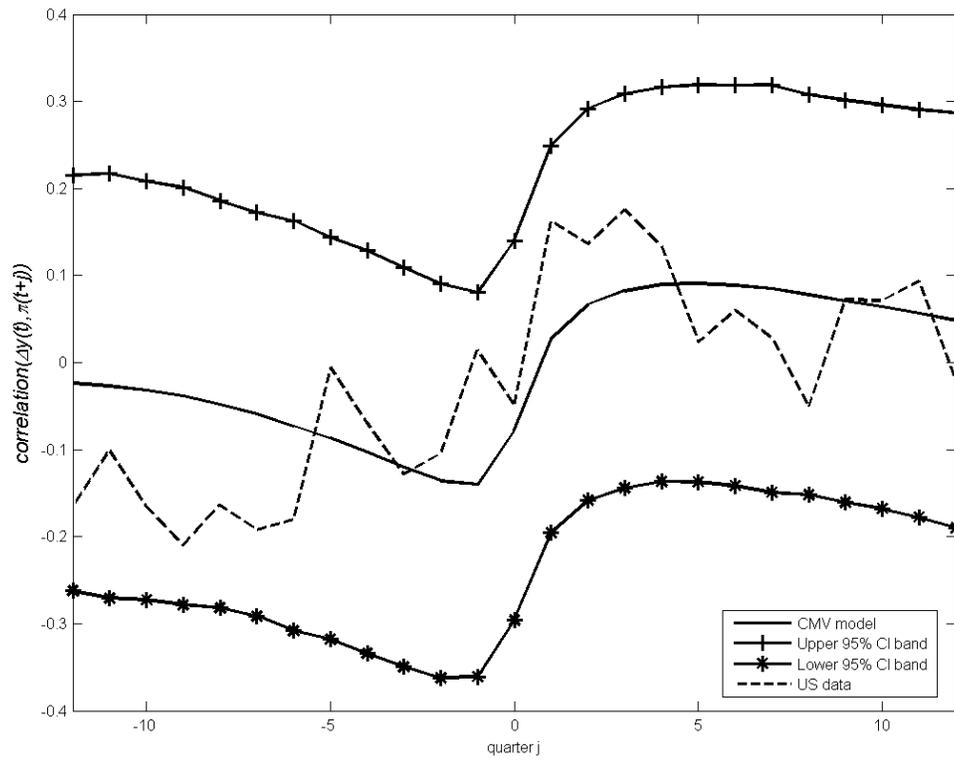


Figure 5: Dynamic cross-correlation between output growth,  $\Delta y_t$ , and inflation,  $\pi_{t+j}$ .

## Technical Appendix.

### 1. Labor supply and labor demand of type $i$ .

Households maximize intertemporal utility subject to a budget constraint. Unlike Smets and Wouters (2007), there is a representative household that provides all types of labor services. Thus, instantaneous utility is

$$\left[ \frac{1}{1 - \sigma_c} (C_t - \lambda C_{t-1}^A)^{1 - \sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} \int_0^1 (L_t^s(i))^{1 + \sigma_l} di \right),$$

where  $\sigma_c, \sigma_l > 0$ ,  $C_t$  is current consumption of the household,  $C_{t-1}^A$  is lagged aggregate consumption and  $L_t^s(i)$  is the supply of labor of the type employed in the  $i$ -th firm. The household budget constraint is

$$C_t + I_t + \frac{B_t}{\exp(\varepsilon_t^b)(1 + R_t)P_t} - T_t = \int_0^1 \frac{(1 - u_t(i))W_t(i)L_t^s(i)}{P_t} di + \frac{R_t^k Z_t K_{t-1}}{P_t} - a(Z_t) K_{t-1} + \frac{B_{t-1}}{P_{t-1}} (1 + \pi_t)^{-1} + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}$$

The first order conditions for consumption and labor supply of type  $i$  that result from the household optimizing program are

$$\begin{aligned} (C_t - \lambda C_{t-1}^A)^{-\sigma_c} \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} \int_0^1 (L_t^s(i))^{1 + \sigma_l} di \right) - \Xi_t &= 0, & (C_t^{foc}) \\ -(\sigma_c - 1) L_t(i)^{\sigma_l} \left[ \frac{1}{1 - \sigma_c} (C_t - \lambda C_{t-1}^A)^{1 - \sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} \int_0^1 (L_t^s(i))^{1 + \sigma_l} di \right) + \Xi_t \frac{(1 - u_t(i))W_t(i)}{P_t} &= 0, & (L_t^{foc}(i)) \end{aligned}$$

where  $\Xi_t$  is the Lagrange multiplier of the budget constraint in period  $t$ . Inserting  $(C_t^{foc})$  in  $(L_t^{foc}(i))$  and rearranging terms leads to the optimal supply of  $i$ -type labor

$$L_t^s(i) = \left( \frac{(1 - u_t(i)) \frac{W_t(i)}{P_t}}{C_t - \lambda C_{t-1}^A} \right)^{1/\sigma_l},$$

which in log-linear terms is

$$l_t^s(i) = \frac{1}{\sigma_l} \left( \log W_t(i) - p_t - \frac{1}{1 - u^n} (u_t(i) - u^n) - \frac{1}{(1 - \lambda/\gamma)} (c_t - (\lambda/\gamma) c_{t-1}) \right)$$

Meanwhile, the labor demand of firm  $i$  is obtained from the loglinear version of equation (5),

$$\frac{1 - \alpha}{\alpha} \frac{K_t^d(i)}{L_t^d(i)} = \frac{W_t(i)/P_t}{r_t^k},$$

$$l_t^d(i) = k_t^d(i) - \log W_t(i) + p_t + \log r_t^k.$$

As in Smets and Wouters (2007), the loglinearized production function, with technology shocks  $\varepsilon_t^a$ , is  $y_t(i) = (1 - \alpha) l_t^d(i) + \alpha k_t^d(i) + \varepsilon_t^a$ , which determines the log of firm-specific capital demand

$$k_t^d(i) = \frac{1}{\alpha} \left( y_t(i) - (1 - \alpha) l_t^d(i) - \varepsilon_t^a \right).$$

Substituting the value of  $k_t^d(i)$  from the last expression in the labor demand equation and rearranging terms results in

$$l_t^d(i) = y_t(i) - \alpha (\log W_t(i) - p_t) + \alpha \log r_t^k - \varepsilon_t^a.$$

As discussed in Woodford (2003, p. 168), the Kimball (1995) scheme for the aggregation of goods –also used in the Smets and Wouters (2007)’s model–, yields a log approximation of demand-determined relative output that is inversely related to the relative price,

$$y_t(i) = y_t - \theta \tilde{P}_t(i),$$

where  $\theta > 0$  defines the elasticity of demand and the relative price is  $\tilde{P}_t(i) = \log P_t(i) - \log P_t = \log P_t(i) - \int_0^1 \log P_t(i) di$ . Inserting  $y_t(i) = y_t - \theta \tilde{P}_t(i)$  in the labor demand equation gives

$$l_t^d(i) = y_t - \theta \tilde{P}_t(i) - \alpha (\log W_t(i) - p_t) + \alpha \log r_t^k - \varepsilon_t^a.$$

Summing up across all firms and taking the difference between firm-specific and aggregate values results in a firm-specific labor demand equation

$$l_t^d(i) = -\theta \tilde{P}_t(i) - \alpha \tilde{W}_t(i) + l_t,$$

which introduces  $l_t$  as the log deviation from steady state of demand-determined employment obtained from the aggregation of log deviations on firm-specific labor demand  $l_t = \int_0^1 l_t^d(i) di$ .

## 2. Derivation of equation (15): The dynamics of relative labor-clearing wages.

The equation of the relative nominal wage, (13) in the main text, is:

$$\widetilde{W}_t^*(i) = -\frac{(1-\bar{\beta}\xi_w)}{\sigma_t^{-1}+\alpha} E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j \left( \frac{1+1/\sigma_l}{1-u^n} (u_{t+j} - u^n) + \theta \widetilde{P}_{t+j}(i) \right) - E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_w^j (\iota_w \pi_{t+j-1} + (1-\iota_w) \varepsilon_{t+k}^w - \pi_{t+j}^w) \quad (\text{A1})$$

We want to express the expected stream of relative prices,  $E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j \widetilde{P}_{t+j}(i)$ , as a function of the relative price current value in order to have an expression for  $\widetilde{W}_t^*(i)$  consistent with (14b).

Beginning with  $\widetilde{P}_{t+1}(i)$ , the Calvo aggregation scheme implies

$$E_t^{\xi_w} \widetilde{P}_{t+1}(i) = \xi_p \left( \widetilde{P}_t(i) + \iota_p \pi_t - E_t \pi_{t+1} \right) + (1 - \xi_p) E_t^{\xi_w} \widetilde{P}_{t+1}^*(i), \quad (\text{A2})$$

where the second term is  $E_t^{\xi_w} \widetilde{P}_{t+1}^*(i) = E_t \widetilde{P}_{t+1}^* + \tau_1 \widetilde{W}_t^*(i)$  using (14a) in  $t+1$  conditional on having a labor-clearing wage contract set in  $t$ . Using that information in (A2) yields

$$E_t^{\xi_w} \widetilde{P}_{t+1}(i) = \xi_p \left( \widetilde{P}_t(i) + \iota_p \pi_t - E_t \pi_{t+1} \right) + (1 - \xi_p) \left( E_t \widetilde{P}_{t+1}^* + \tau_1 \widetilde{W}_t^*(i) \right). \quad (\text{A3})$$

The Calvo aggregation scheme implies  $\widetilde{P}_{t+1}^* = \frac{\xi_p}{1-\xi_p} (\pi_{t+1} - \iota_p \pi_t)$ . Taking rational expectations and substituting in (A3), this equation becomes:

$$E_t^{\xi_w} \widetilde{P}_{t+1}(i) = \xi_p \widetilde{P}_t(i) + \tau_1 (1 - \xi_p) \widetilde{W}_t^*(i). \quad (\text{A4})$$

Analogously to (A3),  $E_t^{\xi_w} \widetilde{P}_{t+2}(i)$  is a linear combination of non-adjusted relative prices and optimal relative prices:

$$E_t^{\xi_w} \widetilde{P}_{t+2}(i) = \xi_p \left( E_t^{\xi_w} \widetilde{P}_{t+1}(i) + \iota_p E_t \pi_{t+1} - E_t \pi_{t+2} \right) + (1 - \xi_p) E_t^{\xi_w} \widetilde{P}_{t+2}^*(i),$$

where using (A4) for  $E_t^{\xi_w} \widetilde{P}_{t+1}(i)$  leads to

$$E_t^{\xi_w} \widetilde{P}_{t+2}(i) = \xi_p \left( \xi_p \widetilde{P}_t(i) + \tau_1 (1 - \xi_p) \widetilde{W}_t^*(i) + \iota_p E_t \pi_{t+1} - E_t \pi_{t+2} \right) + (1 - \xi_p) E_t^{\xi_w} \widetilde{P}_{t+2}^*(i). \quad (\text{A5})$$

Recalling (14a) in period  $t+2$  conditional on the lack of wage resetting,  $E_t^{\xi_w} \widetilde{P}_{t+2}^*(i) = E_t \widetilde{P}_{t+2}^* + \tau_1 E_t^{\xi_w} \widetilde{W}_{t+1}^*(i) = E_t \widetilde{P}_{t+2}^* + \tau_1 \left( \widetilde{W}_t^*(i) + \iota_w \pi_t + (1 - \iota_w) E_t \varepsilon_{t+1}^w - E_t \pi_{t+1}^w \right)$ , (A5) becomes

$$E_t^{\xi_w} \widetilde{P}_{t+2}(i) = \xi_p \left( \xi_p \widetilde{P}_t(i) + \tau_1 (1 - \xi_p) \widetilde{W}_t^*(i) + \iota_p E_t \pi_{t+1} - E_t \pi_{t+2} \right) + (1 - \xi_p) \left( E_t \widetilde{P}_{t+2}^* + \tau_1 \left( \widetilde{W}_t^*(i) + \iota_w \pi_t + (1 - \iota_w) E_t \varepsilon_{t+1}^w - E_t \pi_{t+1}^w \right) \right),$$

where using  $\tilde{P}_{t+2}^* = \frac{\xi_p}{1-\xi_p} (\pi_{t+2} - \iota_p \pi_{t+1})$  simplifies to

$$E_t^{\xi_w} \tilde{P}_{t+2}(i) = \xi_p^2 \tilde{P}_t(i) + \tau_1 (1 - \xi_p^2) \tilde{W}_t^*(i) - \tau_1 (1 - \xi_p) (E_t \pi_{t+1}^w - \iota_w \pi_t - (1 - \iota_w) E_t \varepsilon_{t+1}^w). \quad (\text{A6})$$

A generalization of (A4) and (A6) results in the following rule:

$$E_t^{\xi_w} \tilde{P}_{t+j}(i) = \xi_p^j \tilde{P}_t(i) + \tau_1 (1 - \xi_p^j) \tilde{W}_t^*(i) - \tau_1 E_t \sum_{k=1}^{j-1} (1 - \xi_p^{j-k}) (\pi_{t+k}^w - \iota_w \pi_{t+k-1} - (1 - \iota_w) E_t \varepsilon_{t+k}^w),$$

implying the following expected sum of discounted relative prices:

$$\begin{aligned} E_t^{\xi_w} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j \tilde{P}_{t+j}(i) &= \frac{1}{1 - \bar{\beta} \xi_w \xi_p} \tilde{P}_t(i) \\ &+ \tau_1 \left( \frac{\bar{\beta} \xi_w}{1 - \bar{\beta} \xi_w} - \frac{\bar{\beta} \xi_w \xi_p}{1 - \bar{\beta} \xi_w \xi_p} \right) \left( \tilde{W}_t^*(i) - E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_w^j (\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1 - \iota_w) E_t \varepsilon_{t+j}^w) \right). \quad (\text{A7}) \end{aligned}$$

Substituting (A7) in the relative wage equation (A1), we obtain:

$$\begin{aligned} \left( 1 + \frac{\tau_1 \bar{\beta} \xi_w \theta}{\sigma_l^{-1} + \alpha} \left( 1 - \frac{\xi_p (1 - \bar{\beta} \xi_w)}{1 - \bar{\beta} \xi_w \xi_p} \right) \right) \tilde{W}_t^*(i) &= - \frac{\theta (1 - \bar{\beta} \xi_w)}{(\sigma_l^{-1} + \alpha) (1 - \bar{\beta} \xi_w \xi_p)} \tilde{P}_t(i) - \frac{(1 - \bar{\beta} \xi_w) (1 + 1/\sigma_l)}{(\sigma_l^{-1} + \alpha) (1 - u^n)} E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_w^j (u_{t+j} - u^n) \\ &+ \left( 1 + \frac{\tau_1 \bar{\beta} \xi_w \theta}{\sigma_l^{-1} + \alpha} \left( 1 - \frac{\xi_p (1 - \bar{\beta} \xi_w)}{1 - \bar{\beta} \xi_w \xi_p} \right) \right) E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_w^j (\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1 - \iota_w) E_t \varepsilon_{t+j}^w), \end{aligned}$$

that corresponds to equation (15) displayed in the main text.

### 3. Derivation of equation (26) on the dynamics of relative optimal prices.

Equation (25) from section 2 in the main text indicates that the loglinearized relative price depends upon terms on relative wages as follows

$$\tilde{P}_t^*(i) = A(1 - \bar{\beta}\xi_p) E_t^{\xi_p} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j \left( mc_{t+j} + (1 - \alpha) \widetilde{W}_{t+j}(i) + \lambda_{t+j}^p \right) + E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j \left( \pi_{t+j} - \iota_p \pi_{t+j-1}^p \right). \quad (\text{A8})$$

To be consistent with the value of the undetermined coefficient  $\tau_1$  implied by the linear relationship (14a) from section 2, we must relate  $E_t^{\xi_p} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j \widetilde{W}_{t+j}(i)$  to  $\widetilde{W}_{t-1}(i)$ . The Calvo scheme applied for wage setting in period  $t$  results in

$$\widetilde{W}_t(i) = \xi_w \left( \widetilde{W}_{t-1}(i) + \iota_w \pi_{t-1} + (1 - \iota_w) \varepsilon_t^w - \pi_t^w \right) + (1 - \xi_w) \widetilde{W}_t^*(i).$$

Using the proposed conjecture (14b) conditional on optimal pricing in period  $t$  allows us to write  $\widetilde{W}_t^*(i)$  depending upon the aggregate relative value of new labor contracts,  $\widetilde{W}_t^*$ , and also upon the relative optimal price,  $\tilde{P}_t^*(i)$

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \tilde{P}_t^*(i),$$

which can be inserted in the previous expression to reach

$$\widetilde{W}_t(i) = \xi_w \left( \widetilde{W}_{t-1}(i) + \iota_w \pi_{t-1} + (1 - \iota_w) \varepsilon_t^w - \pi_t^w \right) + (1 - \xi_w) \left( \widetilde{W}_t^* - \tau_2 \tilde{P}_t^*(i) \right). \quad (\text{A9})$$

Recalling that  $\widetilde{W}_t^* = \frac{\xi_w}{1 - \xi_w} (\pi_t^w - \iota_w \pi_{t-1} - (1 - \iota_w) \varepsilon_t^w)$  from Calvo-type sticky wages, and cancelling terms in (A9), we obtain

$$\widetilde{W}_t(i) = \xi_w \widetilde{W}_{t-1}(i) - \tau_2 (1 - \xi_w) \tilde{P}_t^*(i). \quad (\text{A10})$$

Repeating the procedure one period ahead for  $E_t^{\xi_p} \widetilde{W}_{t+1}(i)$ , we have

$$E_t^{\xi_p} \widetilde{W}_{t+1}(i) = \xi_w \left( \widetilde{W}_t(i) + \iota_w \pi_t + (1 - \iota_w) E_t \varepsilon_{t+1}^w - E_t \pi_{t+1}^w \right) + (1 - \xi_w) E_t^{\xi_p} \widetilde{W}_{t+1}^*(i). \quad (\text{A11})$$

Using (14b) conditional on no-optimal pricing in  $t + 1$  yields

$$E_t^{\xi_p} \widetilde{W}_{t+1}^*(i) = \widetilde{W}_{t+1}^* - \tau_2 \left( \tilde{P}_t^*(i) + \iota_p \pi_t - E_t \pi_{t+1} \right),$$

which can be inserted in (A11) together with (A10) and also  $\widetilde{W}_{t+1}^* = \frac{\xi_w}{1 - \xi_w} (\pi_{t+1}^w - \iota_w \pi_t - (1 - \iota_w) \varepsilon_{t+1}^w)$  to obtain (after dropping terms that cancel out)

$$E_t^{\xi_p} \widetilde{W}_{t+1}(i) = \xi_w^2 \widetilde{W}_t(i) - \tau_2 (1 - \xi_w^2) \tilde{P}_t^*(i) + \tau_2 (1 - \xi_w) (E_t \pi_{t+1} - \iota_p \pi_t). \quad (\text{A12})$$

A generalization of (A10) and (A12) for a  $t + j$  future period gives the following expression

$$E_t^{\xi_p} \widetilde{W}_{t+j}(i) = \xi_w^{j+1} \widetilde{W}_{t-1}(i) - \tau_2 (1 - \xi_w^{j+1}) \widetilde{P}_t^*(i) + \tau_2 \left(1 - \xi_w^{j-k+1}\right) E_t \sum_{k=1}^j (\pi_{t+k} - \iota_p \pi_{t+k-1}). \quad (\text{A13})$$

Using (A13), the expected sum of the stream of conditional relative wages becomes

$$E_t^{\xi_p} \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j \widetilde{W}_{t+j}(i) = \frac{\xi_w}{1 - \bar{\beta} \xi_w \xi_p} \widetilde{W}_{t-1}(i) - \tau_2 \left( \frac{1}{1 - \bar{\beta} \xi_p} - \frac{\xi_w}{1 - \bar{\beta} \xi_w \xi_p} \right) \widetilde{P}_t^*(i) \\ + \tau_2 \left( \frac{1}{1 - \bar{\beta} \xi_p} - \frac{\xi_w}{1 - \bar{\beta} \xi_w \xi_p} \right) E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j (\pi_{t+j} - \iota_p \pi_{t+j-1}). \quad (\text{A14})$$

Substituting (A14) in (A8) yields

$$\left( 1 + \tau_2 A (1 - \alpha) \left( 1 - \frac{(1 - \bar{\beta} \xi_p) \xi_w}{1 - \bar{\beta} \xi_w \xi_p} \right) \right) \widetilde{P}_t^*(i) = \frac{A(1 - \alpha)(1 - \bar{\beta} \xi_p) \xi_w}{1 - \bar{\beta} \xi_w \xi_p} \widetilde{W}_{t-1}(i) \\ + A (1 - \bar{\beta} \xi_p) E_t \sum_{j=0}^{\infty} \bar{\beta}^j \xi_p^j \left( mc_{t+j} + \lambda_{t+j}^p \right) \\ + \left( 1 + \tau_2 A (1 - \alpha) \left( 1 - \frac{(1 - \bar{\beta} \xi_p) \xi_w}{1 - \bar{\beta} \xi_w \xi_p} \right) \right) E_t \sum_{j=1}^{\infty} \bar{\beta}^j \xi_p^j (\pi_{t+j} - \iota_p \pi_{t+j-1}),$$

which is equation (26) displayed in the main text.

#### 4. New-Keynesian model with unemployment as excess supply of labor and variable capital.

Set of log-linearized dynamic equations:

- Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \quad (\text{A15})$$

where  $c_y = \frac{C}{Y} = 1 - g_y - i_y$ ,  $i_y = \frac{I}{Y} = (\gamma - 1 + \delta) \frac{K}{Y}$ , and  $z_y = r^k \frac{K}{Y}$  are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at  $\delta = 0.025$  and  $g_y = 0.18$ .

- Consumption equation:

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (R_t - E_t \pi_{t+1}) + \varepsilon_t^b, \quad (\text{A16})$$

where  $c_1 = \frac{\lambda/\gamma}{1+\lambda/\gamma}$ ,  $c_2 = \frac{[(\sigma_c-1)wL/C]}{\sigma_c(1+\lambda/\gamma)}$  and  $c_3 = \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}$ .<sup>26</sup>

- Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \quad (\text{A17})$$

where  $i_1 = \frac{1}{1+\bar{\beta}}$ , and  $i_2 = \frac{1}{(1+\bar{\beta})\gamma^2\varphi}$  with  $\bar{\beta} = \beta\gamma^{(1-\sigma_c)}$ .

- Arbitrage condition (value of capital,  $q_t$ ):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \quad (\text{A18})$$

where  $q_1 = \bar{\beta}\gamma^{-1}(1 - \delta) = \frac{(1-\delta)}{(r^k+1-\delta)}$ .

- Log-linearized aggregate production function:

$$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a), \quad (\text{A19})$$

where  $\phi_p = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$  and  $\alpha$  is the capital-share in the production function.<sup>27</sup>

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<sup>26</sup>Notice that we introduce the risk premium shock  $\varepsilon_t^b$  as expansionary, akin to a positive preference shock, whereas Smets and Wouters (2007) introduce this same shock with the opposite sign, thus contractionary.

<sup>27</sup>From the zero profit condition in steady-state, it should be noticed that  $\phi_p$  also represents the value of the steady-state price mark-up.

- Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \quad (\text{A20})$$

- Capital utilization:

$$z_t = z_1 \log r_t^k, \quad (\text{A21})$$

where  $z_1 = \frac{1-\psi}{\psi}$ .

- Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i, \quad (\text{A22})$$

where  $k_1 = \frac{1-\delta}{\gamma}$  and  $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right) (1 + \bar{\beta}) \gamma^2 \varphi$ .

- Price mark-up (negative of the log of the real marginal cost):

$$\mu_t^p = mpl_t - w_t = \alpha (k_t^s - l_t) + \varepsilon_t^a - w_t. \quad (\text{A23})$$

- New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p, \quad (\text{A24})$$

where  $\pi_1 = \frac{\iota_p}{1+\bar{\beta}\iota_p}$ ,  $\pi_2 = \frac{\bar{\beta}}{1+\bar{\beta}\iota_p}$ , and  $\pi_3 = \frac{1}{1+\bar{\beta}\iota_p} \left[ \frac{(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)(1+\Theta)} \right]$  with  $\Theta = \tau_2 \left( 1 - \frac{(1-\bar{\beta}\xi_p)\xi_w}{1-\bar{\beta}\xi_p\xi_w} \right)$ .

The coefficient of the curvature of the Kimball goods market aggregator is fixed in the estimation procedure at  $\varepsilon_p = 10$  as in Smets and Wouters (2007).<sup>28</sup>

- Optimal demand for capital by firms

$$-(k_t^s - l_t) + w_t = \log r_t^k. \quad (\text{A25})$$

- Unemployment rate equation, using  $u_t = u^n + (1 - u^n)(l_t^s - l_t)$  and the aggregation of log supply of labor at firm level,  $L_t^s(i) = \left( \frac{(1-u_t(i)) \frac{W_t(i)}{P_t}}{C_t - \lambda C_{t-1}^A} \right)^{1/\sigma_l}$ ,

$$\begin{aligned} u_t - u^n &= (1 - u^n)(l_t^s - l_t) = \frac{(1-u^n)}{\sigma_l} \left( w_t - \frac{1}{1-u^n} (u_t - u^n) - \frac{1}{(1-\lambda/\gamma)} (c_t - (\lambda/\gamma) c_{t-1}) \right) - (1 \\ \left(1 + \frac{1}{\sigma_l}\right) (u_t - u^n) &= \frac{(1-u^n)}{\sigma_l} w_t - \frac{(1-u^n)}{\sigma_l(1-\lambda/\gamma)} (c_t - (\lambda/\gamma) c_{t-1}) - (1 - u^n) l_t, \end{aligned}$$

where  $l_t^s$  denotes log-fluctuations of the aggregate labor supply and  $l_t$  denotes log-fluctuations of demand-determined aggregate labor.

<sup>28</sup>Using Dixit-Stiglitz output aggregators as in Smets and Wouters (2003) or Christiano *et al.* (2005), the slope coefficient on the prime mark-up changes to  $\pi_3 = \frac{1}{1+\bar{\beta}\gamma(1-\sigma_c)\iota_p} \left[ \frac{(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p(1+\Theta)} \right]$ .

- Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 (u_t - u^n) + w_5 (\varepsilon_t^w - \bar{\beta} E_t \varepsilon_{t+1}^w), \quad (\text{A27})$$

where  $w_1 = \frac{1}{1+\bar{\beta}}$ ,  $w_2 = \frac{1+\bar{\beta}\iota_w}{1+\bar{\beta}}$ ,  $w_3 = \frac{\iota_w}{1+\bar{\beta}}$ ,  $w_4 = \frac{1}{1+\bar{\beta}} \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)(1+\sigma_l^{-1})}{\xi_w(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}$  with  $\Lambda = \frac{\tau_1 \bar{\beta} \xi_w \theta}{\sigma_l^{-1} + \alpha} \left( 1 - \frac{\xi_p (1 - \bar{\beta} \xi_w)}{1 - \bar{\beta} \xi_w \xi_p} \right)$ , and  $w_5 = \frac{1 - \iota_w}{1 + \bar{\beta}}$ .

- Monetary policy rule, a Taylor-type rule for nominal interest rate management:

$$R_t = \rho R_{t-1} + (1 - \rho) [r_\pi \pi_t + r_Y (y_t - y_t^p)] + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^R. \quad (\text{A28})$$

**Potential (natural-rate) variables** are obtained assuming flexible prices, flexible wages and shutting down price mark-up and wage indexation shocks. They are denoted by adding a superscript “p”.

- Flexible-price condition (no price mark-up fluctuations,  $\mu_t^p = mpl_t - w_t = 0$ ):

$$\alpha (k_t^{s,p} - l_t^p) + \varepsilon_t^a = w_t^p. \quad (\text{A29})$$

- Flexible-wage condition (no wage cost-push shock fluctuations,  $(1 - u^n) \frac{W_t}{P_t} = mrs_t$  and  $l_t^{s,p} = l_t^p$ ):

$$w_t^p = \sigma_l l_t^p + \frac{1}{1-\lambda/\gamma} (c_t^p - \lambda/\gamma c_{t-1}^p). \quad (\text{A30})$$

- Potential aggregate resources constraint:

$$y_t^p = c_y c_t^p + i_y i_t^p + z_y z_t^p + \varepsilon_t^g. \quad (\text{A31})$$

- Potential consumption equation:

$$c_t^p = c_1 c_{t-1}^p + (1 - c_1) E_t c_{t+1}^p + c_2 (l_t^p - E_t l_{t+1}^p) - c_3 (R_t^p - E_t \pi_{t+1}^p) + \varepsilon_t^b. \quad (\text{A32})$$

- Potential investment equation:

$$i_t^p = i_1 i_{t-1}^p + (1 - i_1) E_t i_{t+1}^p + i_2 q_t^p + \varepsilon_t^i. \quad (\text{A33})$$

- Arbitrage condition (value of potential capital,  $q_t^p$ ):

$$q_t^p = q_1 E_t q_{t+1}^p + (1 - q_1) E_t r_{t+1}^{k,p} - (R_t^p - E_t \pi_{t+1}^p) + c_3^{-1} \varepsilon_t^b. \quad (\text{A34})$$

- Log-linearized potential aggregate production function:

$$y_t^p = \phi_p (\alpha k_t^{s,p} + (1 - \alpha) l_t^p + \varepsilon_t^a). \quad (\text{A35})$$

- Potential capital (with one period time-to-build):

$$k_t^{s,p} = k_{t-1}^p + z_t^p. \quad (\text{A36})$$

- Potential capital utilization:

$$z_t^p = z_1 \log r_t^{k,p}. \quad (\text{A37})$$

- Potential capital accumulation equation:

$$k_t^p = k_1 k_{t-1}^p + (1 - k_1) i_t^p + k_2 \varepsilon_t^i. \quad (\text{A38})$$

- Potential demand for capital by firms ( $\log r_t^{k,p}$  is the log of the potential rental rate of capital):

$$-(k_t^{s,p} - l_t^p) + w_t^p = \log r_t^{k,p}. \quad (\text{A39})$$

- Monetary policy rule (under flexible prices and flexible wages):

$$R_t^p = \rho R_{t-1}^p + (1 - \rho) [r_\pi \pi_t^p] + \varepsilon_t^R. \quad (\text{A40})$$

### Equations-and-variables summary

- Set of equations:

Equations (A15)-(A40) determine solution paths for 26 endogenous variables. The subset (A29)-(A40) is introduced to solve the potential (natural-rate) block of the model.

- Set of variables:

Endogenous variables (26):  $y_t, c_t, i_t, z_t, l_t, R_t, \pi_t, q_t, \log r_t^k, k_t^s, k_t, \mu_t^w, \mu_t^p, w_t, y_t^p, c_t^p, i_t^p, z_t^p, l_t^p, R_t^p, \pi_t^p, q_t^p, \log r_t^{k,p}, k_t^{s,p}, k_t^p,$  and  $w_t^p$ .

Predetermined variables (12):  $c_{t-1}, i_{t-1}, k_{t-1}, \pi_{t-1}, w_{t-1}, R_{t-1}, y_{t-1}, y_{t-1}^p, c_{t-1}^p, i_{t-1}^p, k_{t-1}^p,$  and  $R_{t-1}^p$ .

Exogenous variables (8): AR(1) technology shock  $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ , AR(1) risk premium shock  $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ , AR(1) exogenous spending shock cross-correlated to technology innovations  $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$ , AR(1) investment shock  $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ , AR(1) monetary policy shock  $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$ , ARMA(1,1) price mark-up shock  $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$ , and ARMA(1,1) wage mark-up shock  $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$ .

## 5. Model comparison.

In order to recover the Smets and Wouters (2007) model, we must introduce the following modifications in equations (A24), (A26) and (A27):

i) New-Keynesian Phillips Curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3^{SW} \mu_t^p + \varepsilon_t^p, \quad (\text{A24}')$$

where  $\pi_1 = \frac{\iota_p}{1+\beta\iota_p}$ ,  $\pi_2 = \frac{\bar{\beta}}{1+\beta\iota_p}$ , and  $\pi_3 = \frac{1}{1+\beta\iota_p} \left[ \frac{(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)} \right]$ . Notice the changes in the slope coefficient  $\pi_3^{SW} > \pi_3$ .

ii) Wage mark-up (log difference between the marginal rate of substitution between working and consuming and the real wage) which replaces the rate of unemployment present in the CMV model:

$$\mu_t^w = w_t - mrs_t = w_t - \left( \sigma_l l_t + \frac{1}{1-\lambda/\gamma} (c_t - \lambda/\gamma c_{t-1}) \right) \quad (\text{A26}')$$

iii) Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4^{SW} \mu_t^w + \varepsilon_t^w, \quad (\text{A27}')$$

where  $w_1 = \frac{1}{1+\beta}$ ,  $w_2 = \frac{1+\bar{\beta}\iota_w}{1+\beta}$ ,  $w_3 = \frac{\iota_w}{1+\beta}$ , and  $w_4^{SW} = \frac{1}{1+\beta} \left[ \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)} \right]$  with the curvature of the Kimball labor aggregator fixed at  $\varepsilon_w = 10.0$  and a steady-state wage mark-up fixed at  $\phi_w = 1.5$  as in Smets and Wouters (2007). Notice the new slope coefficient  $w_4^{SW}$ .

In addition, the coefficient  $c_2$  from equations (A16) and (A32) suffers a slight change to accommodate the fact that there is a wage mark-up in the SW model. In turn, the coefficient to be used in the SW model is  $c_2 = \frac{[(\sigma_c-1)wL/(\phi_w C)]}{\sigma_c(1+\lambda/\gamma)}$ .

The set of variables is identical in the two models except for the replacement of the rate of unemployment,  $u_t$ , instead of the wage mark-up  $\mu_t^w$ . In turn there are 26 equations and 26 endogenous variables in both models.