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Is the US fiscal deficit sustainable?
A fractionally integrated and cointegrated approach

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ABSTRACT

The sustainability of fiscal deficits has received in recent years increasing attention from economists. Empirical work has concentrated on both the univariate properties of debt and the cointegration properties of public revenues and expenditures. In this paper, we examine if sustainability of the US fiscal deficit holds by means of studying the univariate properties of the difference between public revenues and expenditures. However, instead of using classical approaches based on I(1) or I(0) integration techniques, we use a methodology based on fractional processes. The results show that the public deficit in the US is an I(d) process with d slightly smaller than 1, implying that fiscal deficit is mean reverting, and thus, sustainable, though the adjustment process towards equilibrium will take a very long time.

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1. Introduction

The sustainability of fiscal deficits has been receiving increasing attention from economists, since it will determine the need for future discretionary policy actions. On the one hand, the concept of sustainability relies on the fact that governments need enough resources to ensure their ability to carry out their functions, so that its analysis helps to determine whether a current fiscal policy can be maintained in the long run with the ongoing ability to generate financial resources. On the other hand, it has clear implications for other macroeconomic variables, since a non-sustainable fiscal policy involves the risk of future interest rate rises leading to a slowdown in economic growth.

Many papers have studied the issue of fiscal policy sustainability and they have empirically tested the present value borrowing constraint. Examples of such a growing literature are Hamilton and Falvin (1986), Trehan and Walsh (1988), Kremers (1988), Wilcox (1989), Hakkio and Rush (1991), Tanner and Liu (1994), Quintos (1995), Haug (1991), Ahmed and Rogers (1995), Uctum and Wickens (1997) and Martin (2000). In most of these papers the main tools used to analyse the sustainability of budget deficit are stationarity tests for the stock of public debt and cointegration tests between public expenditures and revenues.

A sustainable fiscal policy is one that would cause the discounted value of debt to go to zero at the limit so that the present-value borrowing constraint would hold. According to the literature, this condition holds when there is a long-run cointegrating relationship between public expenditures and public revenues. However, depending on the cointegrating vector, we can define two different degrees of sustainability: ‘strong’ and ‘weak’ sustainability (see, e.g., Quintos, 1995). In this paper, we re-examine the issue of ‘strong’ sustainability of the fiscal deficit, since as is shown in the following section, the ‘weak’ sustainability condition is inconsistent with the government’s ability to market its debt in the long-run. For this purpose,

we study in detail the order of integration of real tax revenues and real government spending in the US economy, and the difference between these two variables, by means of using a procedure due to Robinson (1994a) for testing I(d) statistical models. This method has several distinguishing features compared with other procedures for testing unit and/or fractional roots. In particular, the tests have a standard null limit distribution and they are the most efficient ones when directed against the apropiate alternatives.

The paper is organized as follows. Section 2 contains some economic foundations of fiscal deficit sustainability. Section 3 presents the testing procedure of Robinson (1994a). In Section 4, the tests are applied to the US economy. Finally, Section 5 contains some concluding comments.

2. Deficit sustainability model

Following Quintos (1995) and Martin (2000), the government budget constraint is the starting point to derive the present value of budget constraint,

$$\Delta B_t = G_t - R_t, \tag{1}$$

where B_t is the real market value of federal debt, G_t is real interest rate inclusive expenditure, R_t is real tax revenues and $\Delta = (1 - L)$ is the first difference operator. The quantity in (1) thus defines the real interest inclusive deficit. Defining i_t as the real interest rate and assuming to be stationary around a mean i, and GE_t as real expenditure exclusive of interest payments, we can write down

$$G_t = GE_t + i_t B_{t-1}, \tag{2}$$

where the second term in the right hand side of (2) represents interest payments on the level of debt accumulated at the end of the previous period. Further, defining

$$EXP_t = GE_t + (i_t - i)B_{t-1}, \tag{3}$$

we can express debt as

$$B_t = (1+i)B_{t-1} + EXP_t - R_t \,, \tag{4}$$

or alternatively as

$$B_t = \left(\frac{1}{1+i}\right)\left(R_{t+1} - EXP_{t+1}\right) + \left(\frac{1}{1+i}\right)B_{t+1} \,, \tag{5}$$

and sorting out (5), via forward substitution, we obtain:

$$B_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+i}\right)^{j+1}\left(R_{t+j+1} - EXP_{t+j+1}\right) + \lim_{j\to\infty}\left(\frac{1}{1+i}\right)^{j+1} B_{t+j+1} \,. \tag{6}$$

Defining E_t (.) as an expectation conditional on information at time t, intertemporal budget balance, or deficit sustainability, holds if and only if

$$\lim_{j\to\infty} E_t\left(\frac{1}{1+i}\right)^{j+1} B_{t+j+1} = 0 \,, \tag{7}$$

since this implies that the current value of outstanding government debt is equal to the present value of future budget surpluses. In other words, the deficit is sustainable if and only if the stock of debt held by the public is expected to grow no faster on average than the mean real interest rate, which can be viewed as a proxy for the growth rate of the economy.

The econometric literature for testing this type of models has focused on cointegrated methods (see, e.g., Trehan and Walsh, 1988). Quintos (1995) proceeds by taking first differences in (6), yielding

$$\Delta B_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+i}\right)^{j+1}\left(\Delta R_{t+j+1} - \Delta EXP_{t+j+1}\right) + \lim_{j\to\infty}\left(\frac{1}{1+i}\right)^{j+1} \Delta B_{t+j+1} \,. \tag{8}$$

Sustainability is then associated with the condition

$$\lim_{j\to\infty} E_t\left(\frac{1}{1+i}\right)^{j+1} \Delta B_{t+j+1} = 0 \tag{9}$$

which, in turn, imposes conditions on the statistical properties of the interest inclusive deficit. Condition (9) can be tested by means of using stationary tests on ΔB_t, or alternatively, by testing stationarity on G_t-R_t. In other words, assuming that G_t and R_t are I(1) variables and given the cointegrating vector (1,-1), we can test if G_t and R_t are cointegrated in the sense that the cointegrating residuals are I(0) stationary. Quintos (1995) shows that defining the equation

$$R_t = a_1 + b_1 G_t + u_t \,, \tag{10}$$

the deficit is 'strongly' sustainable if the I(1) processes R_t and G_t are cointegrated and $\beta_1 = 1$, while it is 'weakly' sustainable if R_t and G_t are cointegrated and $0 < \beta_1 < 1$. However, he also argues that the weak sustainability condition has serious policy implications because a government that continues to spend more than it earns has a high risk of default and would have to offer higher interest rates to service its debt, that is, this condition is inconsistent with the government's ability to market its debt in the long run. Thus, in this paper, we will only test for the 'strong' sustainability condition looking at the univariate properties of R_t - G_t.

The above approach of testing sustainability assumes that the individual series, R_t and G_t are both I(1) nonstationary. We start our analysis by examining this hypothesis. However, instead of using classical approaches based on autoregressive (AR) models, we use new statistical methods developed by Robinson (1994a) for testing I(d) statistical models. For the purpose of the present paper, we define an I(0) process $\{u_t, t = 0, \pm 1, \ldots\}$ as a covariance stationary process, with spectral density function that is positive and finite at the zero frequency. In this context, we say that $\{x_t, t = 0, \pm 1, \ldots\}$ is I(d) if

$$(1 - L)^d \; x_t \; = \; u_t \,, \qquad t \; = \; 1, 2, \ldots, \tag{11}$$

where the polynomial in (11) can be expressed in terms of its Binomial expansion such that

$$(1 \; - \; L)^d \; = \; \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j \, L^j \; = \; 1 \; - \; d\,L \; + \; \frac{d\,(d-1)}{2} L^2 \; - \; \ldots.$$

for all real d. Clearly, the unit root case corresponds to d = 1 in (11). If d > 0, x_t is said to be long memory, so-named because of the strong association between observations widely separated in time. This type of processes was initially introduced by Granger (1980) and Hosking (1981), (though earlier work by Adenstedt, 1974, and Taqqu, 1975, shows an awareness of its representation) and they were theoretically justified in terms of aggregation of ARMA series by Robinson (1978) and Granger (1980). Similarly, Croczek-Georges and Mandelbrot (1995), Taqqu et. al. (1997) and Lippi and Zaffaroni (1999) also use aggregation to motivate long memory processes, while Parke (1999) uses a closely related discrete time error duration model. The fractional differencing parameter d plays a crucial role from both theoretical and empirical viewpoints. Thus, if d < 0.5, x_t is covariance stationary and mean-reverting, with the effect of the shocks dying away in the long run. If d ∈ [0.5, 1), x_t is not longer covariance stationary but it is still mean reverting, while d ≥ 1 implies nonstationarity and non-mean-reversion. In the following section, we present a testing procedure due to Robinson (1994a) for testing this type of models.

3. Testing for fractional integration and cointegration

There exist many different ways of testing unit-root models. Perhaps, the most common ones are the tests due to Fuller (1976), Dickey and Fuller (1979). They consider processes of form:

$$(1 - r L) y_t = m + u_t, \tag{12}$$

which, under the null hypothesis:

$$H_o : r = 1, \tag{13}$$

becomes the random walk model if u_t is white noise. The tests are based on the auxiliary regression of form:

$$(1 - L)\, y_t = p\, y_{t-1} + m + u_t, \qquad (14)$$

and the test statistic is the "t-value" corresponding to π in (14). Due to the non-standard asymptotic distributional properties of the "t-values" under the null hypothesis: H_o: $\pi = 0$, Dickey and Fuller (1979) provide the fractiles of simulated distributions which give us the critical values to be applied when testing the null against the alternatives: H_a: $\pi < 0$. The tests can be extended to allow for autocorrelated disturbances and then, the auxiliary regression (14) may be augmented by lagged values of $(1\text{-}L)y_t$, and also with other deterministic paths, like a linear time trend, though this unfortunately changes the distribution of the test statistic. Another limitation of these tests is that they lose validity if the disturbances are not white noise or AR processes. This was observed by Schwert (1987) who found that Dickey-Fuller critical values can be misleading even for large sample sizes in case of a mixed ARIMA process. He proposed the use of tests of Said and Dickey (1984), which approximate the ARMA structure by an AR. Also, Phillips (1987) and Phillips and Perron (1988) consider tests which employ a nonparametric estimate of the spectral density of u_t at the zero frequency, for example, a weighted autocovariance estimate. More recently, Kwiatkowski et al. (1992) observed that taking the null hypothesis to be I(1) rather than I(0) might itself lead to a bias in favour of the unit root hypothesis; they proposed an I(0) test which formulates the null as a zero variance in a random walk model, while Leybourne and McCabe (1994) extended the tests to the case where the null was an AR(k) process and the alternative was an integrated ARMA (ARIMA) model with AR order k and unit MA order. Their test differs from that of Kwiatkowski et al. (1992) in its treatment of autocorrelation under the null hypothesis, its critical values appearing more robust to certain forms of autocorrelation.

Conspicuous features of the above methods for testing unit roots are the non-standard nature of the null asymptotic distributions which are involved, and the absence of Pitman

efficiency theory. However, these properties are not automatic, rather depending on what might be called a degree of "smoothness" in the model across the parameters of interest, in the sense that the limit distribution do not change in an abrupt way with small changes in the parameters. Thus, they do not hold in case of unit root tests against AR alternatives such as (12). This is associated with the radically variable long run properties of AR processes around the unit root. Under (12), for $|\rho| > 1$, y_t is explosive; for $|\rho| < 1$, y_t is covariance stationary; and for $\rho = 1$, it is nonstationary but non-explosive. In view of these abrupt changes, the literature on fractional processes have become a rival class of alternatives to the AR model in case of unit-root testing. Thus, Robinson (1994a) proposes a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : d = d_o, \tag{15}$$

in a model given by

$$(1 - L)^d x_t = u_t, \tag{16}$$

where d_o can be any real number and where u_t is I(0). The x_t in (16) can be the time series we observe, though it may also be the errors in a regression model of form:

$$y_t = b' z_t + x_t, \tag{17}$$

where $\beta = (\beta_1, \ldots, \beta_k)'$ is a (kx1) vector of unknown parameters, and z_t is a (kx1) vector of deterministic regressors that may include, for example, an intercept, (e.g., $z_t \equiv 1$), or an intercept and a linear time trend, (in case of $z_t = (1,t)'$). Specifically, the test statistic proposed by Robinson (1994a) is then given by:

$$\hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{s}^2}, \tag{18}$$

where T is the sample size and

$$\hat{a} = \frac{-2p}{T}\sum_{j=1}^{T-1} y(l_j)\, g(l_j;\hat{t})^{-1} I(l_j)$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1} y(l_j)^2 - \sum_{j=1}^{T-1} y(l_j)\hat{e}(l_j)' \times \left(\sum_{j=1}^{T-1}\hat{e}(l_j)\hat{e}(l_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{e}(l_j) y(l_j)\right)$$

$$y(l_j) = \log\left|2\sin\frac{l_j}{2}\right|; \qquad \hat{e}(l_j) = \frac{\partial}{\partial t}\log g(l_j;\hat{t}); \qquad l_j = \frac{2p\ j}{T}.$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$, where

$$\hat{u}_t = (1-L)^{d_o} y_t - \hat{b}\, w_t, \qquad w_t = (1-L)^{d_o} z_t; \qquad \hat{b} = \left(\sum_{t=1}^{T} w_t\, w_t'\right)^{-1} \sum_{t=1}^{T} w_t (1-L)^{d_o} z_t,$$

and g above is a known function coming from the spectral density of u_t:

$$f(l_j;t) = \frac{s^2}{2p} g(l_j;t).$$

Note that these tests are purely parametric and therefore, they require specific modelling assumptions to be made regarding the short memory specification of u_t. Thus, for example, if u_t is white noise, $g \equiv 1$, and if u_t is AR(1) of form: $u_t = t\ u_{t-1} + e_t$, $g(l_j;t) = \left|1 - t\, e^{i l_j}\right|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are function of τ.

Robinson (1994a) showed that under certain regularity conditions,

$$\hat{r} \to_d N(0,1) \qquad as \quad T \to \infty. \tag{19}$$

Thus, we are in a classical large-sample testing situation and the conditions on u_t in (19) are far more general than Gaussianity, with a moment condition only of order 2 required. An approximate one-sided 100α%- level test of H_o (15) against the alternative: H_a: $d > d_o$ ($d < d_o$) will reject H_o (15) if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$), where the probability that a standard normal variate exceeds z_α is α. Furthermore, he shows that the above test is efficient in the Pitman sense, i.e.,

that against local alternatives of form: H_a: $d = d_o + \delta T^{-1/2}$, with $\delta \neq 0$, the limit distribution is normal with variance 1 and mean which cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Empirical applications based on this version of Robinson's (1994a) tests can be found in Gil-Alana and Robinson (1997) and Gil-Alana (2000).

4. Empirical results on US fiscal sustainability

The existence of US fiscal sustainability is examined in this section by means of using fractionally integrated and cointegrated techniques. In particular we use the methodology described in Section 3.

We produce results based on the same data set as in Martin (2000) and Quintos (1995). The data set comprises quarterly US data on real revenues (R_t) and real government expenditure (G_t), inclusive of interest paid on debt, over the period 1947(2) to 1992(3).

(Insert Figures 1 and 2 about here)

Figures 1 and 2 contain respectively plots of the original series and their first differences, along with their corresponding correlograms and periodograms. We see in Figure 1 that the original series increase both with the sample, and the nonstationary nature of the series seem to assess themselves in view of the correlograms, with values decaying very slowly, and throughout the periodograms, with a large peak around the smallest frequency. Looking at the plots based on the first differenced data, in Figure 2, we observe that they may be both now stationary, though the correlograms still show significant values even at some lags relatively far away from zero, which may be an indication that fractional orders of integration, smaller than or higher than 1, may be more appropriate than first differences.

Denoting the original series y_t, we employ throughout model (16) and (17), with z_t = (1, t)′, t ≥ 1, (0, 0)′, otherwise, i.e.,

$$y_t = a + b\,t + x_t, \qquad t = 1, 2, \ldots \tag{20}$$

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{21}$$

testing H_o (15) for values d_o = 0, (0.25), 2, and different types of disturbances. Initially, we assume that α = β = 0 a priori, (i.e., we do not include any regressors in the regression model). Then, we also consider the cases of an intercept, (α unknown and β = 0 a priori), and an intercept and a linear time trend, (α and β unknown). Thus, for example, if u_t is white noise and d_o = 1, the differences $(1 - L)y_t$ behave, for t > 1, like a random walk when β = 0, and a random walk with a drift when β ≠ 0. However, we also consider the possibility of the disturbances being weakly autocorrelated. In particular, we take AR(1), AR(2), and the Bloomfield's (1973) exponential spectral model. This is a non-parametric approach of modelling the I(0) disturbances in which u_t is exclusively specified in terms of its spectral density function, which is given by:

$$f(l\,;t) = \frac{s^2}{2p} \exp\left(2\sum_{r=1}^{m} t_r \cos(l\,r)\right). \tag{22}$$

The intuition behind this model is the following. Suppose that u_t follows an ARMA process of form:

$$u_t = \sum_{r=1}^{p} f_r\, u_{t-r} + e_t - \sum_{r=1}^{q} q_r\, e_{t-r},$$

where ε_t is a white noise process and all zeros of $\phi(L)$ lying outside the unit circle and all zeros of $\theta(L)$ lying outside or on the unit circle. Clearly, the spectral density function of this process is then

$$f(l\,;j) = \frac{s^2}{2p}\left|\frac{1-\sum_{r=1}^{q} q_r\, e^{irl}}{1-\sum_{r=1}^{p} f_r\, e^{irl}}\right|^2, \tag{23}$$

where φ corresponds to all the AR and MA coefficients and σ^2 is the variance of ε_t. Bloomfield (1973) showed that the logarithm of an estimated spectral density function is often found to be a fairly well-behaved function and can thus be approximated by a truncated Fourier series. He showed that (22) approximates (23) well where p and q are of small values, which usually happens in economics. Like the stationary AR(p) model, the Bloomfiled (1973) model has exponentially decaying autocorrelations and thus we can use a model like this for u_t in (16). Formulae for Newton-type iteration for estimating the τ_l are very simple (involving no matrix inversion), updating formulae when m is increased are also simple, and we can replace $\hat{A}$ below (18) by the population quantity

$$\sum_{l=m+1}^{\infty} l^{-2} = \frac{p^2}{6} - \sum_{l=1}^{m} l^{-2},$$

which indeed is constant with respect to the τ_j (unlike what happens in the AR case). The Bloomfield (1973) model, confounded with fractional integration has not been very much used in previous econometric models, (though the Bloomfield model itself is a well-known model in other disciplines, e.g., Beran, 1993), and one by-product of this work is its

emergence as a credible alternative to the fractional ARIMAs which have become conventional in parametric modelling of long memory.[1]

(Insert Tables 1 and 2 about here)

The test statistic reported across Table 1 (and also in Tables 2 and 3) is the one-sided statistic given by $\hat{r}$ in (18). Thus, for a given d_o, significantly positive values of $\hat{r}$ are consistent with orders of integration higher than d_o, whereas significantly negative ones imply orders of integration smaller than that hypothesized under the null. A noticeable feature observed across the table is the fact that if the disturbances are white noise, the values of $\hat{r}$ monotonically decrease with d_o, as we should expect in view of the previous discussion since they are one-sided statistics. Thus, for example, we would wish that if H_o (15) is rejected with d = 0.75 in favour of alternatives of form d > 0.75, an even more significant result in this direction should be obtained when d = 0.50 or 0.25 are tested. However, we observe in the table that, if we impose AR u_t, there is a lack of this property for small values of d. This lack of monotonicity could be explained in terms of model misspecification as is argued, for example, in Gil-Alana and Robinson (1997). However, it may also be due to the fact that the AR coefficients are Yule-Walker estimates and thus, though they are smaller than one in absolute value, they can be arbitrarily close to 1. A problem then may occur in that they may be capturing the order of integration by means, for example, of a coefficient of 0.99 in case of using AR(1) disturbances. Imposing Bloomfield (1973) u_t, monotonicity is again achieved for all type of regressors.

Starting with the real revenues, in Table 1, we see that if u_t is white noise, the only non-rejection value of d takes place when d_o = 1, and this happens for the three cases of no regressors, an intercept, and an intercept and a linear time trend. Similarly, allowing weakly

[1] Amongst the few empirical applications found in the literature are Gil-Alana and Robinson (1997).

autocorrelated disturbances, the unit root cannot be rejected, and if they follow the Bloomfield's (1973) exponential spectral model, there are also non-rejections with d = 0.75. The last column of the table reports the confidence intervals of those values of d_o where H_o (15) cannot be rejected at the 95% significance level. We see that they are generally large and include the unit root in all cases. Table 2 reports the results for the government expenditure and they are similar to those given in Table 1. Thus, the unit root null hypothesis cannot be rejected for any type of disturbances and independently of the inclusion or not of deterministic components in (20). The only difference observed in this table, compared with Table 1 occurs in case of the Bloomfield disturbances. Thus, if we do not include regressors, H_o (15) cannot be rejected if d = 1 and 1.25; with an intercept, the only non-rejection value takes place at d = 1; and including an intercept and a linear trend, H_o (15) cannot be rejected for d = 0.75 and 1. In view of all this, it seems clear that both individual series posses a unit root. Moreover, several other unit root tests based on autoregressive alternatives (such as the ones suggested by Dickey and Fuller, 1979, and Phillips and Perron, 1988) were also performed on these series, obtaining in all cases evidence in favour of a unit root.

Next, we examine the order of integration of the difference between the revenues and the government expenditures. In doing so, we can determine if fiscal deficits are stationary or nonstationary, and more importantly, if there exists mean reversion in its behaviour.

(Insert Figure 3 about here)

Figure 3 contains the plots of the differenced series ($R_t - G_t$), and the first differences, again with their corresponding correlograms and periodograms. These plots seem to indicate that the original series is nonstationary. However, the correlogram of the first differences show significant values and the periodogram has a value close to zero at the zero frequency, suggesting that the series may be now overdifferenced.

(Insert Table 3 about here)

Table 3 reports values of the same statistic as in Tables 1 and 2 but based on the $R_t - G_t$ series. Once more, it was observed a lack of monotonicity in the value of $\hat{r}$ with respect to d in case of AR disturbances. Thus, we only report across the table, the results based on white noise and Bloomfield disturbances. Starting with white noise u_t, we see that the only non-rejection value takes place with d = 1, implying that, in this context of white noise disturbances, the order of integration is similar to that of the individual series and thus, there is no cointegration for a given vector (1, -1). However, a very different picture is obtained in case of autocorrelated disturbances. If u_t is Bloomfield (1), H_o (15) cannot be rejected with d = 1 but also with d = 0.75, and the confidence intervals widely oscillates between 0.54 and 1.20. Imposing Bloomfield (2) disturbances, the degree of integration seems to be smaller and the confidence intervals range between 0.18 and 0.68. Therefore, the order of integration of the series substantially vary depending on if the disturbances are or not autocorrelated, and we find some evidence of fractional cointegration if u_t is autocorrelated.

In view of the mixed conclusions obtained in Table 3, it might also be of interest to estimate the order of integration of the series by means of using semiparametric procedures. In doing so, we do not have to take care about the underlying I(0) disturbances. In other words, we just consider a process like (16) with I(0) disturbances. We propose in this article the use of the Quasi Maximum Likelihood Estimate (QMLE) of Robinson (1995a), which we are now to describe.

It is basically a local "Whittle estimate" in the frequency domain, considering a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$d_1 = \arg\min_d \left(\log \overline{C(d)} - 2\, d\, \frac{1}{m} \sum_{j=1}^{m} \log I_j \right), \qquad (24)$$

$$for \;\; d \in (-1/2\,,\; 1/2)\,; \quad \overline{C(d)} \;=\; \frac{1}{m}\sum_{j=1}^{m} I(l_j)\; l_j^{\,2d}, \quad l_j = \frac{2\,p\;j}{T}, \;\; \frac{m}{T} \to 0.$$

Under finiteness of the fourth moment and other conditions, Robinson (1995a) proves the asymptotic normality of this estimate, while Lobato (1999) extended it to the multivariate case.

There also exist several other semiparametric procedures for estimating the fractional differencing parameter, for example, the log-periodogram regression estimate (LPE), initially proposed by Geweke and Porter-Hudak (1983), and modified later by Künsch (1986) and Robinson (1995b), and the averaged periodogram estimate (APE) of Robinson (1994b). However, we have decided to use in this article the QMLE, firstly because of its computational simplicity and also, because several Monte Carlo experiments carried out, for example, by Gil-Alana (2001), showed that, in finite samples, the QMLE has better statistical properties compared with the other procedures.

(Insert Figure 4 about here)

Since the series appear to be nonstationary, we carry out the analysis based on the first differenced data, adding then 1 to the estimated values of d to obtain the proper order of integration. The results of d_1 in (24) for the whole range of values of m are displayed in the upper part of Figure 4. We see that the estimates are very sensitive to the choice of m, especially if m is small. The second plot of the figure displays the results when m is constrained between 50 and 150. We see that the values oscillate around 0, and taking a shorter interval for m, (from 75 and 125), the values are in most cases slightly below 0, implying that the order of integration of the original series is 1 or slightly smaller than 1. This is consistent with the results given in Table 3, implying that there is a small degree of mean reversion in its behaviour.

5. Concluding comments

In this article we have examined the US fiscal deficit by means of using fractionally integrated techniques. Using a version of the tests of Robinson (1994a) for testing unit and fractional roots, the results show that the US real revenues and government expenditure are both integrated of order 1 variables, which is in line with most of empirical work of deficit models. Looking at the results based on the differences between both variables, the results are mixed. Thus, if the underlying disturbances are white noise, the unit root null hypothesis cannot be rejected, implying that there is no cointegration between revenues and government expenditure for a given cointegrating vector (1, -1). From an economic point of view, this result suggests that the US fiscal deficit is not sustainable, at least in its strong sense, and is in line with the results obtained by Hakkio and Rush (1991) or Quintos (1995), who find evidence of sustainability only for a sub-sample ending in 1980. However, imposing autocorrelated (Bloomfield) disturbances, the order of integration appears to be higher than 0 but smaller than 1, suggesting that a certain degree of fractional cointegration exists between both variables. Using a semiparametric procedure for estimating the fractional differencing parameter d, (QMLE, Robinson, 1995a) on the differenced series, the results suggest that d is slightly smaller than 1, implying that fiscal deficits are mean reverting, though the adjustment process towards equilibrium will take a very long time. This finding implies a long run equilibrium relationship between public revenues and public expenses, which we interpret as evidence of 'strong' sustainability of the fiscal policy, in line with the studies by Hamilton and Flavin (1986), Trehan and Walsh (1988) and Martin (2000).

We should mention that the parametric approach of Robinson (1994a) used in this paper generates simply computed diagnostics for departures from any real d and thus, it is not

surprising that, when fractional hypotheses are entertained, some evidence supporting them appears, because this might happen even when the unit-root model is highly suitable. In that respect, the bulk of the hypotheses presented across Tables 1-3 are rejected, suggesting that the optimal properties of the tests may be supported by reasonable performance against non-local departures. In addition, the use of other methods for estimating and testing the fractional differencing parameter d, like the QMLE of Robinson (1995a) produces similar results in terms of a small degree of mean reversion in the US deficit.

The procedures implemented in this article can also be used to estimate and test the order of integration on the residuals from the cointegrating regression in (10). In other words, they can be performed in a similar way as in Engle and Granger (1987), testing the null hypothesis of no cointegration against the alternative of (fractional) cointegration. However, a problem with this procedure appears in that the residuals used are not actually observed but obtained from minimizing the residual variance of the cointegrating regression and, in finite samples, the residual series might be biased towards stationarity. Thus, we would expect the null to be rejected more often than suggested by the normal size of Robinson's (1994a) tests. Therefore, the empirical size of these tests for cointegration in finite samples has to be obtained using a simulation approach. In that respect, we have preferred to use the procedures based on observed data and test for cointegration, imposing the cointegrating vector (1, -1). The results seem to indicate that the US deficit is nonstationary but with a small component of mean reversion, with shocks affecting to the series dissappearing in the long run.

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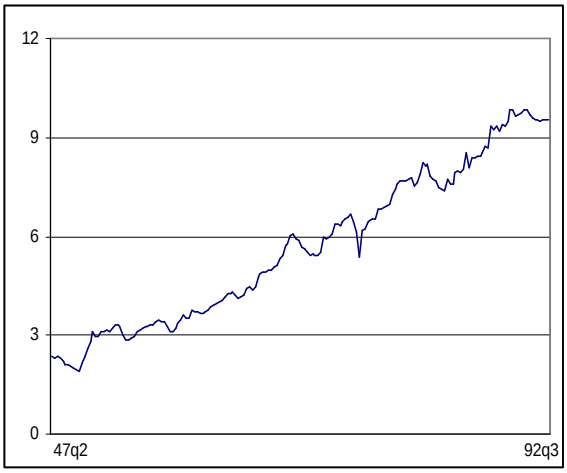
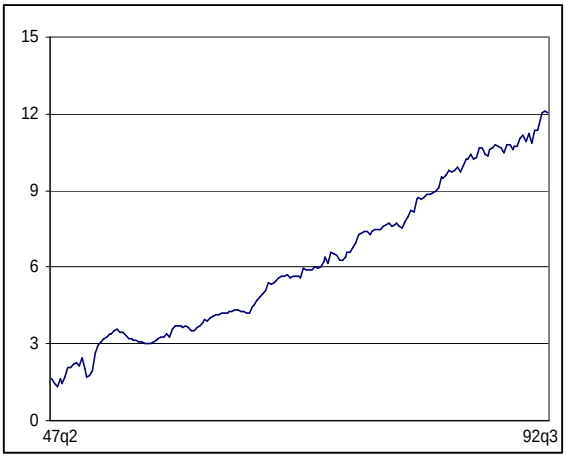
FIGURE 1

US real revenues and government expenditure, with their corresponding correlograms and periodograms

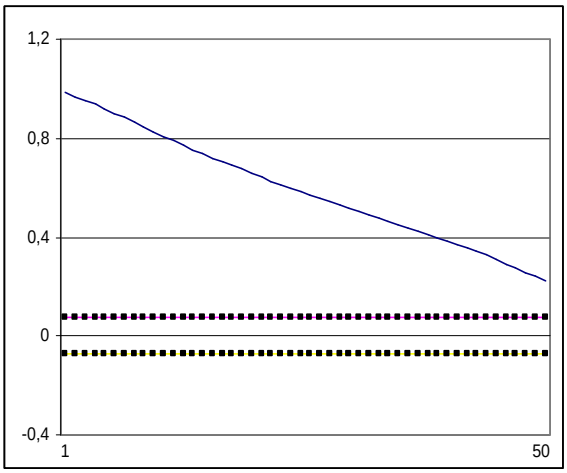
Real revenues (R_t)



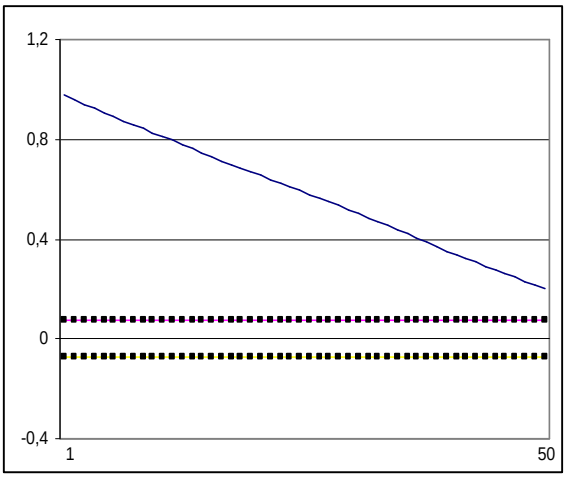
Real government expenditure (G_t)



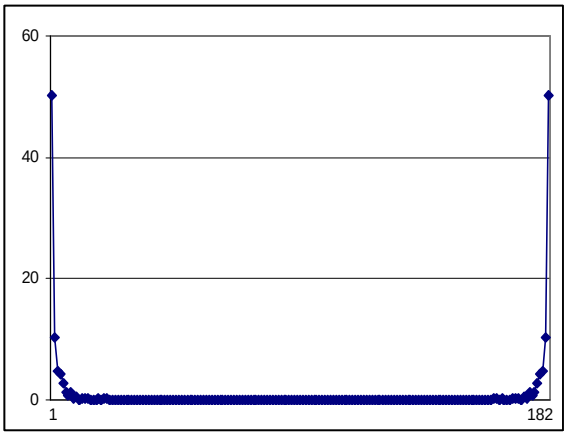
Correlogram R_t



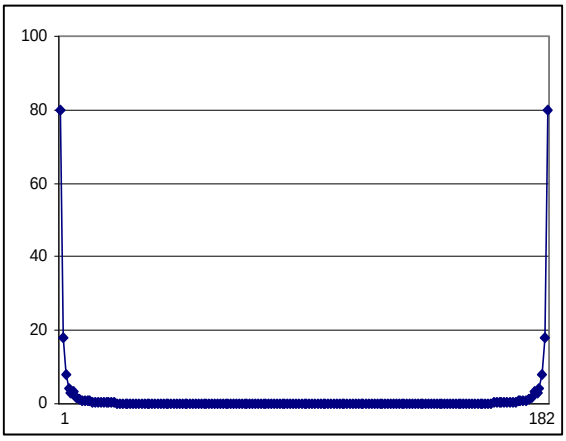
Correlogram G_t



Periodogram R_t



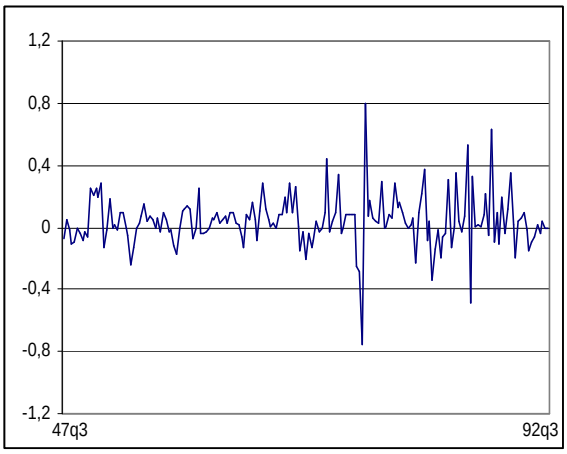
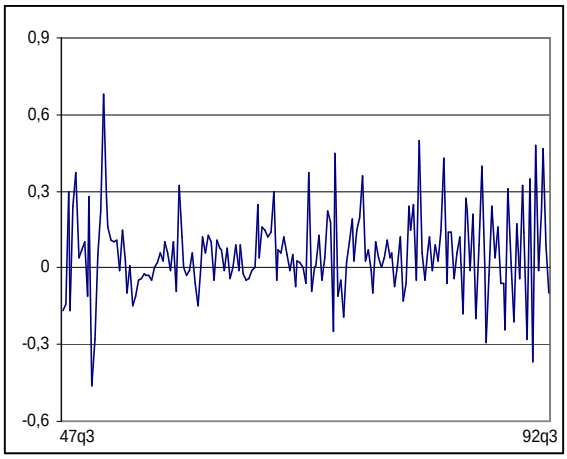
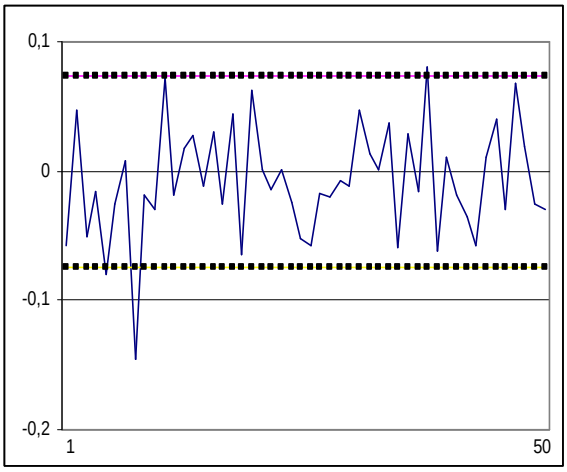
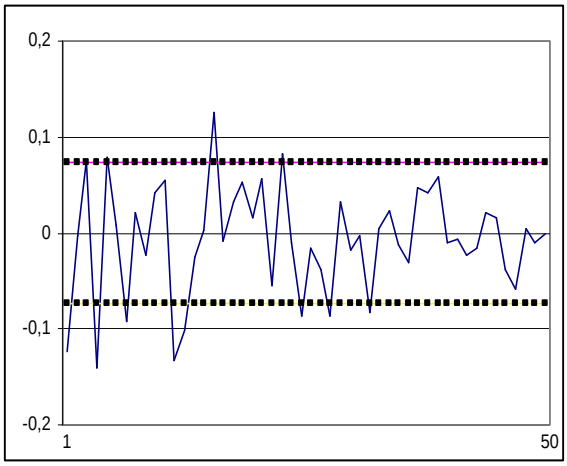
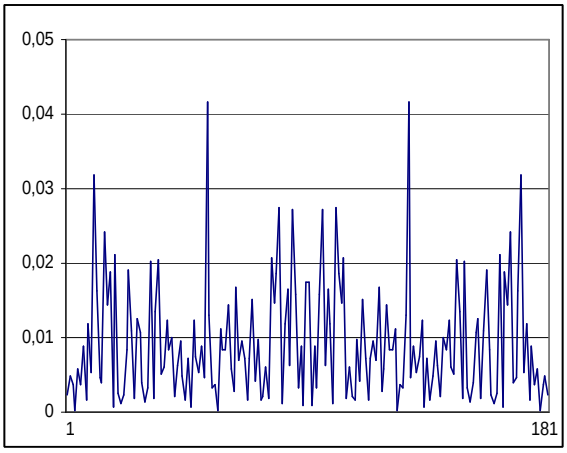
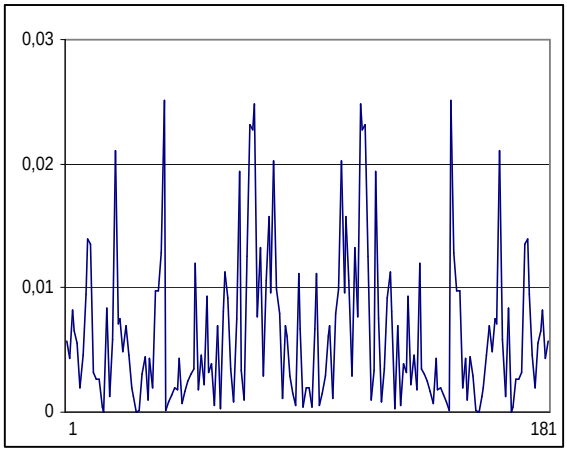
Periodogram G_t



Note: The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.074.

FIGURE 2

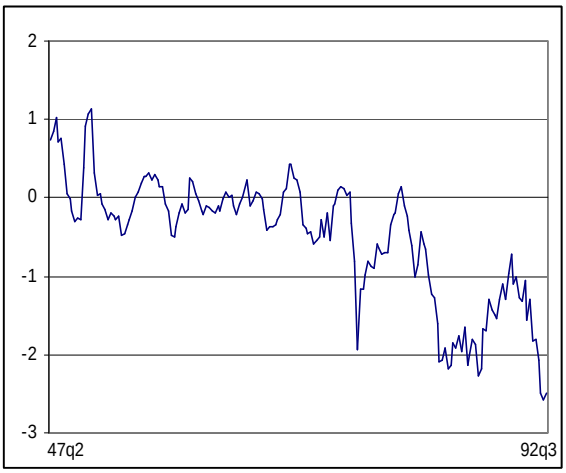
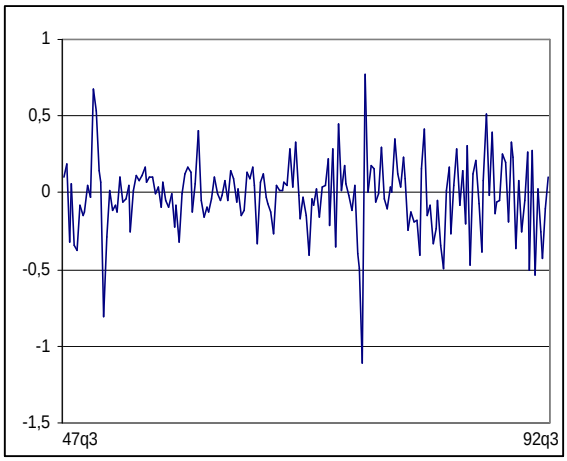
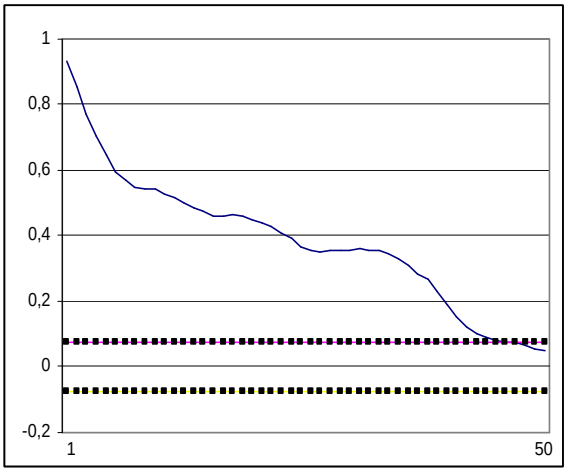
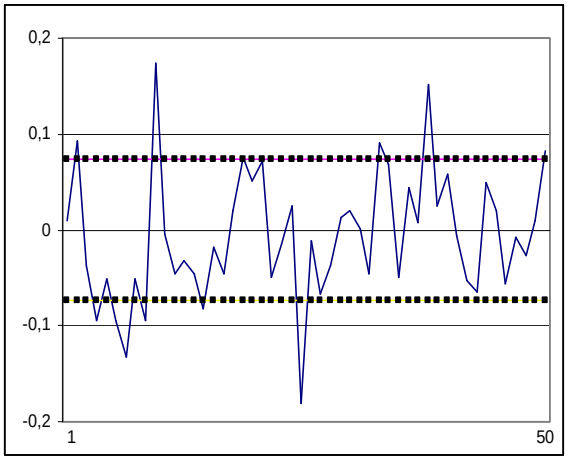
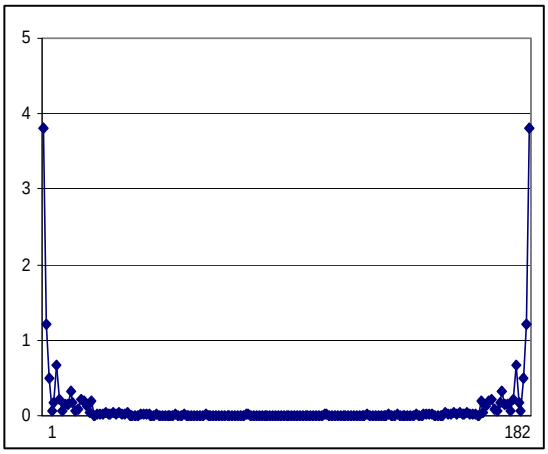
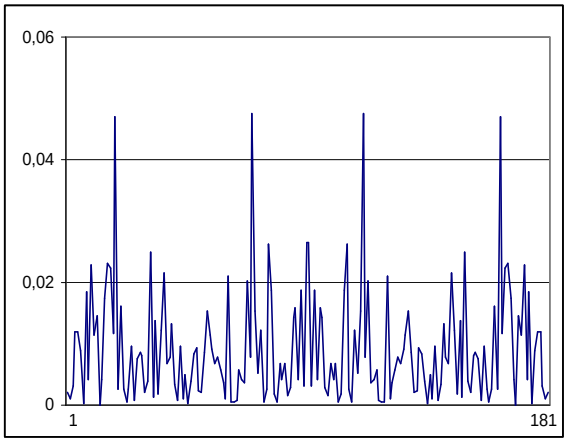
First differenced series, with their corresponding correlograms and periodograms

$(1 - L)\ R_t$	$(1 - L)\ G_t$
	
Correlogram $(1 - L)\ R_t$	Correlogram $(1 - L)\ G_t$
	
Periodogram $(1 - L)\ R_t$	Periodogram $(1 - L)\ G_t$
	

Note: The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.074.

FIGURE 3

Plots of Rt – Gt and its first differences, with their corresponding correlograms and periodograms

$(R_t - G_t)$	$(1 - L)\ (R_t - G_t)$
	
Correlogram $(R_t - G_t)$	Correlogram $(1 - L)\ (R_t - G_t)$
	
Periodogram $(R_t - G_t)$	Periodogram $(1 - L)\ (R_t - G_t)$
	

Note: The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.074.

TABLE 1										
Testing H_o (15) in (20) and (21) with $\hat{r}$ given by (18) in R_t										
i) $\alpha = \beta = 0$										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. interval
White noise	33.10	32.35	12.97	2.25	**-1.34**	-3.43	-4.68	-5.47	-5.99	[0.79 - 1.02]
AR (1)	0.08	-1.50	-6.41	-3.16	**-1.15**	-1.26	-2.17	-3.13	-3.94	***
AR (2)	0.58	1.36	-2.08	-2.69	**-1.56**	-0.47	-0.34	-0.92	-1.76	***
Bloomfield (1)	19.17	18.34	6.57	**0.39**	**-1.47**	-2.34	-3.00	-3.60	-3.90	[0.66 – 1.05]
Bloomfield (2)	8.33	8.24	4.62	**0.70**	**-0.81**	-1.68	-2.64	-2.70	-2.77	[0.60 – 1.03]
ii) α unknown and $\beta = 0$										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	33.10	29.59	21.65	5.65	**-0.75**	-3.23	-4.66	-5.53	-6.07	[0.87 - 1.07]
AR (1)	0.08	-1.72	-3.55	0.83	**-0.53**	-1.66	-2.74	-3.61	-4.28	***
AR (2)	0.58	-0.73	-2.19	-0.11	**-1.14**	-1.71	-2.30	-2.88	-3.38	***
Bloomfield (1)	18.17	15.34	10.83	3.02	**-0.53**	-1.81	-2.73	-3.29	-3.82	[0.83 – 1.21]
Bloomfield (2)	12.33	11.12	4.37	2.07	**1.03**	-1.90	-2.06	-2.37	-4.86	[0.81 – 1.04]
iii) α and β unknown										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	22.19	14.69	8.28	3.07	**-0.77**	-3.22	-4.66	-5.53	-6.07	[0.84 - 1.07]
AR (1)	-0.16	-5.73	-3.24	1.46	**-0.52**	-1.67	-2.73	-3.61	-4.28	***
AR (2)	0.08	-1.97	-2.77	-0.03	**-1.13**	-1.71	-2.30	-2.88	-3.38	***
Bloomfield (1)	10.18	5.54	2.82	**0.85**	**-0.58**	-1.76	-2.73	-3.28	-3.81	[0.67 – 1.21]
Bloomfield (2)	6.32	3.22	2.26	**1.46**	**-1.06**	-1.94	-2.05	-2.36	-3.83	[0.68 – 1.05]

Note: In bold: The non-rejection values of the null hypothesis at the 95% significance level.

TABLE 2										
Testing H_o (15) in (20) and (21) with $\hat{r}$ given by (18) in G_t										
i) $\alpha = \beta = 0$										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	33.64	32.67	22.27	4.29	**-1.53**	-4.03	-5.24	-5.87	-6.22	[0.84 – 1.01]
AR (1)	-0.17	-0.10	-1.40	2.17	**-0.09**	-2.06	-3.60	-4.57	-5.16	***
AR (2)	0.99	3.40	-0.88	0.29	**0.33**	-0.07	-0.87	-1.71	-2.34	***
Bloomfield	17.98	17.69	11.90	3.64	**0.32**	**-1.43**	-2.76	-3.44	-3.94	[0.87 – 1.27]
Bloomfield (2)	9.46	9.29	6.47	3.78	**-0.97**	**-1.46**	-2.02	-2.76	-3.51	[0.86 – 1.26]
ii) α unknown and $\beta = 0$										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	33.64	28.71	20.91	5.84	**-1.37**	-3.62	-4.84	-5.59	-6.07	[0.86 – 1.02]
AR (1)	-0.17	0.06	-1.40	2.76	**-0.72**	-2.03	-3.04	-3.85	-4.49	***
AR (2)	0.99	-0.45	-1.52	-0.06	**-1.02**	-1.50	-1.92	-2.34	-2.72	***
Bloomfield (1)	17.38	14.36	10.03	3.03	**-0.91**	-2.14	-2.80	-3.34	-3.71	[0.82 – 1.14]
Bloomfield (2)	9.46	9.17	5.13	2.33	**-1.76**	-2.73	-2.91	-3.82	-4.72	[0.81 – 1.11]
iii) α and β unknown										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	31.59	23.16	11.59	3.00	**-1.39**	-3.61	-4.84	-5.60	-6.10	[0.82 – 1.02]
AR (1)	-0.48	-5.54	-1.06	2.93	**-0.72**	-2.02	-3.03	-3.88	-4.58	***
AR (2)	0.86	-2.03	-1.90	-0.08	**-1.02**	-1.50	-1.92	-2.39	-2.88	***
Bloomfield (1)	16.53	11.06	5.70	**1.50**	**-0.77**	-2.11	-2.80	-3.38	-3.83	[0.74 – 1.13]
Bloomfield (2)	10.62	5.23	2.57	**0.47**	**-1.83**	-2.69	-2.92	-3.06	-3.39	[0.68 – 1.11]

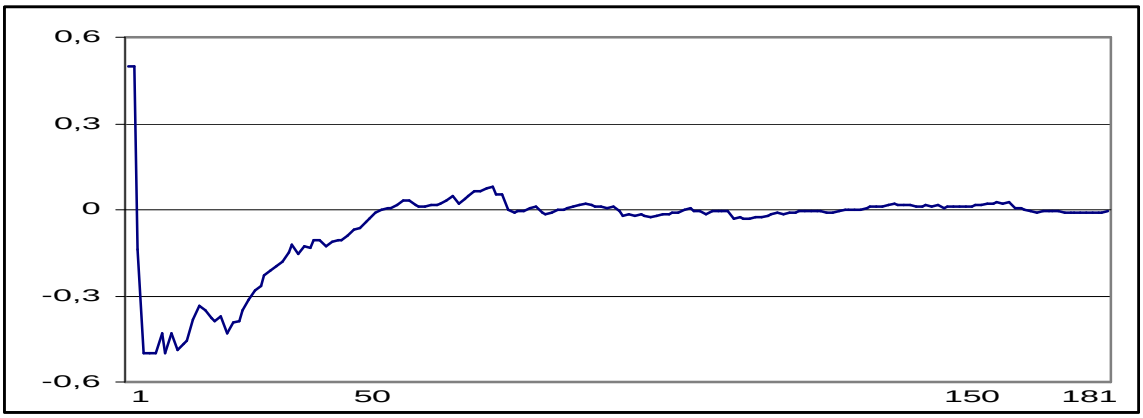
Note: In bold: The non-rejection values of the null hypothesis at the 95% significance level

TABLE 3										
Testing H_o (15) in (20) and (21) with $\hat{r}$ given by (18) in $(R_t - G_t)$										
i) $\alpha = \beta = 0$										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	26.05	16.66	7.90	2.76	**-0.50**	-2.74	-4.22	-5.19	-5.84	[0.83 – 1.11]
Bloomfield (1)	12.37	6.67	1.88	**-0.21**	**-1.24**	-2.16	-2.69	-3.25	-3.61	[0.54 – 1.10]
Bloomfield (2)	5.97	1.93	**-1.26**	-2.23	-3.05	-3.63	-3.71	-3.80	-3.62	[0.30 – 0.57]
ii) α unknown and $\beta = 0$										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	26.05	16.98	7.88	3.04	**-0.42**	-2.83	-4.34	-5.29	-5.90	[0.85 – 1.11]
Bloomfield (1)	12.37	6.67	1.85	**0.25**	**-0.93**	-1.97	-2.70	-3.21	-3.60	[0.54 – 1.18]
Bloomfield (2)	5.97	2.20	**-1.30**	**-1.24**	-1.88	-2.48	-3.06	-3.10	-4.08	[0.31 – 0.67]
iii) α and β unknown										
u_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	Conf. Interval
White noise	19.14	12.97	7.66	3.13	**-0.42**	-2.83	-4.34	-5.30	-5.91	[0.85 – 1.11]
Bloomfield (1)	7.66	4.11	1.89	**0.34**	**-0.93**	-1.98	-2.70	-3.23	-3.55	[0.54 – 1.19]
Bloomfield (2)	2.82	**0.35**	**-0.66**	-2.11	-1.88	-2.49	-3.06	-3.12	-4.20	[0.18 – 0.68]

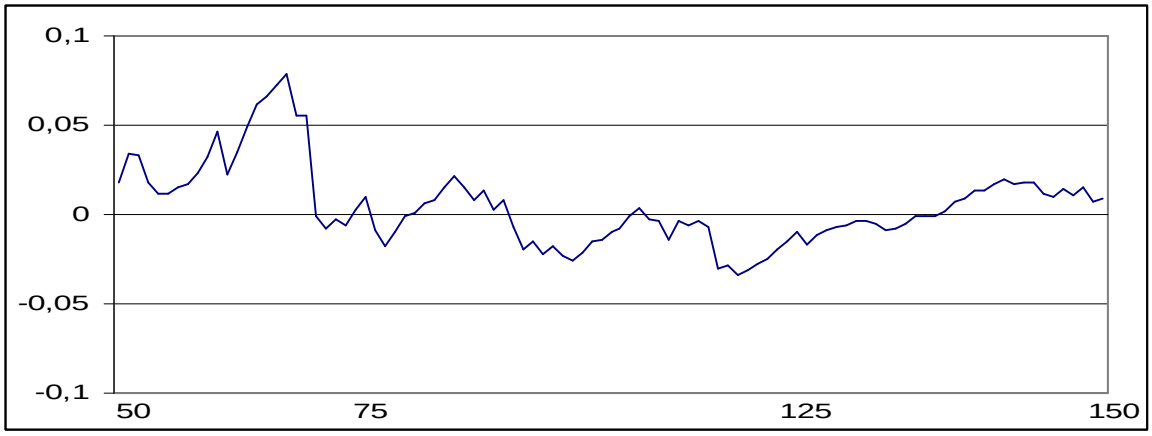
Note: In bold: The non-rejection values of the null hypothesis at the 95% significance level.

FIGURE 4

QMLE(Robinson, 1995a) in (1 – L) (R_t – G_t)



QMLE (Robinson, 1995a) for a shorter range of values of m



QMLE (Robinson, 1995a) for an even shorter range of values of m