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This is a weighty and daring book. It proposes a new philosophy of mathematics, based on a detailed knowledge of the most recent work in advanced mathematics, and constructed in explicit contrast with the traditional analytical approach. A brief yet illuminating introduction summarizes the spirit of the work in four theses: a) contemporary mathematics must not be reduced to Set Theory, and neither must it to elementary mathematics; b) real contemporary work in mathematics is the source of new, as yet unattended, philosophical problems; c) philosophy of mathematics should be synthetic rather than analytic, in order to capture the dialectical tensions that are present in actual mathematical activity; d) there is in fact a fruitful exchange between philosophy and mathematics that must be fostered in order to promote both mathematical creativity and philosophical reflection. Going beyond the dualistic schemas that have traditionally oppressed the philosophy of mathematics, such as the dichotomies realism/idealism, necessity/contingency, unity/plurality, among others, Zalamea undertakes a study of the living and evolving mathematics from an unbiased philosophical perspective.

The eleven chapters are distributed in three parts: I. The General Framework of Contemporary Mathematics. II. Case studies. III. Outlines of a synthesis.

The first part contains three chapters. In chapter one "The Specific Characteristics of Modern and Contemporary Mathematics" the author follows Albert Lautman in identifying five features of advanced mathematics, as opposed to elementary mathematics: complex hierarchy of theories, richness of models, methodological and conceptual unity, dynamic stile of work, and theoretical linkage between levels. The author shows the consequences and philosophical potential of these features, which are common to modern and contemporary mathematics, but also lists another five features which are specific to the mathematical work developed along the last fifty years: structural impurity of arithmetic, geometrization, schematization, structural fluxion, and reflexivity. These specific features imply important "inversions" in the ways of doing mathematics, and they strongly distinguish contemporary mathematics from the elementary subset that has been the focus of the traditional philosophy of mathematics. In sum, the author tries to make evident the inadequacy of any approach that intends to construct a serious philosophy of mathematics without paying attention to the rich developments of real mathematics in the last fifty years. The second chapter "Advanced Mathematics in the Treatises of Mathematical Philosophy" goes through the few philosophical works that have taken into account the "wild core" of advanced mathematics: Albert Lautman occupies a prominent place as the only philosopher who has achieved a both fine and wide understanding of real mathematics, but also the insights of a number of authors who have come close to it are studied: George Pólya, Imre Lakatos, Javier de Lorenzo, Raymond L. Wilder, Morris Kline, Philip Kitcher, Thomas Tymoczko, Saunders MacLane, Gian-Carlo Rota, Alain Badiou, Penelope Maddy, Gilles Châtelet, Frédéric Patras, and David Corfield. The chapter ends with a contrast between these new trends and "traditional" philosophy of mathematics. The latter is seen as a reductive approach ("analytical, logical and Anglo-Saxon"), and it is summarized in the motto "more philosophy, less mathematics". This does not mean that the author fails to recognize the contribution of analytic philosophy of mathematics, he is simply denouncing an impoverishing unwillingness to see the wealth of philosophical problems that can be discovered in contemporary mathematics, problems that go beyond logical discussions on the foundations of elementary mathematics. The third chapter "Towards a Synthetic Philosophy of Contemporary Mathematics" outlines the epistemological "minimal" framework of Zalamea's own proposal, which is a combination of C. S. Peirce's pragmaticism with the mathematical Category Theory. Peirce's "pragmatic maxim" is presented as the rule that will teach the philosopher to integrate the multiplicity of contemporary mathematics into a unity which does not dissolve the differences. The awareness of alternative points of view and the possibility of comparison between them are the keys of an anti-foundationalist yet anti-relativist programme. Accordingly, Category Theory is presented as the technical refinement of Peirce's methodology, as a tool for a synthetic and contextual study of mathematical objects. This will allow a "fuller and more faithful" view of real mathematical activity.

The second part covers chapters 4 to 7, where a number of case studies in contemporary mathematics are intended to provide some actual material to which the philosophical reflection will be applied in the third part (according to the motto "as much mathematics as philosophy"). The whole chapter 4 is devoted to Alexander Grothendieck, pioneer of contemporary mathematics and whose work, of a "high mathematical creativity", develops a "relativistic" mathematics where the objects are no more fixed but evolve along time; chapter 5 deals with four mathematicians that the author considers to be revolutionaries in the sphere of abstract ideas (Serre, Langlands, Lawvere, Shelah); chapter 6 is about another four whose contribution belongs mainly to the application of abstract mathematics to the physical world (Atiyah, Lax, Connes, Kontsevich); finally, chapter 7 is devoted to four authors who stand out by their effort to find some points of stability inside the relativity and flux which are characteristic of contemporary mathematics (Freyd, Simpson, Zilber, Gromov).

The third part includes chapters 8 to 11. After the general framework (pages 21 to 73) and the detailed case-studies (pages 77-150) offered in the first two parts, only 54 pages contain Zalamea's philosophy of mathematics, in the third part (pages 153-207). As explicitly stated in the title of this final and nuclear part, he can just offer some "outlines", but he manages to present a very complete and serious view. He covers the ontology, epistemology and phenomenology of contemporary mathematics, plus some concluding remarks of a wider scope, using the case-studies of the previous section to rigorously support his thesis.

Chapter 8 "Fragments of a Transitional Ontology" proposes an "Einsteinian turn" for the philosophy of mathematics: mathematical objects are not fixed, but they are rather nets and processes which evolve in different contexts; their place is not an abstract logical space, but a structure of proto-geometries which are prior to logic. Zalamea wants to overcome a number of classic twofold (exclusive) divisions, and he discovers three great tendencies in contemporary mathematics which are already taking this line: a) the general tendency to understand the "positive" as a mere limit of the "negative" (for example, the commutative as a limit of the non-commutative); b) Sheaf Theory, which allows for a dialectical movement between the local and the global; c) Category Theory, which helps to find connections between the particular and the universal. The philosophical morals that Zalamea discovers behind these recent trends are: first, classical approaches (which are seen as merely ideal ontologies of fixed, discrete objects) are just the limit of non-classical ones (which capture the real mathematical world of changing continuous objects); second, a logic of "vicinities" instead of a logic of points allows for the inclusion of opposites that will overcome partial accounts; third, the very idea of logic has to be broadened, towards a (Peircean) lattice of partial "flows of truth" against the background of a continuous nature.

In accordance with this dynamic ontology, chapter 9 "Comparative Epistemology and Sheafification" proposes a "fluctuating" epistemology where a multiplicity of perspectives can be integrated in a kind of epistemological sheaf. The result will be a tempered relativism with no trace of incommensurability between the different partial epistemologies. Under the hypothesis of a continuity between the ontological and the epistemological level, the different epistemological perspectives are deemed to be just different breaks in the continuum, which can be easily reintegrated. In contrast with the usual logification of epistemology, Zalamea proposes a geometrization of epistemology, in order to better capture the transformability of mathematical objects. The chapter ends with a substantiation, again with plenty of examples taken from real mathematics, of Zalamea's conviction about a strong advantage of a synthetic epistemology over the analytical ones: only a synthetic approach will allow to balance and overcome three polarities that seem irreconcilable from the analytical perspective: analysis / synthesis, idealism / realism, intension / extension.

Chapter 10 "Phenomenology of Mathematical Creativity" makes a significant contribution to the philosophy of mathematics, as it goes beyond the logical aspects of mathematical activity, in an effort to understand both discovery and invention. Mathematical creativity is seen as a spiral movement along a multiplicity of evolutionary layers of knowledge. As philosophical tools for understanding these kind of processes, Zalamea proposes the Platonic dynamic theory of the mobility of concepts and Merleau-Ponty's notion of sedimentation. The style of mathematical creators is also studied, and it is described as a "back-and-forth" between different polarities (analysis-synthesis, differentiation-integration, etc.) along with the corresponding kinds of "mediations", which have been neglected by analytic philosophy of mathematics. In sum, the evolution of mathematical thinking is seen as a (Peircean) "continual increase of the embodiment of the idea-potentiality".

A concluding chapter "Mathematics and Cultural Circulation" explores both the internal evolution of mathematical thinking and its relations with other forms of cultural expression. In conclusion, seeing contemporary mathematics as the result of a liberation process, which has expanded human capacities (technical, imaginative, rational), the author's hope is that a good (synthetic) understanding of the new mathematical developments will help to transform philosophy itself.

The book is not easy to read (even to those who understand Spanish). This is partly due to the pieces of advanced mathematics that it contains (which can be hard for the outsider), partly due to the use of Peircean categories and terminology (which could sound bizarre if one is not used to them), and partly due to the author's writing style (whose richness often forces one to read twice in order to capture all the contents and nuances). But this new synthetic and open-minded approach is no doubt worthy of attention, and philosophers who dare to make an effort will surely reap the reward.

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