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### **Do Spanish Stock Market Prices Follow a Random Walk?**

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## **ABSTRACT**

In this article we test the random walk hypothesis in the Spanish daily stock market prices by means of using fractionally integrated techniques. We use a version of the tests of Robinson (1994) that permit us to test  $I(d)$  statistical models. The results show that though fractional degrees of integration are plausible in some cases, the confidence intervals are generally narrow, including the unit root in all cases. Therefore, there is very little evidence of fractional integration, despite the length of the series, implying that the standard practice of taking first differences when modelling stock prices is adequate. In addition, the tests cannot reject that the underlying  $I(0)$  disturbances are white noise, supporting thus the (weakly) efficient market hypothesis in the Spanish stock market.

Keywords: Stock market; Unit roots; Long memory; Market efficiency.

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## **1. Introduction**

An important issue in the empirical analysis of financial time series is whether holding period returns on a risky asset are serially independent, which is required by the efficient market hypothesis on its weak form, (i.e., the current stock prices fully reflect all the past stock prices information). Although a precise formulation of an empirically refutable efficient market hypothesis must obviously be model-specific, historically the majority of such tests have focused on the forecastability of common stock returns. Within this paradigm, which has been broadly categorized as the “random walk” theory of stock prices, the evidence is mixed.

In short horizon returns, using variance-ratio tests, Lo and MacKinley (1988) rejected that stock prices follow random walks for daily and weekly returns, but they found no evidence against the random walk hypothesis for monthly returns. They also found that portfolio returns of the NYSE and the AMEX stocks for the time period 1962-1985 exhibit significant positive first-order autocorrelations while security returns present negative first-order autocorrelations although statistically and economically insignificant, as previously documented by French and Roll (1986). Lo and MacKinley (1990) showed that the different autocorrelation sign between portfolios and stocks may be explained by lead-lag positive autocorrelations across securities. Poterba and Summers (1988) found negative autocorrelation in monthly returns for a NYSE value-weighted index during the period 1926-1985 while Lo and MacKinley (1988) obtained positive autocorrelation in a value-weighted index formed by NYSE and AMEX monthly stocks for the shorter period 1962-1985.

Regarding long-horizon returns, Fama and French (1988), using regression techniques, and Poterba and Summers (1988), using variance ratios, conclude that stock

prices present mean reversion. In this case, the autocorrelations become negative for the two-year returns, reach their minimum values for the three to five year horizons and then decay to zero, though these results are mainly due to the first part of the sample, (i.e., 1926-1941) of the NYSE stocks returns. The findings of Fama and French (1988) and Poterba and Summers (1988) might be due to time varying on expected returns or, as De Bondt and Thaler (1985, 1987), Jegadeesh (1991) and Jegadeesh and Titman (1993, 2001) suggest, to investor overreaction or underreaction, which causes stock price swings away their fundamental values. The above empirical evidence is also consistent with Summer's (1986) proposition that the stock prices have temporary components which decay slowly to zero. By contrast, using a generalised form of rescaled range (R/S) statistic, Lo (1991) found no evidence against the random walk hypothesis. Using annual data and allowing for fractional alternatives, Caporale and Gil-Alana (2002) reported that US stock returns are close to being an  $I(0)$  series, and pointed out that their degree of predictability depends on the process followed by the error term.

In this article we revisit this issue by means of using fractionally integrated techniques. In particular, we examine if the stock market prices in Spain are  $I(1)$ , (with or without an autocorrelated structure underlying the process). However, instead of using techniques based on autoregressive (AR) processes, we consider the possibility of long memory. Long-range dependent time series exhibit an unusually high degree of persistence so that observations in the remote past are non-trivially correlated with observations in the distant future, even as the time span between the two observations increases.

The outline of the article is as follows: Section 2 briefly describes the procedure used in the paper for testing unit and fractional roots. In Section 3, this procedure is applied to the Spanish stock market prices while Section 4 contains some concluding comments.

## 2. The testing procedure

In the following,  $P_t$  denotes the stock price at time  $t$ , while the lower case indicates the logarithmic values. Our maintained hypothesis is given by the recursive relation

$$p_t = \mu + p_{t-1} + \varepsilon_t, \quad (1)$$

where  $\mu$  is an arbitrary drift parameter and  $\varepsilon_t$  is a white noise process.

There exist many different ways of testing model (1). Perhaps, the most common ones are the tests due to Fuller (1976), Dickey and Fuller (1979). They consider processes of form:

$$(1 - \rho L)p_t = \mu + \varepsilon_t, \quad (2)$$

which, under the null hypothesis:

$$H_o : \rho = 1, \quad (3)$$

becomes the random walk model (1). The tests are based on the auxiliary regression of form:

$$(1 - L)p_t = \pi p_{t-1} + \mu + \varepsilon_t, \quad (4)$$

and the test statistic is the “t-value” corresponding to  $\pi$  in (4). Due to the non-standard asymptotic distributional properties of the “t-values” under the null hypothesis:  $H_o: \pi = 0$ , Dickey and Fuller (1979) provide the fractiles of simulated distributions which give us the critical values to be applied when testing the null against the alternatives:  $H_a: \pi < 0$ . The tests can be extended to allow for autocorrelated disturbances and then, the auxiliary regression (4) may be augmented by lagged values of  $(1-L)p_t$ , and also with other deterministic paths, like a linear time trend, though this unfortunately changes the distribution of the test statistic. Another limitation of these tests is that they lose validity if the disturbances are not white noise or AR processes. This was observed by Schwert

(1987) who found that Dickey-Fuller critical values can be misleading even for large sample sizes in case of a mixed ARIMA process. He proposed the use of tests of Said and Dickey (1984, 1985), which approximate the ARMA structure by an AR. Also, Phillips (1987) and Phillips and Perron (1988) consider tests which employ a nonparametric estimate of the spectral density of  $u_t$  at the zero frequency, for example, a weighted autocovariance estimate. More recently, Kwiatkowski et al. (1992) observed that taking the null hypothesis to be  $I(1)$  rather than  $I(0)$  might itself lead to a bias in favour of the unit root hypothesis; they proposed an  $I(0)$  test which formulates the null as a zero variance in a random walk model, while Leybourne and McCabe (1994) extended the tests to the case where the null was an  $AR(k)$  process and the alternative was an integrated ARMA (ARIMA) model with AR order  $k$  and unit MA order. Their test differs from that of Kwiatkowski et al. (1992) in its treatment of autocorrelation under the null hypothesis, its critical values appearing more robust to certain forms of autocorrelation.

Conspicuous features of the above methods for testing unit roots are the non-standard nature of the null asymptotic distributions which are involved, and the absence of Pitman efficiency theory. However, these properties are not automatic, rather depending on what might be called a degree of “smoothness” in the model across the parameters of interest, in the sense that the limit distribution do not change in an abrupt way with small changes in the parameters. Thus, they do not hold in case of unit root tests against AR alternatives such as (2). This is associated with the radically variable long run properties of AR processes around the unit root. Under (2), for  $|\rho| > 1$ ,  $p_t$  is explosive; for  $|\rho| < 1$ ,  $p_t$  is covariance stationary; and for  $\rho = 1$ , it is nonstationary but non-explosive. In view of these abrupt changes, the literature on fractional processes have become a rival class of alternatives to the AR model in case of unit-root testing. Thus, Robinson (1994) proposes a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : d = d_o, \quad (5)$$

in a model given by

$$(1 - L)^d x_t = u_t, \quad (6)$$

where  $d$  can be any real number and where  $u_t$  is an  $I(0)$  process, defined for the purpose of the present paper, as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. The  $x_t$  in (6) can be the time series we observe, (in our case, prices,  $p_t$ ), though it may also be the errors in a regression model of form:

$$p_t = \beta' z_t + x_t, \quad (7)$$

where  $\beta = (\beta_1, \dots, \beta_k)'$  is a  $(k \times 1)$  vector of unknown parameters, and  $z_t$  is a  $(k \times 1)$  vector of deterministic regressors that may include, for example, an intercept, (e.g.,  $z_t \equiv 1$ ), or an intercept and a linear time trend, (in case of  $z_t = (1, t)'$ ). Specifically, the test statistic proposed by Robinson (1994) is given by:

$$\hat{r} = \left( \frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \quad (8)$$

where  $T$  is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j)$$

$$\hat{A} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left( \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \lambda_j = \frac{2\pi j}{T}.$$

$I(\lambda_j)$  is the periodogram of  $\hat{u}_t$ , where

$$\hat{u}_t = (1 - L)^{d_o} y_t - \hat{\beta} w_t, \quad w_t = (1 - L)^{d_o} z_t; \quad \hat{\beta} = \left( \sum_{t=1}^T w_t w_t' \right)^{-1} \sum_{t=1}^T w_t (1 - L)^{d_o} z_t,$$

and  $g$  above is a known function coming from the spectral density of  $u_t$ :

$$f(\lambda_j; \tau) = \frac{\sigma^2}{2\pi} g(\lambda_j; \tau).$$

Note that these tests are purely parametric and therefore, they require specific modelling assumptions to be made regarding the short memory specification of  $u_t$ . Thus, for example,

if  $u_t$  is white noise,  $g \equiv 1$ , and if  $u_t$  is AR(1) of form:  $u_t = \tau u_{t-1} + \varepsilon_t$ ,

$g(\lambda_j; \tau) = |1 - \tau e^{i\lambda_j}|^{-2}$ , with  $\sigma^2 = V(\varepsilon_t)$ , so that the AR coefficients are function of  $\tau$ .

Robinson (1994) showed that under certain regularity conditions,

$$\hat{r} \rightarrow_d N(0, 1) \quad \text{as } T \rightarrow \infty. \quad (9)$$

Thus, we are in a classical large-sample testing situation and the conditions on  $u_t$  in (9) are far more general than Gaussianity, with a moment condition only of order 2 required. An approximate one-sided  $100\alpha\%$ - level test of  $H_0$  (5) against the alternative:  $H_a: d > d_0$  ( $d < d_0$ ) will reject  $H_0$  (5) if  $\hat{r} > z_\alpha$  ( $\hat{r} < -z_\alpha$ ), where the probability that a standard normal variate exceeds  $z_\alpha$  is  $\alpha$ . Furthermore, he shows that the above test is efficient in the Pitman sense, i.e., that against local alternatives of form:  $H_a: d = d_0 + \delta T^{-1/2}$ , with  $\delta \neq 0$ , the limit distribution is normal with variance 1 and mean which cannot (when  $u_t$  is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Empirical applications based on this version of Robinson's (1994) tests can be found in Gil-Alana and Robinson (1997) and Gil-Alana (2000), and other versions of his tests, based on seasonal, (quarterly and monthly), and cyclical data, are presented in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001a) respectively.

### 3. Empirical evidence in the Spanish stock market prices

The time series data analysed in this section correspond to the daily structure of the Spanish stock market prices, (IBEX 35), obtained for the time period 4-January-1994 to 26-November-2001. The Spanish stock market index IBEX 35 is a value-weighted index that includes the thirty five most traded stocks of the Spanish stock market. Every semester the effective trading volumes of all stocks are studied in order to adjust the stocks and their weights that will form the index in the next semester. This index was created on December 31<sup>st</sup> 1989. Until 2000, the IBEX 35 stock weights were based in their market values but since 2000 they are based only in their free float capital.<sup>1</sup>

Figure 1 contains plots of the log-transformed series and its first differences along with their corresponding correlograms and periodograms. We observe that the series appears to be nonstationary and this can also be viewed across the correlogram, (with values decaying very slowly), and throughout the periodogram, (with a large peak around the smallest frequency). The first differenced series may have now a stationary appearance, though we still observe in the correlogram significant values even at some lags relatively far away from zero, which may be an indication that other types of differencing, greater than or smaller than one, might be more appropriate than first differences.

Denoting the log-transformed series  $p_t$ , we employ throughout model (6) and (7), with  $z_t = (1, t)'$ ,  $t \geq 1$ , 0 otherwise, i.e.,

$$p_t = \alpha + \beta t + x_t, \quad t = 1, 2, \dots \quad (10)$$

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (11)$$

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<sup>1</sup> The possibility of structural breaks was examined and we could not find any evidence in favour of this hypothesis.

testing  $H_0$  (5) for values  $d_0 = 0, (0.25), 2,$  and different types of disturbances. In Table 1(i), we assume that  $\alpha = \beta = 0$  a priori, (i.e., we do not include any regressors in the regression model). Tables 1(ii) and (iii) assume respectively an intercept, ( $\alpha$  unknown and  $\beta = 0$  a priori), and an intercept and a linear time trend, ( $\alpha$  and  $\beta$  unknown). Thus, for example, if  $u_t$  is white noise and  $d_0 = 1$ , the differences  $(1 - L)p_t$  behave, for  $t > 1$ , like a random walk when  $\beta = 0$ , and a random walk with a drift when  $\beta \neq 0$ . However, we also consider the possibility of the disturbances being weakly autocorrelated. In particular, we take AR(1), AR(2), and also the Bloomfield's (1973) exponential spectral model.<sup>2</sup> This is a non-parametric approach of modelling the I(0) disturbances in which  $u_t$  is exclusively specified in terms of its spectral density function, which is given by:

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} \exp\left(2 \sum_{r=1}^m \tau_r \cos(\lambda r)\right). \quad (12)$$

The intuition behind this model is the following. Suppose that  $u_t$  follows an ARMA process of form

$$u_t = \sum_{r=1}^p \phi_r u_{t-r} + \varepsilon_t - \sum_{r=1}^q \theta_r \varepsilon_{t-r},$$

where  $\varepsilon_t$  is a white noise process and all zeros of  $\phi(L)$  lying outside the unit circle and all zeros of  $\theta(L)$  lying outside or on the unit circle. Clearly, the spectral density function of this process is then

$$f(\lambda; \varphi) = \frac{\sigma^2}{2\pi} \left| \frac{1 - \sum_{r=1}^q \theta_r e^{ir\lambda}}{1 - \sum_{r=1}^p \phi_r e^{ir\lambda}} \right|^2, \quad (13)$$

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<sup>2</sup> Moving average (MA) and jointly ARMA components were also examined but the estimated coefficients corresponding to the MA structure were insignificant in all cases.

where  $\varphi$  corresponds to all the AR and MA coefficients and  $\sigma^2$  is the variance of  $\varepsilon_t$ . Bloomfield (1973) showed that the logarithm of an estimated spectral density function is often found to be a fairly well-behaved function and can thus be approximated by a truncated Fourier series. He showed that (12) approximates (13) well where  $p$  and  $q$  are of small values, which usually happens in economics. Like the stationary AR( $p$ ) model, the Bloomfield (1973) model has exponentially decaying autocorrelations and thus we can use a model like this for  $u_t$  in (11). Formulae for Newton-type iteration for estimating the  $\tau_l$  are very simple (involving no matrix inversion), updating formulae when  $m$  is increased are also simple, and we can replace  $\hat{A}$  below (8) by the population quantity

$$\sum_{l=m+1}^{\infty} l^{-2} = \frac{\pi^2}{6} - \sum_{l=1}^m l^{-2},$$

which indeed is constant with respect to the  $\tau_j$  (unlike what happens in the AR case). The Bloomfield (1973) model, confounded with fractional integration has not been very much used in previous econometric models, (though the Bloomfield model itself is a well-known model in other disciplines, e.g., Beran, 1993), and one by-product of this work is its emergence as a credible alternative to the fractional ARIMAs which have become conventional in parametric modelling of long memory.<sup>3</sup>

The test statistic reported across Table 1 is the one-sided statistic given by  $\hat{r}$  in (8). Thus, for a given  $d_0$ , significantly positive values of  $\hat{r}$  are consistent with orders of integration higher than  $d_0$ , whereas significantly negative ones imply orders of integration smaller than that hypothesized under the null. A noticeable feature observed across the table is the fact that if the disturbances are white noise, the values of  $\hat{r}$  monotonically decrease with  $d_0$ , as we should expect in view of the previous discussion since they are

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<sup>3</sup> Amongst the few empirical applications found in the literature are Gil-Alana and Robinson (1997), Velasco and Robinson (2000) and more recently Gil-Alana (2001b).

one-sided statistics. Thus, for example, we would wish that if  $H_0$  (5) is rejected with  $d = 0.75$  in favour of alternatives of form  $d > 0.75$ , an even more significant result in this direction would be obtained when  $d = 0.50$  or  $0.25$  are tested. However, we observe in the table that, if we impose AR  $u_t$ , there is a lack of this property for small values of  $d$ . This lack of monotonicity could be explained in terms of model misspecification as is argued, for example, in Gil-Alana and Robinson (1997). However, it may also be due to the fact that the AR coefficients are Yule-Walker estimates and thus, though they are smaller than one in absolute value, they can be arbitrarily close to 1. A problem then may occur in that they may be capturing the order of integration by means, for example, of a coefficient of 0.99 in case of using AR(1) disturbances. Imposing Bloomfield (1973)  $u_t$ , monotonicity is again achieved for all type of regressors.

The most noticeable feature observed across Table 1 is the fact that the only value of  $d$  where  $H_0$  (5) cannot be rejected corresponds to the unit root case, (i.e.,  $d = 1$ ), and this is obtained independently of the inclusion or not of deterministic regressors and of the different types of disturbances underlying the process. The last column of the table reports the confidence intervals of those values of  $d$  where  $H_0$  (5) cannot be rejected at the 95% significance level. We see that they are very narrow and they are all centred around the unit root case. In view of this, it seems clear that the Spanish stock market prices possesses a unit root.<sup>4</sup>

In view of the similarities observed in the results presented across Tables 1(ii) and (iii), it might also be of interest a joint test of the null hypothesis:

$$H_0 : d = d_0 \text{ and } \beta = 0 \quad (14)$$

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<sup>4</sup> Moreover, several other unit root tests based on autoregressive alternatives (such as the ones suggested by Dickey and Fuller, 1979, and Phillips and Perron, 1988) were also performed, obtaining in all cases evidence in favour of a unit root.

in (10) and (11). This possibility is not addressed by Robinson (1994) but a LM test of (14) against the alternative,

$$H_a : d \neq d_o \text{ or } \beta \neq 0 \quad (15)$$

is suggested in Gil-Alana and Robinson (1997). It was shown in that paper that the test statistic takes the form:

$$\hat{R} = \hat{r}^2 + \frac{\left( \sum_{t=1}^T \tilde{u}_t w_{2t} \right)^2}{\left[ \sum_{t=1}^T w_{2t}^2 - \left( \sum_{t=1}^T w_{1t} w_{2t} \right)^2 / \sum_{t=1}^T w_{1t}^2 \right]} \quad (16)$$

$$w_{1t} = (1 - L)^d \mathbf{1}; \quad w_{2t} = (1 - L)^d S_t; \quad \tilde{u}_t = (1 - L)^d y_t - \tilde{\beta}_1 w_{1t};$$

$$\tilde{\beta}_1 = \left( \sum_{t=1}^T w_{1t}^2 \right)^{-1} \sum_{t=1}^T w_{1t} (1 - L)^d y_t; \quad \hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^T \tilde{u}_t^2, \quad \text{and } \hat{r}^2 \text{ calculated as in (8) but using}$$

the  $\tilde{u}_t$  just defined. We can compare (16) with the upper tail of the  $\chi_2^2$  distribution. Then, rejections of (14) for a given  $d_o$ , which was non-rejected before, will give us some evidence that the linear time trend is required when modelling this series. The results for the same  $d_o$  values as in Table 1 are given in Table 2.

We see that there are non-rejection values for the three types of disturbances and, similarly to Tables 1(ii) and (iii), they correspond to the unit root case, with an asymmetry in favour of small values of  $d$ . Notice that even for  $d = 2$  the hypothesis is less strongly rejected than for small  $d$ ; this accords with the similarity in the corresponding statistics between Tables 1 (ii) and (iii). On the whole, the results seem to indicate that when the appropriate (first) differencing order is used, the time trend is unrequired and thus, a model

like (1) with white noise or weakly (Bloomfield) autocorrelated disturbances seems to be adequate to explain the Spanish stock market prices.<sup>5</sup>

Another issue is to determine if there is an underlying autocorrelated structure in the disturbances. In view of the preceding results, it seems clear that the series follows an integrated process of order 1, with an intercept, and with white noise or Bloomfield disturbances. However, and though is not reported across the paper, in case of the Bloomfield model, the estimated coefficients in the spectral density function in (12) were in both cases ( $m = 1, 2$ ), relatively close to zero, implying that if an autocorrelated structure is present in the data, its degree of dependence would be extremely weak, this being probably the reason why the tests cannot reject the null with  $d = 1$  and white noise  $u_t$ . In view of all this, we can conclude by saying that the random walk hypothesis cannot be rejected in the Spanish stock market prices using fractionally integrated techniques.

#### **4. Concluding comments**

In this article we have examined the random walk hypothesis in the Spanish stock market prices. However, instead of using classical tests based on AR alternatives, which usually have non-standard limit distribution and lack of efficiency theory, we have employed a version of Robinson's (1994) tests in which the unit root is embedded in a fractional model. The tests have standard asymptotic distribution and are the most efficient ones when directed against the appropriate alternatives. The results based on the daily structure of the IBEX 35 show that the  $I(1)$  hypothesis cannot be rejected in the Spanish stock market and though the confidence intervals include fractional degrees of integration, these intervals are generally narrow and centred around the unit root case. In addition, the

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<sup>5</sup> In case of including an intercept in the regression model (10), the coefficients were significantly different from 0 for all values of  $d$ . Note that these estimates are based on the null differenced model which is short memory under the null and, in case of  $d = 1$ , it only affects to the first observation.

underlying  $I(0)$  disturbances may incorporate an autocorrelated structure, though the results based on the Bloomfield's (1973) exponential spectral model seem to indicate that the dependence between them is extremely weak, and thus, the random walk null hypothesis cannot be rejected in the Spanish stock market.

It is also important to note that the tests of Robinson (1994) do not impose Gaussianity on the series. This is only required to show its efficiency properties, and several Monte Carlo experiments conducted by Robinson (1994) showed that the tests perform well even in non-Gaussian environments. It would perhaps be worthwhile proceeding to get point estimates of  $d$ . Note that the approach used in this paper generates simply computed diagnostics for departures from any real  $d$  and thus, it is not surprising that, when fractional hypotheses are entertained, some evidence supporting them appears, because this might happen even when the unit-root model is highly suitable. In that respect, the bulk of the hypotheses presented across the paper are rejected and the confidence intervals corresponding to the non-rejection values are extremely narrow in all cases, suggesting that the optimal local power properties of the tests may be supported by reasonable performance against non-local departures. In addition, the use of other methods for estimating and testing the fractional differencing parameter  $d$ , like the semiparametric procedures of Geweke and Porter-Hudak (1982) or Robinson (1995a,b) may be too sensitive to the choice of the bandwidth parameter number, while Robinson's (1994) parametric procedure proposed here produces simple and clear results, with strong evidence in favour of the unit root models. In any case, the fact that the results we have obtained are robust to the model chosen for the disturbances, though, suggests that these semiparametric methods would produce very similar results to ours. Other issues such as heteroscedasticity or the potential strong dependence in volatility will be addressed in future papers.

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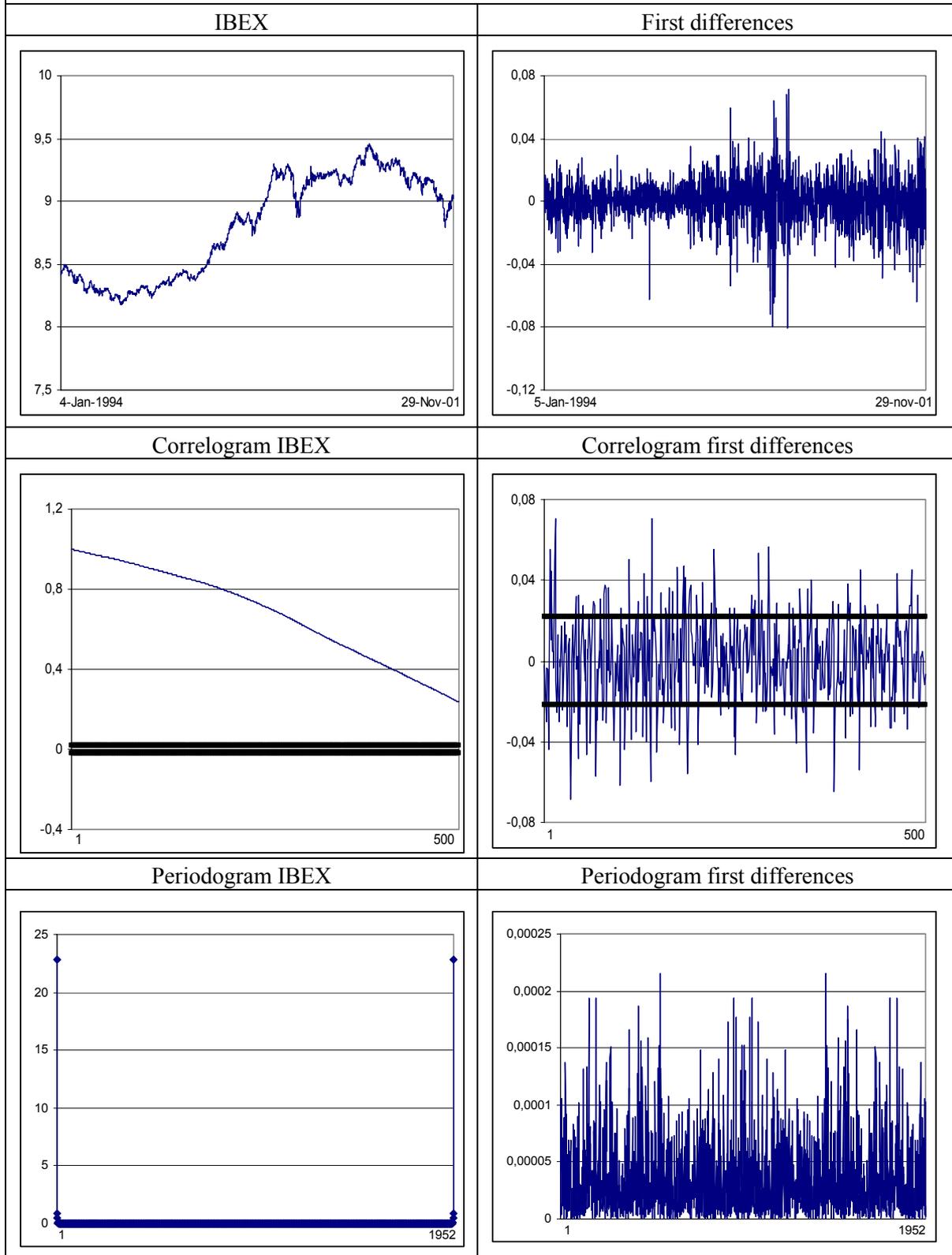
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**FIGURE 1**

Log of the Spanish stock market prices (IBEX) with their corresponding correlograms and periodograms



The large sample standard error under the null hypothesis of no autocorrelation is  $1/\sqrt{T}$  or roughly 0.022

<b>TABLE 1</b>										
Testing $H_0$ (5) in (10) and (11) with $\hat{r}$ given by (8) in the log of the Spanish stock market index										
i) $\alpha = \beta = 0$										
$u_t / d_0$	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. interval
White noise	195.94	125.02	77.37	24.01	<b>-0.17</b>	-9.31	-13.58	-16.00	-17.55	[0.97 - 1.01]
AR (1)	-3.17	-11.07	-19.76	-38.67	<b>0.10</b>	4.16	-3.13	-7.68	-10.76	***
AR (2)	-3.08	-18.18	-26.55	-47.55	<b>-0.58</b>	-13.59	5.74	2.07	-3.45	***
Bloomfield (1)	125.95	66.60	39.44	13.70	<b>-0.27</b>	-5.99	-8.88	-10.69	-12.15	[0.95 - 1.04]
Bloomfield (2)	97.84	42.03	30.83	9.40	<b>-0.36</b>	-7.37	-10.51	-11.10	-12.83	[0.99 - 1.01]
ii) $\alpha$ unknown and $\beta = 0$										
$u_t / d_0$	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	195.94	183.26	113.58	26.44	<b>-0.64</b>	-9.42	-13.49	-15.85	-17.41	[0.97 - 1.02]
AR (1)	-3.17	-11.10	-21.35	10.55	<b>-0.91</b>	-7.13	-10.35	-12.43	-13.96	***
AR (2)	-3.23	-6.74	-10.09	9.67	<b>-0.50</b>	-5.81	-8.66	-10.51	-11.91	***
Bloomfield (1)	125.71	110.07	63.07	15.39	<b>-1.05</b>	-6.92	-9.66	-11.11	-12.10	[0.94 - 1.01]
Bloomfield (2)	97.81	86.00	40.85	12.16	<b>-1.36</b>	-8.23	-10.75	-11.75	-12.09	[0.98 - 1.00]
iii) $\alpha$ and $\beta$ unknown										
$u_t / d_0$	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	164.46	150.20	95.86	25.81	<b>-0.64</b>	-9.42	-13.49	-15.85	-17.41	[0.97 - 1.02]
AR (1)	-3.50	-12.87	-19.98	26.69	<b>-0.91</b>	-7.11	-10.35	-12.43	-13.96	***
AR (2)	5.08	-14.87	-16.79	16.90	<b>-0.49</b>	-5.79	-8.66	-10.51	-11.91	***
Bloomfield (1)	100.62	86.56	51.71	14.71	<b>-1.05</b>	-6.92	-9.66	-11.11	-12.10	[0.94 - 1.01]
Bloomfield (2)	71.24	68.84	37.75	11.43	<b>-1.36</b>	-8.23	-10.74	-11.75	-12.11	[0.98 - 1.00]

In bold: The non-rejection values of the null hypothesis at the 95% significance level.

<b>TABLE 2</b>										
Testing $H_0$ (14) in (10) and (11) with $\hat{R}$ given by (16)										
$u_t / d_0$	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	
White noise	29053	24761	9876.4	666.97	<b>1.93</b>	94.17	194.34	271.44	305.11	
Bloomfield (1)	14124	9981.0	3071.8	296.54	<b>3.11</b>	49.07	109.45	126.56	170.11	
Bloomfield (2)	8750.8	5939.9	1627.5	139.66	<b>3.87</b>	69.77	119.53	148.33	164.76	

The critical value corresponding to the  $\chi_2^2$  is 5.99 at the 95% significance level.