
ENTROPY, QUANTUM MECHANICS, AND INFORMATION IN COMPLEX SYSTEMS A PLEA FOR ONTOLOGICAL PLURALISM

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Abstract

Complex dynamical systems (CDS) are providing a new way of understanding nature. Complexity needs the interplay of different levels in reality and a redefinition of the epistemic approaches in the transition from lower to upper levels. Traditionally, the concept of emergence has tried to bring together the philosophical explanations for the differentiation and crossover between levels. However, it is highly controversial whether emergence is merely epistemic or truly ontic. In this article, I present the philosophical challenges posed by the emergence of CDS regarding conventional epistemic reductionism. I make a strong case for ontic emergence and, therefore, an ontologically plural universe focusing on three conundrums of current Science: the Second Law of Thermodynamics, the measurement problem in Quantum Mechanics, and the symbol grounding problem. I conclude that all of them are interwoven, point towards rejection of naturalistic monism, and suggest the presence of a transcendent *logos* in nature.

Keywords: entropy, Quantum Mechanics, symbol grounding problem, CDS, ontological pluralism

1. Introduction

The literature on complex dynamical systems (CDS) is nowadays providing a new paradigm for understanding nature, as human beings perceive it. Even if a straightforward definition of CDS is hard to find, I will tentatively characterize them as open systems: (1) consisting of at least two distinct levels of description — that I label micro and macro — which remain highly correlated at the same time; (2) whose available phase-space at the micro level is restricted to a tiny specific region — thus being far from thermal equilibrium; and (3) possessing some emergent properties that cannot be predicated upon any of their subsystems. In the essence of this novel conceptualization of entities stands the interplay between different levels of reality defining the way in which specific

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systems are being looked at. In some sense this is not surprising. In our everyday experience, we humans deal with different protocols in order to access reality.

We behave differently when dealing with stones, plants, animals, single human beings or societies. The world we inhabit seems to be composed of different interrelated levels of organization. But, from a realist point of view, are those levels ontologically distinct (theory-independent) or are they related to our species-dependent way of knowing?

For CDS enthusiasts, Biology could involve many critical networks, nestled together in hierarchies that generate ever more complex phenomena. But is this emergence of complexity truly real? Supporters of ontological reductionism claim that it is not — complexity being either an illusion or epiphenomenal, i.e. reducible to more basic constituents of nature. Whereas this view is compatible with the aforementioned multilevel experience, it assumes a perfect many-to-one correspondence between a specific level and its immediate upper level. The relationship between the former and the latter is statistics. Ontological reductionism thus allows for a compatibilist view of epistemically different levels of knowledge. For instance, there is no contradiction between the use of the concept of free will and the fact that our behaviour arises from the movement of the molecules of our body, which could be, at the molecular level, perfectly deterministic [C. Rovelli, *Free Will, Determinism, Quantum Theory and Statistical Fluctuations: A Physicists' Take*, in Edge, 2013, <http://edge.org/conversation/free-will-determinism-quantum-theory-and-statistical-fluctuations-a-physicists-take>]. A sort of coarse-graining procedure in the phase-space of finest levels' available states should allow the univocal epistemic building up of upper levels' states. Since ontological reductionism tends to consider the basic building blocks of nature as fundamental, the many-to-one correspondence means that fine-grained levels are necessary and sufficient conditions for the emergence of coarse-grained levels. It is a layered view of nature. "The world is divided into discrete strata, with fundamental Physics as the base level, followed by Chemistry, Biology, and Psychology (and possibly Sociology)." [1]

While the literature on the emergence of complexity has extensively debated this issue, it has rarely focused on specific gaps which, at our present scientific knowledge, do not allow the applicability of ontological reductionism. The main goal of this paper is to present three of these gaps that, in my opinion, should be explained away by the reductionist party lest its claim become flawed. The paper is organized as follows: first, I shall briefly introduce the main philosophical views regarding the emergence phenomenon; I shall discuss later the epistemic and ontic relevance of the three gaps precluding ontological reductionism; finally, I present some concluding remarks that explain why it is highly unlikely that, even though the three actual gaps are explained, ontological reductionism might get rid of future gaps and become a plausible view of reality. All this suggests that human access to reality has some intrinsic, ontologically grounded limits that, notwithstanding, permit real knowledge and, even more importantly, that nature is laden with both an inner and transcendent logos.

2. Kinds of emergence

When one speaks of complexity of a particular natural system, one refers to a system being studied under at least two different viewpoints, namely, one micro and one macro-level perspective. These two perspectives are connected in the sense of being correlated at the same time. The integrity and identity of a complex system are therefore fundamentally related to dynamical connectivity among its parts [2]. For example, when a living organism is considered to be complex, we assume that at its macroscopic level it consists of a very specific set of microstates — usually an attractor in phase-space — among the pool of microstates that would in principle be accessible were it in thermal equilibrium. A defining characteristic of CDS is therefore being far from thermodynamic equilibrium.

Roughly speaking, we can distinguish two broad understandings of emergent entities (properties or substances) arising out of more fundamental entities and yet novel or irreducible with respect to them: (a) epistemological emergence, which reflects the unpredictable macroscopic outcomes of the world's dynamics and focus on diachronic relationships between matter in pre- and post-complexity stages, regardless the metaphysical origin of macroscopic unpredictability; and (b) ontological emergence, which involves the appearance of primitive high-level causal interactions that are additional to those of the more fundamental levels. Each new layer is a consequence of the appearance of an interacting range of novel qualities [1].

Whereas epistemological emergence judges emergent properties as explanatory relevant but epiphenomenal, ontological emergence implies downward causation from upper levels to lower ones. Some epistemological emergentists deem it possible to represent novel macroscopic phenomena within a more comprehensive physical theory. Then, we need only augment the theory to include variables for the precise structural conditions in which the novel phenomena occur. However, the instrumentalist flavour of such prescription is evident since it says nothing regarding the fundamentality of the levels of description. On the other hand, it is unclear whether all emergent phenomena can be phrased in terms of degrees of freedom of the same pre-defined basic level.

Whether there are any instances of ontological emergence and the way in which emergent properties might consistently covariate with fundamental properties at the basal level is highly controversial at present. However, well-known scientists have pointed out the existence of 'protected' properties, e.g. the Quantum Hall and the Josephson Effect that are well understood in terms of high level principles and cannot be deduced from microscopic laws. They are true and lead to exact results even irrespective of the eventual Theory of Everything (TOE) at play [1]. "Emergent universality causes sample variations and imperfections to heal away as the size grows." [3] In this article, I strongly make the case for ontological emergence and downward causation in nature. If that is the case, ontological emergence and downward causation must be taken into

account in order to improve our scientific knowledge of the (allegedly) most fundamental levels of reality.

3. Top-down causality in complex dynamical systems

Can we identify CDS in light of their evolutionary capacity? Since CDS's identity must include both their dynamic etiology and their potential, how are we to address the problem of identity as permanence with respect to them? A new way of conceptualizing the emergence of complexity seems unavoidable, according to which boundaries of dynamical systems are best conceptualized as sites of phase changes where a different phase portrait can suddenly appear. CDS are the (not spatial but dynamical) locus of emergent properties [2], in which the novelty defines the system and its degrees of freedom. The mechanical paradigm of defining the phase space for the whole problem *a priori* is no longer valid [4].

On the other hand, there seems to be a true downward causation of top-level constraints on the evolution of some CDS. Somehow, these constraints act as a self-tuning tool, functionally contextualizing the system's dynamics [5]. The emergent higher level phenomena exert top-down influence on the very components that make them up [6-8]. But is it truly ontological causation? Juarrero explains how context-sensitive constraints embody changes in probability distributions of the underlying microstates and, somehow, create information. In the system, the set of available microstates is narrowed down at the lower level while a new set of available macrostates appears at an upper level. Complexification creates heretofore nonexistent possibilities and new directions for the system. Top-down causality is thus "a process of selection, a kind of semiosis that interprets — from the point of view and for the benefit of the higher level whole — which among all the possible component addition, deletion, or replacement alternatives best satisfies the requirements of the higher level" [7]. Does it mean an alteration of the underlying physical process? It does in the sense of modifying the accessible phase space, creating attractors and making overall non-ergodic the initially non-constrained dynamics.

Nevertheless, there still seems to be some problems regarding the definition of constraints and top-down causality. Juarrero deems constraints to be "relational properties that parts acquire in virtue of being unified—not just aggregated — into a systematic whole" [6]. But who or what decides on the actual presence of a systematic whole? Conditions for the emergence of CDS are equally ambiguous, "a complex dynamical system emerges when the behavior of each molecule suddenly depends both on what the neighboring molecules are doing and on what went before" [6]. But, in general, that occurs in many-body systems that need not be complex like, e.g. gases in a volume.

Obviously, the interplay between hierarchical levels is essential in order to understand CDS. The evolutionary advantage of such systematic hierarchical differentiation is that the whole can access states that the independent parts cannot, introducing true novelty in nature. These phase changes are in principle

unpredictable; the only way to explain them is with a retrospective narrative that retraces the actual leap [6]. But this also means that we should rethink our understanding of scientific knowledge, since it presents intrinsic limitations. A new philosophy of science must confront the investigation of the role played by constraints, perturbations and symmetry breaking processes in nature. For the time being, CDS point towards a shift to narrative hermeneutics as scientific explanation. Narrative interpretation is the best available explanation, not deduction, because of the historicity of the interpreter and of the facts. Phase changes embody essentially incompressible information, i.e. there exists no law or algorithm more concise than the process itself that can capture and describe what happened [6]. As a matter of fact, there are some basic laws of the universe that only are unraveled to us via history.

4. Can physics-laden evolution be a complete theory?

Since we are mainly interested in the emergence of complexity, one possible question is whether this phenomenon might be purely explained in terms of evolution. The evolutionary framework should eventually explain all the bottom-up processes responsible of the emerging complexity. Nevertheless, in real life, the reference of any scientific theory to epistemic levels whose interconnection is far from trivial makes this route extremely impractical. For instance, all predictive models that we know deal with a ‘physics in a box’ situation [9], where the world is split into a dynamical system and a static environment which serves as a generalized boundary condition for tackling the problem. Would we have obtained similar results had the partition been made differently? It is highly unclear whether our actual knowledge is able to account for the identity of complex adaptive processes. More work is needed in fundamental Physics and a TOE before claiming that evolution theory can explain nature from scratch.

Scientific study of CDS poses serious challenges to the mainstream view of evolution theory. Since we cannot predict necessary and sufficient causes that achieve an adaptation, we cannot have an efficient cause law for how the adaptation will eventually be achieved. Biological adaptations suggest a process that is “both partially beyond sufficient efficient cause natural law, yet, importantly, very much context dependent and non-random” [S.A. Kauffman, *Five Problems in the Philosophy of Mind*, Edge, 2009, <https://edge.org/conversation/five-problems-in-the-philosophy-of-mind>]. The biosphere is continually searching for new structures of increasing complexity depending on its actual state and its adjacent possibilities of actualization in new upper levels.

The history of the cosmos thus appears to be interspersed with the onset of specific context constraints, enabling the emergence of complex structures as stars, living and intelligent beings. *Constraints suppress part of the underlying phase-space, break its symmetry, and organize it differently.* The lesson to be learned is that every time a new complex structure emerges, a sort of phase-transition is happening anew, which cannot be described with the degrees of

freedom corresponding to the former situation. A new conceptualization is needed [10]. A new set of degrees of freedom comes to the fore that turns out to be as fundamental as those of the lower levels.

In words of Hermann Weyl, “the truth as we see it today is this: The laws of nature do not determine uniquely the one world that actually exists” [11]. Conditions at the lowest levels are not necessary, because one could eventually reproduce the upper levels with different ingredients (just think about a computer working with water instead of electricity). We can have the same software with different hardware. But more importantly, conditions at the lowest level are not sufficient either. As far as I know, we need at least to invoke three irreducible procedures to retrieve the classical world from the microscopic physical realm: the Second Law of Thermodynamics (linked to the arrow of time); the quantum wave-function reduction (irrespective of whether using decoherence or consciousness direct causation) and the emergence of information. I shall study all of them and their interconnectedness more in detail in the following paragraphs.

5. The difficulties with entropy

The Second Law of Thermodynamics states that the entropy of a closed system is a non-decreasing function of (increasing) time. Since time and entropy increase in parallel, it is commonly accepted that entropy signals the arrow of time in the Universe. Quite paradoxically, the Second Law cannot be deduced from the fundamental laws of Physics because of the latter’s symmetry in time. We do not have yet a commonly-accepted fundamental theory explaining the origin of time-reversal symmetry breaking.

There is a certain amount of subjectivity in the definition of entropy, which involves a fuzzy distinction between micro and macrostates. However, this physical concept is extraordinarily robust for calculations when the Boltzmann’s definition is used, due to the overall huge phase-space volumes involved in defining macrostates and especially the thermal state. “These enormous factors reveal a remarkable fact about our universe that *does* seem to be clearly objective and ‘out there’ (...) despite the admittedly confusing issues of subjectivity that are involved in our concept of ‘entropy’, these serving merely to cloud the central mystery that underlines the profound usefulness of this remarkable physical notion.” [12]

Admittedly, the recourse to entropy is not always necessary to solve a physical problem. But it is when non-equilibrium Thermodynamics is at play — as the case turns out to be with CDS. Understanding Thermodynamics just from a statistical point of view misses the critical point signaled by ‘non-equilibrium’ Physics. Since fundamental microscopic laws refer to microstates, they do not favor any particular macrostate. They are blind regarding the latter. But they are also blind regarding the arrow of time, which seems to have different status from the rest of spatial dimensions featuring the Universe. Actually, the Second Law is a consequence of an initial state of the universe having an extremely low

entropy [13, 14], independently of any psychological argument. The very fact that entropy is increasing depends upon the actuality of one end “of the evolution curve in phase space being constrained to a very tiny coarse-graining region, and this is the case for only a very minute fraction of possible universe histories. It is the very tininess of the coarse-graining region (...) that our evolution curve appears to have encountered that needs explaining.” [12, p. 53]

Smolin presses the argument up to the point of making time the central notion to understand the behavior of the Universe. Even the laws of nature should evolve over time. If everything can be connected to everything else, there must be a global time. Space itself could be epiphenomenal, but time has to be real. The arrow of time is marked by this long-lasting non-equilibrium universe whose mathematical description is time-asymmetric. “These properties are extraordinarily unlikely, were the solution that describes our universe to be picked randomly.” [9, p. 203] Statistics cannot be the explanation because of the well-known Boltzmann brain paradox, i.e. evolution is statistically less likely than the random emergence of brains were the former a sheer fluctuation of a universe overall in equilibrium.

One possible line of research is the anti-thermodynamic nature of gravity, which could be related to the principle of driven self-organization invoked by Smolin and Kauffman when dealing with CDS. Contrary to what happens with other fundamental interactions, the coalescence of massive objects because of gravity increases their entropy; black holes being the most entropic objects in the Universe [12, p. 75; 14, p. 715]. On the other hand, phenomena of life and consciousness seem to be related to high-dimensional patterns and attractors in phase-space that lower local entropy [7]. Whatever the case, on every scale and at every level of complexity, time is fundamental and the future is open. The Second Law of Thermodynamics implies a ‘subjective’ distinction of micro and macrostates in the system and a selection of perspective according to which the relevant magnitudes for the problem are defined. Until today, this law is fundamental and purports the inescapable conclusion — even for multiverse approaches — that “the initial conditions of our Universe appear to have been finely tuned to produce a Universe that is asymmetric in time” [9, p. 206]. Of course, a multiverse picture with an anthropic principle in order to choose the right initial conditions may be invoked, but such argument risks circularity. In Tegmark’s words, after trying to explain the quantum factorization and the consciousness problems ‘from scratch’, either the initial conditions (described by the initial density matrix of the Universe) or the dynamics (described by the Hamiltonian) are ‘special’ [M. Tegmark, *Consciousness as a State of Matter*, arXiv:1401.1219 [quant-ph], 2014, <http://arxiv.org/abs/1401.1219>].

6. Quantum physics and the emergence of the ‘classical world’

The characteristic interplay between micro and macrostates for CDS turns out to be characteristic for the entropy problem and also for the emergence of the ‘classical’ world in quantum mechanics (QM). As it is well-known, standard

interpretation of QM basically considers two different processes: the deterministic, predictable and computable evolution of the wave-function according to the Schrödinger equation, once the initial conditions have been settled; and the indeterministic, unpredictable and non-computable wave-function collapse after a measurement into one of the possible outcomes regarding that specific measurement — then becoming an actual event in the ‘emerging classic world’. How can the discontinuous and probabilistic wave-function collapse come about as a result of the interaction (measurement) between two parts of the physical reality? This is the QM measurement paradox. Quantum entanglement among different classical possibilities — which can no longer be separated — is responsible for exotic interference patterns, wave-like behaviours of particles in our classical world. There are top-down influences of the globally entangled wave-function onto its local parts — which do not assume an individuation or localization independent of the whole — determining both the possible results of specific measurements and the emergence of a well-defined physical reality (having classical physical properties). The process of decoherence is currently the favourite physical model to explain the transition from the quantum world of possibilities to the classical world of actual events [15]. Interaction of the quantum system with its environment somehow acts like a classical measuring device: the system is ‘partially measured’ by its environment hence the gradual onset of decoherence leading the system into a classical mixed state of classical probabilities, not quantum possibilities that superimpose [<https://edge.org/conversation/five-problems-in-the-philosophy-of-mind>]. This is sufficient to obtain classical statistical predictions that can be successfully tested, but does not obviously solve the fundamental problem of why dealing differently with the system and the environment [14, p. 803; 16], nor the problem of ontological indeterminism in nature’s fundamental levels.

6.1. The problem of decoherence

Whereas recourse to decoherence initially avoids invoking consciousness to solve the measurement paradox of QM and points to breaking the link between human consciousness and the wave-function’s collapse [17], it leaves untouched the problem of decoherence itself, merely postponing the influence of the conscious observer in selecting the relevant variables of the problem [16]. More importantly, the task of identifying a system by the observer is non-computable. An observation is a process of decoherence that occurs when the Kolmogorov complexity of at least one of the systems involved approaches the Kolmogorov complexity of the external observer. But, in general, Kolmogorov complexity is not computable and, therefore, non-algorithmic. This suggests that quantum mechanical systems exist only with respect to a particular class of observers, whose task of system identification is prior to QM itself [18].

Zurek argues in favour of an einselection of a preferred basis (of observables) by the environment. The criteria for such a procedure would be minimization of entropy in the information transfer between the different

systems involved and maximization of stability along time, in a sort of Quantum Darwinism [19]. The predictability sieve shows that only quantum states that are robust in spite of decoherence (and effectively ‘classical’) have predictable consequences. Classical reality would be equivalent to predictability; therefore, the predictability sieve would have been extremely useful in evolution, allowing forecasts about upcoming states of the world [15]. However, since the ontological features of the state vectors are acquired through the epistemological information transfer of einselection, we are still left with essential questions about the ontological character of information and entropy, and of the splitting between system and environment; not to mention the problem of indeterminism at the fundamental level as Zurek himself acknowledges.

Modal interpretations of QM, for their part, still have a problem with the a priori decomposition of system and environment. But, even more, since modal interpretations supply actualization rules that pick out from the set of all observables of a quantum system the subset of definite-valued properties making it possible to derive rigorous results [20], further *cognitive* prescriptions must be added to the theory. In the consistent histories approach, on the other hand, the situation is not better: there is no criterion by which certain consistent coarse-grained histories are preferred over others that might be equally consistent [21]. In other words, a prescription on how to go from the wave-function to the measured-value state is always needed. If both of these states are ontological — and the effect of quantum interference is strong enough evidence to support such view — fundamental cognitive prescriptions seems to be unavoidable in order to explain the link between these two levels of reality. Despite the efforts of the Quantum Darwinism program [19], the origin of classicality still depends on definitions of observation and communication [22].

6.2. Quantum mechanics and logic

In a recent book, Epperson and Zafiris claim, from a Whiteheadian perspective, that logic is irreducible to Physics and clarify the ontological function of logic in quantum causality, thanks to the Boolean contextual basis of particular quantum measurements [21, p. 158; 23]. In that sense, the act of observation is generative of novel facts, not merely revelatory of pre-existing facts — as stated by hidden variables theories. Evaluation and contextualization are deemed ontologically relational features of actuality and potentiality in the world, mediating logic the transition from the latter to the former and vice versa. Moreover, the usual set theoretic framework for QM should then be replaced by a category (sheaf-) theoretic framework capable of maintaining topological correlations that are global. Since ‘system’, ‘detector’, and ‘environment’ are always formally entangled in QM, such that their partitioning is intrinsically relative [24], “quantum mechanical relations between measured system and environment cannot be characterized as purely efficient causal relations”. Decoherence “makes sense only as an intrinsic, locally conceived, reductive process of the global-level description of a quantum system”, and the classical

world “emerges intrinsically from the global quantum level via coarsening the scale of resolution of quantum observable behavior by means of macroscopic Boolean observables locally according to the sheaf-theoretic topological compatibility relation” [21, p. 351].

As said by these authors, “all causal relationships presuppose logical relationships, and logical relationships can have physical significance” because the “classical (merely epistemic) conception of conditional probabilities is entirely invalid in QM”. The “ontological interpretation of QM depicts logical correlations as a physical significant feature of nature itself”. The significance of this perspective for CDS is straightforward, for the reason that quantum entanglement “precludes in a novel way the possibility of defining individual entities independently of the *local logical frames* (Boolean frames) under which their behavior is manifested”. Each “higher level totality contains and is thus conditioned by the facts of the next lowest level, and so on down the series of inclusions” and “every becoming actual occasion/propositional predicative fact is always internally related to a global, objective actual world — an actual system of facts with objective truth values” [21, p. 142]. In short, our epistemic access to the universe is grounded on a prior logical hierarchy of levels continually coming into being in a generative process.

QM touches upon the logical order of nature and descends deeper than the causal order of nature. There “exists a mutually implicative bidirectional relation between the parts and the whole, where the correlation of potentialities plays a crucial role” [23, p. 358]. But then, we assist to the *coup de grâce* to any reductive interpretation of emergence. The realm of potentialities has an *inner logos* which commensurates with our human logos. Somehow, in a very subtle manner, logic shapes the world. For instance, the existence of non-commutativity relations between specific pairs of observables — canonical conjugates of each other — means that there is an intrinsic order in the Universe. This order is not naïve. It always plays its part at the level of correlations of potentialities and, noteworthy, after a specific measurement context has been devised, i.e. after localization of the global quantum event. This order (induced by non-commutativity) is present everywhere and is responsible in last term of the novel quantum effects. Non-commutativity is a logical relation that cannot be emergent, but primary. The obvious question now is, why can this objective logos be experienced subjectively? Why does consciousness appear in the world?

6.3. Quantum mechanics and information

The QM mysteries also bring up the question: Why is the Universe favourable to consciousness? [25] Out of all of the possible factorizations of Hilbert space, why is the particular factorization corresponding to classical space — as we humans perceive it — so special? Why do we perceive our world as a hierarchy of objects that are strongly integrated and relatively independent? Max Tegmark tries to understand consciousness as a state of matter (‘perceptronium’)

characterized by the principles of storage capacity of integrated information, autonomy (independence and complex dynamics), and utility. In his ‘physics-from-scratch’ approach, the maximization of integrated information in the human brain — especially from the QM perspective — or the decomposition of the Universe into maximally independent objects presents, paradoxically, meagre results. The definition of integrated information must be modified or supplemented by additional principles related to information processing in autonomous systems. Tegmark is confident of the existence of some optimal factorization of our universe into integrated and relatively independent parts that explains what “conscious observers perceive, because an integrated and relatively autonomous information complex is fundamentally what a conscious observer is” [<http://arxiv.org/abs/1401.1219>].

In accordance with Zurek’s results for the privileged basis, the states that are most robust toward environment-induced decoherence are those that approximately commute with the interaction Hamiltonian. Such states dominate Quantum Darwinism’s survival of the fittest by multiplying their imprints in the environment — which thereby shapes how decoherence to classicity occurs for all practical purposes [<https://edge.org/conversation/five-problems-in-the-philosophy-of-mind>]. The utility principle, on the other hand, suggests that it is precisely these most stable and predictable states that conscious observers will perceive. However, when studying autonomy from scratch, the state of the system and its dynamics (its density matrix and the Hamiltonian) cannot be considered separately, their interplay being crucial. Moreover, work on Quantum Darwinism has shown that a “state selected at random from the Hilbert space of a many-body system is overwhelmingly likely to exhibit highly non-classical correlations (...). The objectivity of classical reality — the fact that multiple observers can agree on the state of a subsystem after measuring just a small fraction of its environment — implies that the correlations found in nature between macroscopic systems and their environments are exceptional.” [26]

In summary, the particular Hilbert space factorization we observe is very special and unique, since there are countless other Hilbert space factorizations that mix the system and the environment. Either the system state is special or the Hamiltonian is special. On the other hand, Tegmark points out that “the optimal factorization can change at least somewhat with time, since our designation of objects is temporary (...). An obvious way out of this impasse is to bring consciousness back to center-stage”. “It therefore appears that if we can one day solve the quantum factorization problem, then we will find that the emergence of time is linked to the emergence of consciousness”. Conscious observation needs to be taken into account to address the issue properly. We “need a criterion for identifying conscious observers, and then a prescription that determines which factorization each of them will perceive” [<http://arxiv.org/abs/1401.1219>]. Conscious human observers perceive the world in a species-specific way, so that the physics-from-scratch approach fails, being a too fine-grained level that misses the point of human symmetry breaking. One needs something more at an

upper level; one needs to invoke the level of human beings interested in relevant information from the system in order to solve specific problems.

As far as we know, consciousness distinguishes available and relevant information for the subject, using top-down causation at different levels [27]. This selection process, typical of CDS, is obvious insofar as “mental states, whatever we mean by that and whatever the amount of information they imply, contain immensely less information than the information necessary to determine the full physical state of the brain” [<http://edge.org/conversation/free-will-determinism-quantum-theory-and-statistical-fluctuations-a-physicists-take>].

Even if QM “describes a universe in which you can make probabilistic predictions of how systems behave, but in which those systems have as much freedom from determinism as any physical system described by probabilities can have” [9, p. 150], nature itself somehow asks for a conscious observer at the end of the determination process. In addition, the “need to refer to consciousness exists, insofar as only consciousness can distinguish a mere physical correlation, e.g. of an external system with the observer’s eye, from actually available information, i.e. such that the observer is aware of, and can act upon in the future”. This means that “that information is a correlation between the degrees of freedom of the observed and observing systems” [18]. We humans being, from the point of view of Physics, CDS far from equilibrium is a situation clearly favourable to information processing at different levels. But, what exactly is this ‘relative’ concept of information?

7. Information-laden Nature

Is information everywhere in nature independently of conscious observers (metaphysical monism of *it from bit*)? Does this concept reduce to a causality relation or to a correlation between physical magnitudes whenever a cognitive agent is absent? Is information a special interface between the universe and its inhabitants, with QM as a theory of physical information? Floridi, from his philosophy-of-information approach, offers a valid definition of semantic information as “well-formed, meaningful, and truthful data; knowledge is relevant semantic information properly accounted for; humans are the only known semantic engines and conscious inforgs (informational organisms) in the universe who can develop a growing knowledge of reality; and reality is the totality of information (notice the crucial absence of ‘semantic’)”. In the same vein, information “is not about representing the world: it is rather a means to model it in such a way as to make sense of it and withstand its impact” [28]. Therefore, whereas information pervades reality, relevant semantic information is only proper to human beings. What does it mean?

7.1. Knowledge as ‘accounted-for’ information

Questions about how can raw data acquire their meaning or how meaningful data acquire their truth value then arise. As a matter of fact, a datum

is ultimately reducible to some ‘lack of uniformity’, but data must be accessed and elaborated by an information agent with finality, within some context and defined level of abstraction. Truth is encapsulated in information providing distal access to, successful interaction with the modelled system. On the other hand, Floridi defends a subjectivist interpretation of epistemic relevance: no “semantic information is relevant *per se*, relevance being an informee-oriented concept (...). More explicitly, this means grounding relations of causal relevance on relations of epistemic relevance”. But this subjectivist interpretation of relevant information cannot obviously work for artificial agents, nor for differentiating a lucky informee from a knower [28, p. 267]. Knowledge implies correctly accounting for *how come* questions about the available information. Being informed that you are mistaken is different from knowing why you are [29]. Knowledge — especially scientific knowledge — requires the embedding of information in a network of relevant questions and correct answers; an account in which each piece of information is relevant. In that sense, knowledge is intentional and needs a coherent explanatory history behind it.

Accounting for relevant information allows a graded appraisal of epistemic states, as it occurs with scientific knowledge. On the other hand, even if non-human animals do not hold explicit accounts for their information, they do possess sensitive knowledge and implicit accounts of their information embedded in their life time span. All in all, ‘knowing that’ “is grounded on ‘knowing how’, hardly surprising from an evolutionary perspective” [28, p. 287]. Nevertheless, as Floridi recognizes, the sceptical problem remains. He explains what knowledge is, but not how to obtain that knowledge in specific situations. His model turns out to be a pragmatic theory of truth. But a more fundamental theory of truth is needed because there is no normativity in the pure neuronal processes underlying, e.g. mathematical and logical judgments [30]. Both a constitutive and a historical explanation of knowledge are needed [31].

7.2. *Consciousness and knowledge*

The capability of accounting for relevant information seems to disclose an innate power of conscious observers to process whatever kind of information. At the same time, it gives us a hint on what consciousness is. Whereas externally inferable states do not allow us to discriminate between types of inferential agents — what casts serious questions on Turing test’s significance to determine conscious beings — conscious agents can be distinguished from artificial agents and zombies thanks to their ability to exploit the information implicit in questions addressed to them and in their own answers. A “zombie can jump and walk but he cannot infer (let alone be certain) from this that he exists, for *he* does nor (indeed cannot) know that *he himself* is jumping and walking” [28, p. 304]. Dealing with information is therefore different from ‘knowing that’. This subjective processing of information needs consciousness; it is present conscious knowledge that transcends time. Consciousness allows for true knowledge about reality beyond time.

Floridi shows how extracting relevant semantic information from self-answering questions about one's particular states requires agents endowed with advanced semantic capacities in a sort of 'transcendental thinking'; thinking about the conditions of possibility of the very answer. A "self-answering question requires both understanding of the content of, and a detachment from, the question itself. Self-answering questions are part of the *frame problem*." [28, p. 305] Human agents, different from artificial machines and zombies, have a position in the world because they do possess an identity which shapes their relationships with the Universe. They are in the world as here-now oriented subjects [32] not objects. Knowledge is thus inseparable of 'I know' statements. Subjective reflection requires not only semantics, but also consciousness; an overall capacity of reflectivity that, in the case of artificial agents and zombies, cannot be predefined. Even though reflective knowledge never comes to a halt — I can always know that I know that I know... — it is enough for qualifying human consciousness.

Remarkably, Floridi compares consciousness to a mathematical fixed point in an abstract phase space: "it occurs as a decoupling from reality and a collapsing of the referring agent and the referred agent" [28, p. 314]. A new limit is then reached in nature. From the point of view of CDS, this fact points toward the emergence of the level of consciousness and raises strong doubts about its reconciliation with some form of naturalism. In spite of Searle's strong commitment in favour of the possibility of an objective account of consciousness [30, p. 114], the ontology of the mental seems to be an irreducibly first-person ontology.

7.3. The symbol grounding problem

Whereas we do not know of the existence of real zombies, we know of everyday artificial agents capable of handling information. Is it possible to coordinate several artificial agents to obtain semantic systems? Is there any possibility to attach meaning to symbols from scratch? Searle clearly points out that neither semantics nor syntax in computation are intrinsic properties of the grounding Physics. Both of them need an external interpretation [30, p. 17]. Nevertheless, Floridi attempts to unravel this problem searching for an interpretation of the symbols intrinsic to the symbol system itself, which should be able to generate its meaningful elements. The solution, however, cannot be grounded in any form of representationalism, semirepresentationalism or non-representationalism, which do not satisfy the 'zero semantic commitment condition'. All of them surreptitiously introduce pre-established criteria, hard-coded by a supervisor, for categorizing the world [28, p. 139-140].

Since specific meaning can hardly be acquired in isolation or independently of context, level of abstraction and purpose parameters, one clue for the possible solution stems from posing the question of how a primitive, simple, and initial form of intentionality develops (in an autonomous way) from the direct interactions between an artificial agent and its environment. Artificial

evolutionism always presumes the presence of a semantic framework from the programmer to the ‘right’ fitness. On the contrary, Floridi invokes ‘action-based semantics’ (AbS) as an explanation of the particular process that allows the coupling of symbols to meanings. In AbS, artificial agents should relate their symbols to the states in which they are placed by the actions that they perform. The initial association of symbols and meanings is a direct input-output relation that follows only from the performance of actions. However, the critical issue is that AbS needs a system capable of ‘metaprogramming’, operating at *two levels* interacting with each other: “It organizes actions at an *object level*, where it interacts with the external environment. But it can also take actions on its internal states and on its own elaborations. In this case, it operates at a *meta-level*, which takes as data the actions at the object level.” [28, p. 166]

Even if this metaprogramming paradigm is highly suggestive — resembling a proposed partition of brain linguistic functions [33] — it seems to beg the question in different ways: (a) it is unclear whether the input/output structure of data, as well as the concept of internal state, can be well-defined ‘from scratch’, as it is grounded in a system/environment partition which is not fundamental; (b) the transition from an analogical external universe to a discrete world of internal states requires a new, upper, coarse-grained level of abstraction; (c) since the meta-level operation is reserved to a ‘part’ of the system, it must be a somehow hardwired functional process, directly following from the part’s particular physical evolution, embodiment in the system and overall survival [28, p. 172]. However, it cannot be simply assumed that a new level of abstraction stems from the physics, because this is precisely what is at stake. It is hard to speak about levels of abstraction without reference to human interpretation.

Whereas association processes following Hebb’s rule are very likely playing a role in learning processes and social pressure must be the main influence on communication tuning, avoiding the problem of a private language, Floridi does not overcome Wittgensteinian criticism related to meaning-as-use. Intrinsic flaws of his metaprogramming approach beg the question about why the association of a symbol with an internal state of the system *is* a meaning. Most interestingly, the perspective of CDS support the view that meaning cannot be accessed algorithmically and requires a non-algorithmic shift of inter-level, cognitive strategy, namely, a simplification of the relevant degrees of freedom to attain stability in the deterministic chaos province of neurons and the minimization of information loss [34]. In brief, swap of information is needed to obtain a new meaningful level of abstraction.

7.4. Consciousness as integrated information?

According to Tononi’s recent approach, consciousness is one and the same thing as integrated information; it exists as a fundamental quantity whose quality is completely and univocally given by the informational relationships generated by a complex of elements. Integrated information is above and beyond

the information generated independently by the parts of a system and turns out to be ‘intrinsic’ to the system [35]. But how is that possible, since information must be ‘interpreted nature’? One never deals with pure data, but with interpreted data [28, p. 356].

Integrated information should therefore be evaluated from the perspective of the system itself, starting from its elementary, indivisible components and interactions, and not by arbitrarily imposing ‘units’ from the perspective of an observer. But then again, are all systems not observer-dependent? For Tononi, “there will often be a privileged spatio-temporal ‘grain size’ at which a given system forms a complex of highest integrated information — the spatiotemporal scale at which it ‘exists’ the most in terms of integrated information, and therefore of consciousness” [35]. In that sense, consciousness would emerge as a most basic informative, spatio-temporal property, linked to the deepest mysteries of the universe. However, it is highly controversial as to why and how a particular content of consciousness, or *qualia*, must be ascribed to a particular set of physical causal connections. Tononi’s hypothesis does not touch upon the question of the origin of meaning; it simply imposes the identification by fiat of a plausible correlation between *qualia* and integrated information.

7.5. *Information and mind*

What we learn from all these approaches is that the polarity between ontology and epistemology seems to be irreducible. Reality is informational because it consists of differences, or inequalities *de re*, which can be interpreted according to different levels of abstractions by *inforqs* like human beings. Human beings are able to extract relevant information because they have minds; and such relevant information can be updated to knowledge when integrated within a coherent history. Data implies ontic differences that are epistemically exploitable as resources for cognitive processes.

Reality in itself — with its intrinsic multi-level intelligibility — remains an epistemically inexhaustible resource out of which knowledge is formed. Of course objects in the world allow or invite certain epistemic constructs and resist or impede some others, but the ways in which one has epistemic access to the system affect the cognitive outcome. Knowing thus has to do with designing and modelling reality [28, p. 370], with a process of semanticization — coming to mean — of nature. The primacy of an information-laden nature is only perceived by minds able to extract and deal with it at different levels of abstraction. No ontology can be freed of the latter. Information in the Universe and mindful human beings are but two sides of the same coin. Both of them share an internal *logos* allowing the latter’s cognitive access to the former. The price to be paid is that the question of why reality is intelligible for us has no answer within the pure realm of our cognitive skills. In other words, the symbol grounding problem might be unsolvable.

Presently, it is common to admit that our most fundamental scientific theories underdetermine reality. We see that new principles must be invoked in order to deal with emergent levels of complexity in CDS. If we consider reality as the totality of structures dynamically interacting with each other, complexity permits the emergence of cohering cluster of data waiting to be interpreted. Then, the evolution of the universe turns out to be a superb unfolding of sense.

7.6. The case for ontological pluralism

Can we say anything more specific about the kind of emergence that happens in nature? Despite my last considerations, many authors continue to defend a mere epistemological emergence, rejecting ‘ontological levelism’ for different reasons [36, 37]. Emergence is deemed to be a relational concept in comparison between models. Some rule for determining an observable at one level is overlooked at another level for reasons of convenience about the goals and purposes aimed at [28, p. 64]. However, that is not the case for the Second Law of Thermodynamics, the wave-function collapse or the emergence of meaning. Neither of them stems from a coarse-grained transition from lower, more concrete, to upper, more abstract levels; and all of them are omnipresent in our scientific knowledge of reality.

Certainly, all naïve mimetic theory or ‘picture of the world’ view of knowledge must be rejected. Nature does not know about levels of abstraction, being designed by the epistemic agents which experience it. A constructionist view of knowledge is needed [28, p. 77] according to which, the selection of an observable to be measured locally fixes a context for the individuation of events. In QM, any local observable is a partial information carrier; and the totality of all different potential local frames “function as a semantic tool for combining features under the condition of partial compatibility of observable information within the limits determined by the imposition of the uncertainty relations” [21, p. 364]. But it is then hard to disavow the ontological, top-down influence of higher levels of nature in nature via the imposition of constraints. These work “by modifying either a system’s phase space or the probability distribution of events and movements within that space” [6]. The Second Law of Thermodynamics constrains evolution, the wave-function collapse constrains measurement outcomes in Hilbert space, and the relevant information constrains a multidimensional neural state into new patterns of activity.

The case for ontological pluralism even becomes more evident if one regards specifically the symbol grounding problem. Instead of representing meaning in a symbol structure, “a dynamical neurological organization embodies meaning in the topographical configurations — the set of self-organized context-dependent constraints — of its phase space”. Bottom-up propagation combined with top-down attentional amplification results in a whole-brain neural assembly and “ignites into a self-sustained reverberant state of coherent activity that involves many neurons distributed throughout the brain” [7]. Obviously, because of the ontology of CDS, we cannot keep track —

in a single well-defined level — of the interplay between the system's own top-down inhibiting constraints and the alternatives available to its components that gives rise to conscious meaningful content. As Smolin explains, the “problem of qualia, or consciousness, seems unanswerable by Science because it's an aspect of the world that is not encompassed when we describe all the physical interactions among particles. It's in the domain of questions about what the world really is, not how it can be modelled or represented.” [9, p. 269]. In short, it is the paradigm of ontic emergence in an ontologically plural universe.

8. Concluding remarks

“All the ‘fitting’ between mathematics and the regularities of the physical world is done within the minds of physicists who comprehend both.” [38] Fair enough, all normativity is a construction of the human mind, but if human brain working is not different from any other physical or biological system, how does normativity stem from a universe without normativity? Somehow, normativity itself is beyond evolution's epistemic framework [39]. Science itself is beyond evolution's epistemic framework because the “aim of Science is not just the manufacture of new toys: it is the enrichment of the human spirit” [11, p. 128]. Complex dynamical systems (CDS) and the three issues studied in this article — the Second Law of Thermodynamics, the quantum wave-function collapse, and the emergence of meaningful information — point toward an irreducible interplay between different levels of reality.

CDS are partly independent of their parts, which often become replaceable components [6]. It is thus problematic to deny some sort of ontic independence for the upper levels of complexity in nature. New types of entities and qualitatively different regimes emerge as a result of discontinuous and irreversible phase transitions. With the onset of the new regime, the system top-down constrains its lower-level behaviour. A different epistemic method is necessary to approach the emergent level of complexity: a redefinition of degrees of freedom and phase-space turns out to be inevitable. In that sense, the new description is level-dependent and, since it depends on human epistemic interest, human-dependent [16]. As Quantum Mechanics (QM) suggests, knowledge makes a difference in the world because *logical conditioning makes a real difference in the world*. The presence of mind — the subjective side of objective logos — makes a difference in nature.

Despite some tentative of explaining the emergence of indeterminacy and free will via deterministic chaos [<http://edge.org/conversation/free-will-determinism-quantum-theory-and-statistical-fluctuations-a-physicists-take>] and identifying randomness with unpredictability [40], deterministic chaos in itself is unable to explain the emergence of upper levels of complexity. There are actually different sources of determination at different levels because lower levels are not sufficient conditions for upper levels. It is not surprising then that the principle of sufficient reason must be complemented with new principles. In that sense, QM intrinsic indeterminism need not be equivalent to free will; it

simply reflects some intrinsic limitation of physics within the realm accessible to human freedom and spiritual determination. The QM measurement problem might be unsolvable.

Evidently, there are more than clouds on the reductionist horizon and on the possibility that an ultimate TOE can be formulated as a finite number of principles [S.W. Hawking, *Godel and the End of the Universe*, 2002, <http://www.hawking.org.uk/godel-and-the-end-of-physics.html>]. No finite set of efficient causes will describe the becoming of the universe, including the mind. “We do not know all the possibilities in the adjacent possible of the biosphere! Not only do we not know what will happen, we do not even know what can happen.” [<https://edge.org/conversation/five-problems-in-the-philosophy-of-mind>] Of course, science resorts to probability distributions to deal with unknowability, but in many cases we do not even know the set of possibilities. This is hardly shocking inasmuch as real novelties and differences appear in the Universe and new epistemic principles to approach them can be added only *a posteriori*. The alternative therefore is not between pure determinism and pure randomness — as, for example, the emergence of the classical world from decoherence illustrates. We need an a priori cognitive, specifically human identification of the problem — what should be the system and the environment and the relevant degrees of freedom — to scientifically be able to tackle it. Therefore, it is very doubtful that we can scientifically study the universe as a whole from scratch.

In summation, we deal with different levels of reality in CDS and we need to invoke some novel constraints or conditions for understanding the emergence of upper levels from lower ones. If someone wishes to maintain a global microscopic determinism, the emergence of constraints — as the Second Law of Thermodynamics, the quantum wave-function collapse, and information in non-interpreted nature — from much more basic laws should be explained. Otherwise, we can maintain the view that emergence of complexity is purely epistemic, but if that is the case, we cannot longer trust our scientific access to reality. The basic realism of science and science itself are then, in my opinion, undermined. Tegmark is right when affirming that the “quests to better understand the internal reality of our mind and the external reality of our universe will hopefully assist one another” [<http://arxiv.org/abs/1401.1219>], but I would spell it out better with the words of former pope Benedict XVI: “Mathematics, as such, is a creation of our intelligence: the correspondence between its structures and the real structures of the Universe — which is the presupposition of all modern scientific and technological developments, already expressly formulated by Galileo Galilei with the famous affirmation that the book of nature is written in mathematical language — arouses our admiration and raises a big question. It implies, in fact, that the Universe itself is structured in an intelligent manner, such that a profound correspondence exists between our subjective reason and the objective reason in nature. It then becomes inevitable to ask oneself if there might not be a single original intelligence that is the common font of them both.” [41]

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