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Technology Shocks and Hours Worked: A Fractional Integration Perspective

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ABSTRACT
Previous research has found that the response of hours worked to a technology shock crucially depends on whether the variable hours is assumed to be an I(0) or an I(1) variable ex-ante. In this paper we employ a multivariate fractionally integrated model which allows us to determine simultaneously the order of integration of hours worked and the response of hours to a technology shock. We find that hours fall on impact in response to a positive technology shock.

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1 Introduction

What is the effect of a technology shock on the number of hours worked by the private sector at business cycle frequencies? This question lies at the heart of modern Macroeconomics. The reason is that technological innovation has received widespread attention among academics as a source of dynamics for the aggregate economy and employment. At the same time, full employment remains the main goal for policy makers. The question which naturally arises in this context is then: How compatible are these two, at least in the short-run? A host of macro models have tackled this issue from both theoretical and empirical perspectives, but no consensus has emerged in the literature yet. This paper uses fractional integration techniques to give an answer to this central question in Macroeconomics.

Authors disagree on the empirical implications of a technology shock on hours worked per capita. Galí (1999) ignited an empirical literature on the issue when he contradicted the tenets of the real business cycle (RBC) theory, whereby technology shocks are key for business cycle dynamics. Galí (1999) and later on Neville and Ramey (2004) and Galí and Rabanal (2004) (henceforth GR) showed not only that technology shocks were unimportant for business cycle fluctuations but that, contrary to the implications of RBC models, hours worked declined in response to a technology shock. Galí's results have recently been challenged by Christiano, Eichenbaum, and Vigfusson (2003) (henceforth CEV) and Fisher (2002). In a very similar empirical framework to that of Galí, these authors find that hours actually increase after a technology shock. The crucial difference between both sets of studies is that the latter authors, such as CEV, treat the variable hours as stationary, whereas the former authors, such as GR, treat it as a non-stationary variable. From this perspective, the main issue which remains to be determined is the exact order of integration of hours worked per capita: Is it one or zero?

The main contribution of the present paper is to show that there is an alternative way to resolve the technology-hours issue without assuming a given order of integration for standard measures of hours ex-ante. We derive a simple method in a fractional integration framework, which lets the data determine simultaneously the response of hours to a technology shock and the order of integration of hours worked. This method presents three advantages with respect to previous approaches. First, the fractional integration approach allows to discern the order of integration of a given variable without restricting the econometrician to choose between one and zero. The order of integration could be zero, a fraction of one, one or it could even be above one. Second, our approach is agnostic with respect to the level of integration of the variables before including them in a vector autoregressive (VAR) framework. As a result, pre-tests on the orders of integration of the variables are not required. Using an asymptotic local-to-unity econometric approach, Pesavento and Rossi (2005) also propose an econometric agnostic method and find that a positive productivity shock has a negative impact effect on hours. In their framework, the researcher does not have to choose between levels and first differences in hours worked. Our approach is essentially different to theirs in that we allow the order of integration to be any real number.

Third, there is no disagreement between the impulse responses of the variables in levels or first differences in our approach, as the responses in first differences are exactly the same as those implied by the variables in levels by construction. Moreover, the multivariate fractionally integrated model employed in this paper permits us to identify the structural impulse response functions in a similar way to the classic VAR systems, either in levels or differences, with the additional interaction of the binomial expansions implied by the fractional polynomials involved in the model.

For all our data specifications, we find that hours worked decline on impact in response

to a technology shock. We also find that the orders of integration of hours worked identified by the more general fractionally integrated multivariate systems are uniformly lower than their univariate counterparts. Whereas all the univariate frameworks and very stylized multivariate models point at orders of integration of hours close to 1 or even larger, multivariate models which allow for richer and more realistic dynamics identify orders of integration lower than 1. Finally, our multivariate model implies statistically different orders of integration for hours worked across data specifications. While the variable used by GR –non-farm business hours worked per capita– has an order of integration of 0.67, the order of integration of the hours variable used by CEV –total business hours worked per capita– is slightly above zero.

Section 2 revisits the controversial issue in hand, the divergence between the responses of hours worked to a technology shock depending on the order of integration of hours worked. Section 3 develops our econometric framework intended to identify simultaneously the order of integration of the macro variables and the impulse responses to the structural shocks. Section 4 performs univariate tests for the order of integration of productivity and hours from a fractionally integrated perspective. Section 5 employs the multivariate fractionally integrated model derived in section 3 to determine the response of hours worked to a technology shock across data specifications. Section 6 concludes.

2 The Controversy

In this section we revisit the empirical evidence regarding the effect of a technology shock on hours worked. We first describe the data used throughout the paper. Then we report the impulse responses for both Galí and CEV's specifications and comment on the differences across responses.

Both GR and CEV work with quarterly data, which is commonplace in the business cycle literature. While GR use productivity and hours data from the non-farm business sector in his bivariate VARs, CEV use data from all businesses, including farming activities. We perform our analysis throughout the paper with both datasets in order to uncover potential discrepancies across data specifications. Both the non-farm business data and total business data were collected from the Federal Reserve Bank of St. Louis database (FRED). Non-farm business sector productivity is measured as output per hour of all persons (OPHNFB is the ID of the series). Non-farm business hours are computed as the ratio between the non-farm business sector hours of all persons (HOANBS) and the civilian non-institutional population over the age of 16 (CNP16OV). Total business productivity is measured as the output per hour of all persons (OPHPBS) and total business hours per capita are measured as the business hours of all persons (HOABS) divided by the civilian non-institutional population over the age of 16 (CNP16OV). We apply natural logarithms to the resulting productivity and hours series. Our dataset runs from the first quarter of 1948 to the fourth quarter of 2004. We compared our total business productivity and hours per capita series with those employed by CEV.[1] We compared the data and the differences between our total business series and theirs were indeed minimal. Moreover, the impulse responses were essentially the same, despite of the fact that their database ends on the fourth quarter of 2001.

All throughout the paper we work with bivariate VARs, since, as Galí (1999) and CEV show, introducing additional variables to the vector autoregressive systems does not change qualitatively the direction of the key impulse responses. Our empirical framework is similar to that of GR and CEV. This framework is based upon the existence of an infinite moving average representation for the first differences of the productivity ($\triangle x_t$)

[1]We are very grateful Elena Pesavento and Barbara Rossi for kindly providing the data. They, in turn, received the data directly from CEV.

and the hours series ($\triangle^i n_t$), where $i = 0$ corresponds to the CEV specification of hours in levels, and $i = 1$ corresponds to the GR specification with the first difference of hours. In matrix notation:

$$\begin{bmatrix} \triangle x_t \\ \triangle^i n_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^n \end{bmatrix} \tag{1}$$

where the $C^{ik}(L)$ $(i, k = 1, 2)$ elements are polynomials of infinite order dependent on the lag operator L. ε_t^x and ε_t^n are the technology and hours $i.i.d.$ shocks, respectively. In order to recover the structural macro shocks, we first estimate bivariate VAR systems. The order of the VAR is chosen so as to minimize the Schwarz information criterion. In all cases the order chosen was 2. With the estimates of the bivariate VAR(2), we obtain the infinite joint moving average representation of the first differences in productivity and of hours worked as in (1). We then apply the Blanchard and Quah (1989) (BQ) technique to identify the structural shocks. Following both GR and BQ, the identification assumption is that a shock to the hours worked does not affect productivity in the long-run, i.e. that $C^{12}(1) = 0$. This identification strategy is implemented by means of a standard Choleski decomposition.

Figure 1 displays in two panels the impulse responses of hours worked to a technology shock for the GR and CEV's data specifications, respectively. The size of the technology shock is normalized to one. In both panels we compare the responses treating hours both as stationary and as a unit root. Both figures confirm the results reported in the literature. When the hours variable is treated as a unit root, it decreases after a technology shock. Then hours increase and start becoming positive in the third quarter. After several quarters, the first difference of hours reverts to their steady state value. When hours are treated as stationary, a different picture emerges: Hours increase after

a technology shock and display a persistent hump-shaped trajectory, with a slow decay to the steady-state value.

Within this framework, the key issue which remains to be elucidated is then the order of integration of the variable hours worked per capita. Standard unit root tests, such as Dickey and Fuller (1979) (ADF), Phillips and Perron (1988) (PP) or Kwiatkowski, Phillips, Schmidt, and Shin (1992) (KPSS) are unable to reject the hypothesis of a unit root for the level of the series, but cannot reject that the series is stationary in first differences (see both GR and CEV). While this result is robust across data specifications, it is well-known that the power of these tests is small under meaningful alternatives. Diebold and Rudebusch (1991), Hassler and Wolters (1994) and Lee and Schmidt (1996), among others, show that standard unit-root tests have extremely low power if the alternatives are close to the unit-root circle, but also if they are of a fractional form.

In this paper we circumvent the problem of pre-testing the order of integration of the series object of study. Instead of testing for the order of integration of hours before computing the impulse response of hours to a technology shock, we perform both tasks simultaneously. To do so, the next section develops a simple method to estimate empirical macroeconomic systems in a multivariate fractional integration setting.

3 A General Method to Compute Impulse Response Functions in a Multivariate Fractional Integration Framework

In a fractional integration setting, if a variable y_t has an order of integration $d, (d \in R)$, it is denoted as $y_t \sim I(d)$ and can be expressed as:

$$(1-L)^d y_t = \mu_t \quad t = 1, 2, \ldots \tag{2}$$

with $y_t = 0, t \leq 0$. μ_t is an $I(0)$ process, defined as a covariance stationary process, with spectral density function that is positive and finite at the zero frequency. Thus μ_t may be a stationary ARMA process. We can express $(1-L)^d$ as the following binomial expansion:

$$(1-L)^d = \sum_{j=0}^{\infty} \begin{pmatrix} d \\ j \end{pmatrix} (-1)^j L^j = \left(1 - dL + \frac{d(d-1)}{2} L^2 - \frac{d(d-1)(d-2)}{3} L^3 \ldots\right) \tag{3}$$

The representation of y_t in (2) can then be approximated for any real d, as:

$$\left(1 - dL + \frac{d(d-1)}{2} L^2 - \frac{d(d-1)(d-2)}{3} L^3 \ldots\right) y_t = \mu_t. \tag{4}$$

While d captures the long-memory component of the series, μ_t describes the short-run dynamics through its ARMA structure. The literature on fractional models like (2) has recently emerged in macroeconomics and finance. Some examples are Diebold and Rudebusch (1989), Baillie and Bollerslev (1994) and Gil-Alana and Robinson (1997).[2]

[2]The fractional integration literature was pioneered by Granger (1980) and Granger and Joyeaux

The fractional integration framework nests the two standard cases documented in the vast majority of applied work in time series. If $d = 0$, as is the case for hours worked in CEV, the series is a covariance stationary process and possesses 'short memory', with the autocorrelations decaying fairly rapid. If $d = 1$, as is the case for hours worked in GR, the series is a non-stationary $I(1)$ process. But in a fractional framework there are more alternatives available for the order of integration of y_t. If d belongs to the interval $(0, 0.50)$, y_t is still covariance stationary, but both the autocorrelations and the response of a variable to a shock take much longer time to disappear than in a standard $(d = 0)$ stationary case. If $d \in [0.50, 1)$, the series is no longer covariance stationary, but is still mean reverting, with the effect of the shocks dying away in the long run. Thus, the fractional differencing parameter d plays a crucial role for our understanding of the economy, and of the macro dynamics. For instance, as d increases, a stronger policy action is required to bring a variable back to its steady-state.

There exist many procedures for estimating and testing the fractional differencing parameter d in a univariate framework. They can be parametric or semi-parametric and they can be specified in either the time or the frequency domain. In section 4 we describe and employ some of them. However, the main goal of our study is the identification of the structural macroeconomic shocks and the associated impulse response functions in a multivariate setting. We now show how the fractional integration framework captures the joint behavior of a set of macro variables. We first describe the structural multivariate model and then show how the structural shocks can be recovered from an estimable reduced-form model under standard identifying assumptions.

(1980). See Baillie (1996) for a complete review of $I(d)$ processes.

A set of jointly related macroeconomic variables can be described as:

$$ADY_t = \nu_t \tag{5}$$

where A is an $n \times n$ matrix, Y_t is an $n \times 1$ vector of observable macro variables and ν_t is an $n \times 1$ vector of possibly correlated errors. D is an $n \times n$ diagonal matrix which has the following form:

$$D = \begin{pmatrix} (1-L)^{d_1} & 0 & 0 & \dots & 0 \\ 0 & (1-L)^{d_2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & (1-L)^{d_n} \end{pmatrix} \tag{6}$$

where d_i is the order of integration of the variable y_i. We assume, without loss of generality, that the vector of errors ν_t follows a VAR(1) process:

$$\nu_t = G\nu_{t-1} + \varepsilon_t \tag{7}$$

where G is an $n \times n$ matrix and ε_t is an $n \times 1$ vector of structural macro shocks $i.i.d.$ distributed with diagonal variance-covariance matrix Σ. Substituting (7) into (5), one can obtain the following infinite moving average representation for the macro system:

$$Y_t = D^{-1}(I - (A^{-1}GA)L)^{-1}A^{-1}\varepsilon_t \tag{8}$$

where I is the identity matrix of order n. D^{-1} can be easily computed from the binomial expansion in (3), valid for any real d. We therefore need to identify $(2n + 2n^2)$ structural parameters: $2n$ from D and Σ and $2n^2$ from A and G. Equation (8) makes clear that

the (potentially) fractional orders of integration of the macro variables (D) will directly affect the impulse response functions to the structural shocks. Since D is diagonal, the order of integration of each variable (d_i) will only affect the responses of this variable y_i. Notice that our setting generalizes the standard impulse response function framework, where the diagonal values in D are restricted to be 1 or $1-L$ depending on the choice of integration order for a variable, $I(0)$ or $I(1)$ respectively. Moreover, we do not have to impose any a priori assumption about the fractional order of integration of the variables since, as we show below, they are simultaneously estimated with the remaining system parameters. Finally, and unlike the standard VAR impulse response framework displayed in section 2, the impulse response functions are not sensitive to the choice between levels and first differences for the macro variables by construction.

In a multivariate setting, the number of estimation procedures for fractional integration is very limited. Gil-Alana (2003a) and Gil-Alana (2003b) proposed an extension of the univariate tests of Robinson (1994) in the frequency domain, while Nielsen (2005) developed time-domain versions of Gil-Alana's (2003a,b) tests. These methods allow to estimate a reduced-form system such as:

$$DY_t = \zeta_t \tag{9}$$

where ζ_t is an $n \times 1$ stationary $I(0)$ vector of errors. We can further assume that ζ_t follows a VAR(1) process, such as:

$$\zeta_t = F\zeta_{t-1} + \eta_t \tag{10}$$

where F is an $n \times n$ matrix and η_t is a vector of reduced-form errors with variance-covariance matrix V. Substituting (10) into (9), the following infinite moving average

representation can be derived:

$$Y_t = D^{-1}(I - FL)^{-1}\eta_t \tag{11}$$

The relation between structural and reduced-form error terms in (8) and (11) is then given by:

$$A^{-1}\varepsilon_t = \eta_t, \tag{12}$$

whereas the relation between structural and reduced-form matrices is given by:

$$A^{-1}GA = F. \tag{13}$$

The reduced-form model has $\left(n + n^2 + \frac{n(n+1)}{2}\right)$ parameters: n from D, n^2 from F and $\frac{n(n+1)}{2}$ from V. As a result, we need $\frac{n(n+1)}{2}$ additional restrictions in the structural system (5) so that the structural errors can be identified. It is standard in the literature to assume that the variance-covariance of the structural error vector is the identity matrix. Therefore, we need $\frac{n(n-1)}{2}$ additional restrictions. One alternative strategy followed by researchers such as Christiano, Eichenbaum, and Evans (1999) is to assume that the matrix A in the structural model (5) is lower triangular. Another alternative is to assume long-run restrictions, as in BQ or GR. We follow this second strategy in the empirical part of the paper, consistent with the recent technology-hours macro literature. In our framework $n = 2$, so that we only need one long-run restriction. In agreement with both GR and CEV, we assume that a structural shock to hours worked does not affect productivity in the long-run. Notice finally that D, while capturing additional long-memory dynamics, does not alter the standard impulse response identification techniques.

4 Univariate Analysis

This section presents empirical evidence on the fractional order of integration of both labor productivity and hours worked per capita in a univariate setting. This evidence is relevant for two reasons. First, while researchers have applied standard unit root tests to the productivity and hours variables, no study has investigated the fractional order of integration of these variables. The fractional setting is clearly more general, as it allows a given variable to display an order of integration different from one and zero. We apply parametric and semi-parametric fractional integration tests in the frequency domain. Second, we will be able to use the univariate results in a multivariate setting for two purposes: First, if the fractional integration tests manage to pin down the order of integration clearly, then we can directly assume it to be the right one in a multivariate analysis. Second, we can assess whether the estimates of the order of integration of a given variable differ in univariate and multivariate contexts.

The first thing we do is to plot the individual series and their first differences, along with their corresponding correlograms and periodograms. In figure 2 we display the plots for the number of hours worked and the productivity series following the definition in GR, while figure 3 shows the first differenced data. A visual inspection at the graphs of the two series in levels clearly shows that productivity is non-stationary, with the values of the correlogram decaying very slowly, and a large peak in the periodogram at the smallest frequency.[3] If we look at the plots of the data in first differences, we observe significant values in the correlogram at lags relatively far away from zero, which

[3] If a series is $I(d)$ with $d > 0$, (e.g. $I(1)$), the spectral density function, $f(\lambda)$, is unbounded at the origin, that is, $f(0) = \infty$. The periodogram is an asymptotic unbiased estimate of the spectral density and thus, it should reproduce such behavior. On the other hand, if the series is $I(0), 0 < f(0) < \infty$, and the periodogram should be positive and finite at the smallest frequency.

might suggest that the first differences of productivity might still present a component of long-memory behavior.

With respect to the number of hours worked, the correlogram plot in figure 2 is a bit unclear. The values in the correlogram decay slowly (though at a higher rate than for productivity), and the periodogram also presents a peak at the smallest frequency. The plots in figure 3 seem to indicate that the differenced series is I(0), finding no evidence of over-differentiation.[4] Thus, if we have to choose within the paradigm $I(0)/I(1)$ for the specification of the two series, we would conclude from these two figures that both series are non-stationary $I(1)$ processes, though fractional degrees of integration should also be taken into account as plausible specifications for the two series.

Figures 4 and 5 display similar plots for the series used in CEV. The productivity series plots are very similar to those in GR, suggesting that this series is also a non-stationary $I(1)$ process. However, we observe some slight differences for the hours worked. Both the correlograms and the periodograms in the original and the differenced data seem to indicate that this variable is clearly $I(1)$ but this evidence is stronger than in the case of the GR hours series. We observe that the periodogram of the levels of hours shows a large peak at the smallest frequency, while the value at the same zero frequency in the differenced data (figure 5) is clearly positive and finite, suggesting that the series is I(0).

We first use a parametric method proposed by Robinson (1994) to test for the fractional order of integration of the productivity and hours series. This model is described in Appendix A. It is based on the Lagrange Multiplier (LM) principle and uses the Whittle function, which is an approximation to the likelihood function. One advantage of this method is that it allows us to consider fractional orders of integration at any real value d, including both stationary and non-stationary processes. In fact, previous parametric

[4] If the series is over-differenced, $f(0) = 0$.

methods such as Sowell (1992) only allowed for $-0.5 < d < 0.5$, i.e. in the stationary region. Another advantage of the fractional approach by Robinson (1994) is that it does not display an abrupt change in the limit behavior of the tests against the unit root. In fact, the limit distribution is a standard normal for any real value d. In contrast, the classic ADF, PP and KPSS methods have a non-standard limit distribution in the sense that the critical values must be tabulated case by case by means of a Monte Carlo simulation study. For ease of exposition, we rewrite the standard expression for a fractionally integrated process y_t

$$(1-L)^d y_t = \mu_t. \tag{14}$$

Following the approach by Robinson (1994), we test:

$$H_0 : d = d_0, \tag{15}$$

for any given real value d_0, in a model given by:

$$x_t = \alpha + \beta t + y_t, \tag{16}$$

with t as a time trend and y_t given by (14). Note that x_t is the observable macro variable and y_t is now the regression error series which might be fractionally integrated according to (14). We first assume that $\alpha = \beta = 0$ in (16), i.e., there are no deterministic terms, in which case $x_t = y_t$. We also consider the cases of an unknown α and $\beta = 0$ (with an intercept) and both α and β unknown (a linear time trend). The results for the four series are given in tables 1 and 2.[5] In table 1 we assume that the μ_t disturbances are white noise. In table 2 we permit autocorrelation patterns in the error term. Across these tables

[5] A diskette containing the FORTRAN codes for all the programs in this paper is available from the authors upon request.

we report the confidence intervals of those values of d_0 where the null hypothesis cannot be rejected at the 5% level.[6] We also display in the tables the value of d_0 producing the lowest statistic across d's. This value should be an approximation to the maximum likelihood estimate.

Starting with the case of white noise for μ_t, we see that if we do not include regressors, the unit root null hypothesis (i.e., $d_0 = 1$) cannot be rejected for any series. This hypothesis cannot be rejected for either of the two productivity series when an intercept and/or a linear trend is included in the regression model. For the number of hours, the unit root is rejected in favor of higher orders of integration. In what respects to the model with autocorrelated residuals, we first estimated with autoregressive (AR) models. Modelling μ_t in terms of an AR(1) process produced some inconsistencies in the interpretation of the results. For instance, the null hypothesis of $d = 0$ was not rejected in any series; it was rejected for values of d between 0 and 1 and it was again not rejected for values of d close to 1. This lack of consistency can be explained by the fact that the AR coefficients, though lower than 1 in absolute value, can be arbitrarily close to 1 and thus they might be competing with d in describing non-stationarity. Note that other standard unit root testing procedures face the same problem. We solved this problem by using the method of Bloomfield (1973). This method, which can be flexibly applied in the context of Robinson's (1994) tests, does not impose a given parametric model for the $I(0)$ disturbances but implies autocorrelations for μ_t which decay exponentially as in the ARMA case. Moreover, this model is stationary across the whole range of values for the parameter set unlike the AR case. Using this model, the results are very similar across

[6]These intervals were constructed as follows: First, we choose a value of d from a grid, $d_0 = 0, 0.01, \ldots, 2$. Then we compute the test statistic testing the null for this value. If the null is rejected at the 5% level, we discard this value of d. Otherwise, we keep it. An interval is then obtained after considering all the values of d in the grid.

series and most of the non-rejection values for d oscillate around 1. However, for the number of hours, while the unit root is not rejected in case of the GR series, d is larger than 1 using CEV's definition and the null of a unit root is rejected in two of the three cases. This might be consistent with the plots presented in Figures 2-5 where the order of integration for the number of hours in Christiano seems to present a higher degree of dependence. Another important feature observed across the tables is that if we do not include regressors, the lowest statistics occur in all series at values of d lower than 1. However, including deterministic terms, they occur at values slightly higher than 1.

Figure 6 displays the estimates of d based on a semi-parametric "local" Whittle method proposed by Robinson (1995). This method is described in Appendix B. We use the Gaussian Whittle method because of its computational simplicity. Note that this method requires no additional user-chosen numbers in the estimation. The top panel of figure 6 shows the results for the GR series whereas the bottom panel presents the CEV's counterparts. For both series, we display the estimates of d across the whole range of values for the bandwidth number m, along with the 95% confidence interval corresponding to the $I(1)$ hypothesis.[7] Starting with the number of hours, we see that the results are quite unstable. Thus, if the bandwidth number m is lower than $T/4$, most of the estimates of d are within the $I(1)$ interval; however, if $m > T/4$ the values of d are significantly above 1. Alternatively, the results for the productivity series strongly support the hypothesis of a unit root in the two cases.

To sum up, our univariate fractional integration results strongly support the hypothesis of a unit root for the productivity series and lead to some ambiguous conclusions about the order of integration with respect to the number of hours worked. The series for

[7] In the case of the "local" Whittle estimator, the use of optimal values has not been yet theoretically justified. Some authors, such as Lobato and Savin (1998) use an interval of values for m.

hours worked may admit orders of integration higher than 1, implying that even taking first differences, the series may still present a component of long memory behavior. In the next section we will allow for the estimation of the fractional order of integration of the hours series in a multivariate context.

5 Multivariate Analysis

A number of macroeconomic studies try to determine the effect of structural shocks on the dynamic path of economic aggregates. One example of this approach is Bekaert, Cho, and Moreno (2005). Our study also falls into this category, since it tries to elucidate the response of hours worked per capita to a technology shock. In order to answer this question, we need a multivariate system. This section applies the multivariate model derived in section 3 to resolve the technology-hours question. What is interesting about this framework is that it allows the econometrician to estimate jointly the order of integration of the macro variables and the impulse response functions to the structural shocks.

As noted above, we estimate bivariate systems with labor productivity and hours worked. We will assume that the order of integration of the productivity series is 1 throughout the following analysis. Our motivation for this assumption is threefold. First, the univariate tests decisively pointed at 1 as the order of integration of productivity, unlike in the hours case (see, e.g., figure 6). Second, this assumption is uncontroversial for all of the papers in the technology-hours literature. Indeed, GR, CEV and all related papers assume that productivity is integrated of order 1. Third, by assuming that the order of integration of productivity is 1, our approach will estimate more efficiently the order of integration of hours in a multivariate model, the main object of study in the

present paper. Nevertheless, we also computed the procedure allowing both orders of integration to be unknown and the value for the productivity series was close to 1 in practically all cases.

The bivariate models are estimated following the procedure in Gil-Alana (2003a). This method is briefly described in Appendix C. An advantage of this technique is that it is an extension of the univariate tests of Robinson (1994) to the multivariate case and thus, similarly to the univariate case, we do not need to impose a priori any assumption about the orders of integration of the series since they are freely estimated from the real line. In the estimations we present, we proceeded as follows: First, we estimated the model in (5) with white noise disturbances. Then we estimated the model letting the residuals follow an autoregressive process of order one, as in (7).[8]

Table 3 shows the orders of integration for hours worked across data and model specifications. It also displays the associated 95% confidence intervals. The model with white noise residuals yields orders of integration for hours worked statistically higher than 1. In the case of the GR data specification, the order of integration is 1.59 whereas for the CEV data specification, it is 1.05. These results are roughly consistent with the univariate evidence, although it is noteworthy that the order of integration for the GR dataset is now substantially higher than that of the CEV dataset.

The results for the multivariate model which allows for weak time dependence in the error term are also displayed in table 3. It shows that the orders of integration for hours worked across data specifications are approximately one order of integration lower than the case with white noise residuals. This finding suggests that introducing

[8]Multivariate versions of the Bloomfield's (1973) model have not been yet developed. Moreover, they would be of no use to compute impulse responses, given that the original model does not display a parametric formula for the disturbances μ_t.

additional cross-sectional and time series information reduces the order of integration of a given series. Additionally, the order of integration of the GR measure (0.69) is much higher than that of CEV (0.04). Table 4 shows the implied VAR(1) matrix of coefficients for the structural error terms (matrix G in equation (7)). Interestingly, it shows that while the autoregressive coefficient in the productivity equation is close to zero across data specifications, its counterpart in the hours equation is close to one in both cases, especially with the CEV data. In other words, most of the time dependence in the CEV hours variable is now captured by the autoregression, unlike in the GR case.

Figure 7 shows the response of per capita hours to the structural technology shock in the macro system with white noise errors. We display the responses implied by the systems in levels and first differences of the fractionally integrated variables. The responses in levels show a persistent decline in the level of hours worked across hours measures. The decline in hours is larger in the case of the CEV data specification, in line with what was found in the first differences specifications under the standard VAR framework shown in figure 1. The responses in first differences of our fractional integration model are exactly the ones implied by the model in levels. They show that hours worked decline on impact in response to the technology shock. Following this initial impact, the hours in first differences converge monotonically towards their steady-state level.

One limitation of the white noise model is that it implies a null response of labor productivity to the hours shock.[9] In contrast, the model with autocorrelated residuals allows for a free estimation of this response. Another limitation of the model with white noise residuals is that it implies monotonic impulse responses, given its stylized

[9]To see this, notice that in a bivariate system with white noise residuals where $A = [(a_{11} \quad a_{21})^T, (a_{12} \quad a_{22})^T]$, the long-run identifying restriction implies that $-\frac{a_{12}}{a_{11}a_{22}-a_{12}a_{21}} \sum_{j=0}^{\infty} \psi_j^n = 0$, where ψ_j^n are the coefficients in the expansion of the fractional polynomial associated with the hours shocks in the productivity equation. That condition is satisfied if and only if $a_{12} = 0$.

structure. More general models with VAR structures for the residuals allow however for more flexible impulse responses. Figure 8 shows the associated responses for the model with autocorrelated residuals. The response of the level of hours in the GR specification to a technology shock is negative on impact. Following the initial reaction, hours increase and they reach the peak after 15 quarters. Afterwards, hours start to decrease slowly. Regarding the response in levels of the CEV specification, the initial impact is negative. Since then, hours remain negative but they increase slowly through time. Notice that the initial negative response of per capita hours is again larger in the case of the CEV specification.

The responses of the first differences of hours to the technology shock coincide again with those implied by the model in levels. They are qualitatively similar across data specifications. Following a technology shock, the first differences of hours decline. After three quarters, they become positive, reaching a peak after four quarters. Since then, the first differences of hours decline and converge towards zero. The responses in first differences are therefore qualitatively (and quantitatively, in the case of the GR data specification) similar to those shown by Galí and Rabanal (2004), Pesavento and Rossi (2005), Francis and Ramey (2005) and to the responses of hours in first differences shown in figure 1.

Notice that while the order of integration of the CEV measure is similar to the one they apply in their VARs, the implied response of hours to the technology shock is different from the one they obtain. This finding must be related with the interaction of the binomial expansions of the long-memory coefficients with standard vector autoregressive coefficients which, albeit small, have the potential to change the direction and evolution of the impulse responses.

In summary, all our data specifications detect a decline of hours worked on impact

in response to a technology shock. While there are quantitative differences across multivariate models and variables used, this finding seems to be qualitatively robust.

6 Discussion and Conclusions

The goal of this paper was to determine the response of hours worked to a technology shock, a hotly debated question in Macroeconomics today. Our contribution is to derive a unified econometric framework which determines the order of integration of hours worked and the dynamic impulse response function of hours to a technology shock simultaneously. We found that per capita hours fall on impact in response to a technology shock. In this respect, our results support those by GR. Interestingly, even though the estimate of the order of integration of per capita hours used by CEV was close to 0 in our most general multivariate model, the response of hours was found to be negative following a technological innovation.

In a recent paper, Francis and Ramey (2005), based on the intuition of Fernald (2004), construct a measure of per capita hours worked which removes some of the low-frequency fluctuations of the standard measures of hours worked. They find that removing these low-frequency dynamics renders the variable stationary and that this variable decreases on impact in response to a technology shock. Our implied impulse responses are consistent with their findings, despite of the clear methodological differences. We believe that the fractional integration framework presented in this paper is well suited to account for low-frequency dynamics, since it controls for the long-memory of the stochastic processes implied by macro aggregates. Nevertheless, it would be interesting to perform univariate and multivariate fractional integration analysis with the new hours variable proposed by Francis and Ramey (2005).

Another closely related study is Gambetti (2005). He allows for time-varying coefficients in a vector autoregressive framework and finds that hours decline in response to a technology shock under both levels and first differences specifications for hours. The results in this paper are consistent with his findings, even though our setting presents constant coefficients and a more flexible setup for the integration order of hours worked.

The present article raises a number of interesting questions for future research. A first issue is related to the difference in the order of integration of the variable hours estimated in univariate and general multivariate contexts. We found that it was lower in the case of the multivariate models. This finding, in itself, suggests that conditioning on additional information may reduce the memory of a given process. While multivariate tests are not often used to determine the level of integration of a given variable, they are most interesting for macroeconomists, since the macro literature often focuses on the dynamic properties of systems of variables. In this sense, the issue of pre-testing for the order of integration of a given variable in univariate frameworks may be of second importance once we control for the fractional order of integration in a multivariate framework. The study of fractionally co-integrated systems, allowing for a non-diagonal matrix D in (6), seems also a fruitful avenue for future research in this area.

One drawback of our study is that we do not derive the confidence bands associated with the impulse response functions. Confidence intervals are typically derived through bootstrapping or Montecarlo simulations from the error terms given the distribution of the parameters in the model. The difficulty in our setup is that our parameter set does not have a well defined distribution, given that it is obtained via testing procedures and not via estimation methods which yield the standard distribution of the parameters. The literature on impulse response functions in fractionally integrated systems is still in its infancy. In fact, to our knowledge, this is the first theoretical or empirical paper on the

topic. More theoretical econometric work needs to be carried out in order to present results valid at the statistical level. In future research, we intend to address this relevant issue.

Finally, the issue of seasonality should also be taken into account. The productivity and hours variables routinely used in the literature are seasonally adjusted and the use of seasonal adjustment procedures might be obscuring other important features of the data. Montanari, Rosso, and Taqqu (1997) have shown that when seasonality is not considered, estimation based on long-memory such as fractional integration might be biased in favor of higher orders of integration. Fractional seasonal multivariate models might be a solution to deal with this problem.

Appendix

A Robinson's (1994) Univariate Parametric Fractional Integration Test

Robinson (1994) proposes the following parametric test statistic in order to test for the fractional order of integration in the model outlined in equations (14)-(16). It is based on the Lagrange Multiplier (LM) principle, and is given by:

$$\hat{r} = \frac{T^{\frac{1}{2}}}{\hat{\sigma}^2} \hat{A}^{-\frac{1}{2}} \hat{a}, \tag{17}$$

where T is the sample size and

$$\hat{a} = -\frac{2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j)\, g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j) \tag{18}$$

$$\hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j) \tag{19}$$

$$\hat{A} = \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j)\, \hat{\epsilon}(\lambda_j)' \times \left(\sum_{j+1}^{T-1} \hat{\epsilon}(\lambda_j)\, \hat{\epsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j)\, \psi(\lambda_j) \right) \tag{20}$$

$$\psi(\lambda_j) = \log \left| 2\sin \frac{\lambda_j}{2} \right| \tag{21}$$

$$\hat{\epsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log \quad g(\lambda_j; \hat{\tau}) \tag{22}$$

$$\lambda_j = \frac{2\pi j}{T} \tag{23}$$

$$\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau) \tag{24}$$

where T^* is a compact subset of the R^q Euclidean space. $I(\lambda_j)$ is the periodogram of μ_t evaluated under the null, and g above is a known function coming from the spectral density of μ_t, $f = \left(\frac{\sigma^2}{2\pi}\right) g$. Thus, if μ_t is white noise, then $g \equiv 1$, and if it is an AR process of the form $\phi(L)\mu_t = \varepsilon_t$, then $g = |\phi(e^{i\lambda})|^{-2}$, so that the AR coefficients are a function of τ. Based on H_0 (15), under very mild regularity conditions, Robinson (1994) showed that $\hat{r} \rightarrow_d N(0,1)$ as $T \rightarrow \infty$.

B Robinson's (1995) Univariate Semi-Parametric Fractional Integration Test

This appendix describes the semi-parametric method derived by Robinson (1995) in order to estimate the fractional order of integration of the process laid out in equation (14). It is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log\bar{C}(d) - 2d\frac{1}{m}\sum_{j=1}^{m} \log\lambda_j \right) \quad d \in (-0.5, 0.5) \tag{25}$$

$$\bar{C}(d) = \frac{1}{m}\sum_{j=1}^{m} I(\lambda_j)\,\lambda_j^{2d} \tag{26}$$

$$\lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \rightarrow 0 \tag{27}$$

where m is a bandwidth parameter number. Under finiteness of the fourth moment and other mild conditions, Robinson (1995) showed that $\sqrt{m}(\hat{d} - d_0) \rightarrow_d N\left(0, \frac{1}{4}\right)$ as $T \rightarrow \infty$, where d_0 is the true value of d.

C Gil-Alana's (2003a) Multivariate Fractional Integration Test

A simple version of the procedure proposed in Gil-Alana (2003a) consists of testing the null hypothesis:

$$H_0: \quad d \equiv (d_1, d_2, \ldots, d_n) = (d_{10}, d_{20}, \ldots, d_{n0}) \equiv d_0, \tag{28}$$

for any real vector d_0, in the model given by (9), where ζ_t is supposed to be an $I(0)$ vector process with positive definite spectral density function $F(\lambda)$. Thus ζ_t may be white noise but it can also accommodate VAR structures. We assume that ζ_t in (9) is generated by a parametric model of the form:

$$\zeta_t = \sum_{j=0}^{\infty} A_j(\tau)\omega_{t-j} \quad t = 1, 2, \ldots \tag{29}$$

where ω_t is white noise and W is the unknown variance-covariance matrix of ω_t. The spectral density matrix of ζ_t is then:

$$f_\zeta(\lambda;\tau) = \frac{1}{2\pi}\theta(\lambda;\tau)W\theta(\lambda;\tau)^* \tag{30}$$

where $\theta(\lambda;\tau) = \sum_{j=0}^{\infty} A_j(\tau)e^{i\lambda j}$, and θ^* is the complex-conjugate transpose of θ. A number of conditions are required on A and f_ζ in order to derive the test statistic. The main practical implication is that its spectral density matrix must be finite, with eigenvalues bounded away from zero. It can be shown that a Lagrange Multiplier (LM)

test of the H_0 in (28) for (9) takes the form:

$$\tilde{S} = T\tilde{b}^T \left[\tilde{C} - \tilde{D}^T \tilde{E}^{-1} \tilde{D}\right]^{-1} \tilde{b}, \tag{31}$$

where T is the sample size and

$$\tilde{b} = -\frac{1}{T}\sum_{r=1}^{T-1} \psi(\lambda_r) tr\left(I_\zeta(\lambda_r)\tilde{f}(\lambda_r;\tilde{\tau}))\right) \tag{32}$$

$$\tilde{C} = \frac{4}{T}\sum_{r=1}^{T-1} \psi(\lambda_r)\psi(\lambda_r)^T \tag{33}$$

$$\tilde{D}^T = -\frac{1}{T}\sum_{r=1}^{T-1} \psi(\lambda_r)\left[tr\left(\tilde{f}^{-1}(\lambda_r;\tilde{\tau})\frac{\partial \tilde{f}(\lambda_r;\tilde{\tau})}{\partial \tau_1}\right);\ldots;tr\left(\tilde{f}^{-1}(\lambda_r;\tilde{\tau})\frac{\partial \tilde{f}(\lambda_r;\tilde{\tau})}{\partial \tau_q}\right)\right] \tag{34}$$

$$\tilde{E}^{uv} = \frac{1}{2T}\sum_{r=1}^{T-1} tr\left(\tilde{f}^{-1}(\lambda_r;\tilde{\tau})\frac{\partial \tilde{f}(\lambda_r;\tilde{\tau})}{\partial \tau_u}\tilde{f}^{-1}(\lambda_r;\tilde{\tau})\frac{\partial \tilde{f}(\lambda_r;\tilde{\tau})}{\partial \tau_v}\right) \tag{35}$$

where $I_\zeta(\lambda_r)$ is a matrix with the following $(u,v)^{th}$ element:

$$I_{uv}(\lambda_r) = W_u(\lambda_r)\bar{W}_v(\lambda_r); \quad W_u(\lambda_r) = \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T} \tilde{\zeta}_{ut} e^{i\lambda_r t}; \quad \lambda_r = \frac{2\pi r}{T} \tag{36}$$

where $\bar{W}$ denotes the complex conjugate and $\tilde{f}$ is the estimated spectral density matrix of $\tilde{\zeta}_t$, and where $\tilde{\zeta}_t$ are the reduced-form errors. Finally,

$$\tilde{\tau} = \arg\min_{\tau \in T^*}\left(\frac{T}{2}\log|\tilde{f}(\lambda_r;\tau)| + \frac{1}{2}\sum_{r=1}^{T-1} tr\left(\tilde{f}^{-1}(\lambda_r;\tau)I_\zeta(\lambda_r\right),\right) \tag{37}$$

where T^* is a compact subset of the q-dimensional Euclidean space. Extending the conditions in Robinson (1994), Gil-Alana (2003a) shows that, under H_0 (28):

$$\tilde{S} \rightarrow_d \chi^2_n \qquad \text{as} \quad T \rightarrow \infty. \tag{38}$$

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Table 1: **Robinson's (1994) Univariate Test for Fractional Integration: White Noise Disturbances**

Series	No Regressors			Intercept			Linear Time Trend		
GP	[0.91	**0.98**	1.08]	[0.91	**1.01**	1.13]	[0.94	**1.01**	1.10]
GH	[0.91	**0.98**	1.08]	[1.45	**1.60**	1.77]	[1.45	**1.60**	1.77]
CP	[0.92	**0.99**	1.09]	[0.93	**1.03**	1.13]	[0.96	**1.02**	1.10]
CH	[0.91	**0.98**	1.08]	[1.36	**1.49**	1.65]	[1.36	**1.49**	1.65]

This table shows the 95% confidence intervals for the order of integration of a given time series computed through the Robinson's (1994) model:

$$\begin{aligned} x_t &= \alpha + \beta t + y_t \\ (1-L)^d y_t &= \mu_t \end{aligned}$$

where x_t is the macroeconomic variable: GP is the productivity variable used by Galí and Rabanal (2004), GH is the hours variable used by Galí and Rabanal (2004), CP is the productivity variable used by Christiano, Eichenbaum, and Vigfusson (2003) and CH is the hours variable used by Christiano, Eichenbaum, and Vigfusson (2003). α and β are constants, d is the order of integration of each process and μ_t is assumed to be a white noise process. The lowest statistics of the Robinson's (1994) test appear in bold in the middle of the confidence interval.

Table 2: **Robinson's (1994) Univariate Test for Fractional Integration: Autocorrelated Disturbances**

Series	No Regressors			Intercept			Linear Time Trend		
GP	[0.83	**0.97**	1.13]	[0.77	**1.04**	1.27]	[0.92	**1.01**	1.17]
GH	[0.84	**0.97**	1.13]	[0.84	**1.05**	1.36]	[0.85	**1.05**	1.36]
CP	[0.85	**0.96**	1.14]	[0.98	**1.16**	1.33]	[1.00	**1.10**	1.25]
CH	[0.85	**0.97**	1.14]	[1.07	**1.21**	1.35]	[1.05	**1.14**	1.27]

This table shows the 95% confidence intervals for the order of integration of a given time series computed through the Robinson's (1994):

$$\begin{aligned} x_t &= \alpha + \beta t + y_t \\ (1-L)^d y_t &= \mu_t \end{aligned}$$

where x_t is the macroeconomic variable: GP is the productivity variable used by Galí and Rabanal (2004), GH is the hours variable used by Galí and Rabanal (2004), CP is the productivity variable used by Christiano, Eichenbaum, and Vigfusson (2003) and CH is the hours variable used by Christiano, Eichenbaum, and Vigfusson (2003). α and β are constants, d is the order of integration of each process and μ_t is assumed to be an autocorrelated process. The lowest statistics of the Robinson's (1994) test appear in bold in the middle of the confidence interval. These statistics are computed assuming that μ_t follows the model of Bloomfield (1973).

Table 3: **Fractional Order of Integration of Hours Worked: Multivariate Model**

Series	WN			VAR1		
GR	[1.38	**1.59**	1.85]	[0.62	**0.67**	0.74]
CEV	[1.02	**1.05**	1.07]	[0.00	**0.04**	0.07]

This table shows the results for the order of integration of hours worked per capita obtained under the multivariate model described in section 5. The model, in its more general form, can be expressed as:

$$\begin{aligned} ADY_t &= \nu_t \\ \nu_t &= G\nu_{t-1} + \varepsilon_t \end{aligned}$$

The table shows the 95% confidence interval along with the lowest value statistic (in bold) for the order of integration of hours. In the case of the VAR1 CEV model, the confidence interval is a 90% interval. We present the results for both data specifications (Galí and Rabanal (2004) (GR) and Christiano, Eichenbaum, and Vigfusson (2003) (CEV)) and for both the multivariate model with white noise (WN) and VAR(1) (VAR1) residuals. For the white noise model, $G = 0$.

Table 4: **Multivariate Autocorrelated Model: VAR(1) Matrix for Structural Residuals**

$$G^{GR} = \begin{bmatrix} 0.0160 & 0.0037 \\ 0.0300 & 0.7984 \end{bmatrix}$$

$$G^{CEV} = \begin{bmatrix} -0.0175 & 0.0065 \\ -0.7784 & 0.9801 \end{bmatrix}$$

This table shows the implied VAR(1) matrices for the structural residuals of the the multivariate model described in section 5. The model is expressed as:

$$\begin{aligned} ADY_t &= \nu_t \\ \nu_t &= G\nu_{t-1} + \varepsilon_t \end{aligned}$$

G^{GR} is the G matrix obtained with the data in Galí and Rabanal (2004) whereas G^{CEV} is the G matrix obtained with the data in Christiano, Eichenbaum, and Vigfusson (2003).

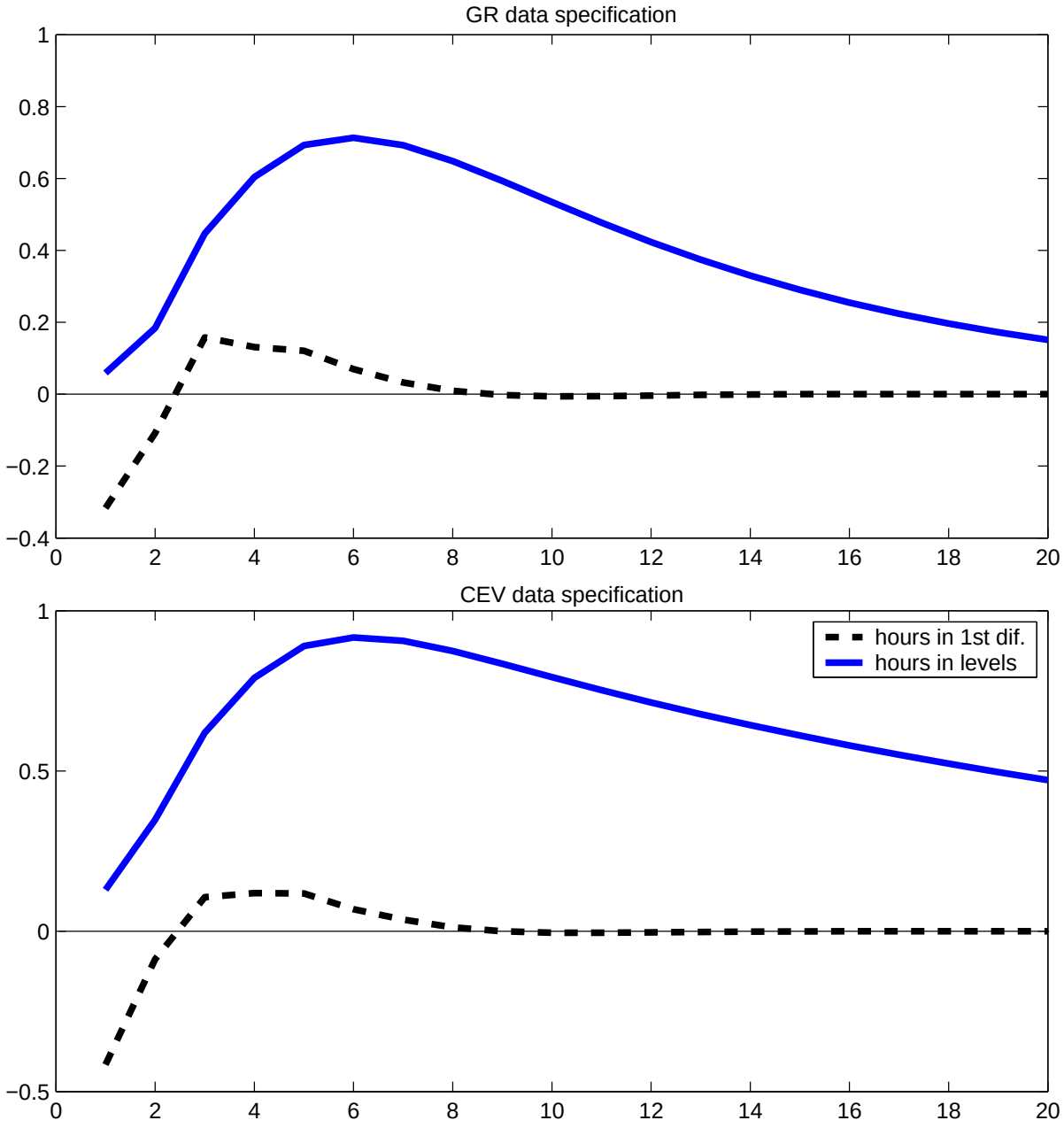
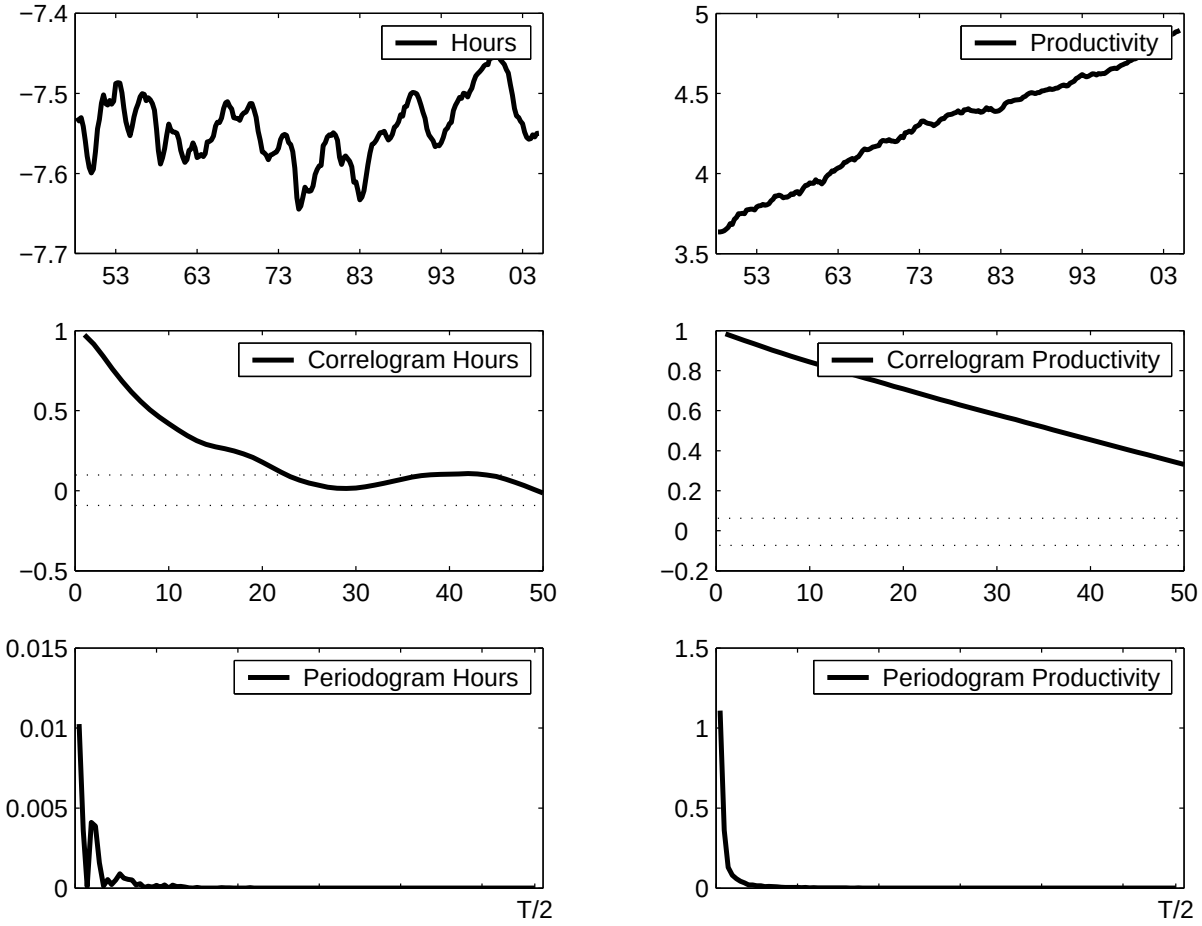


Figure 1: Impulse Response Functions of Hours to a Technology Shock.

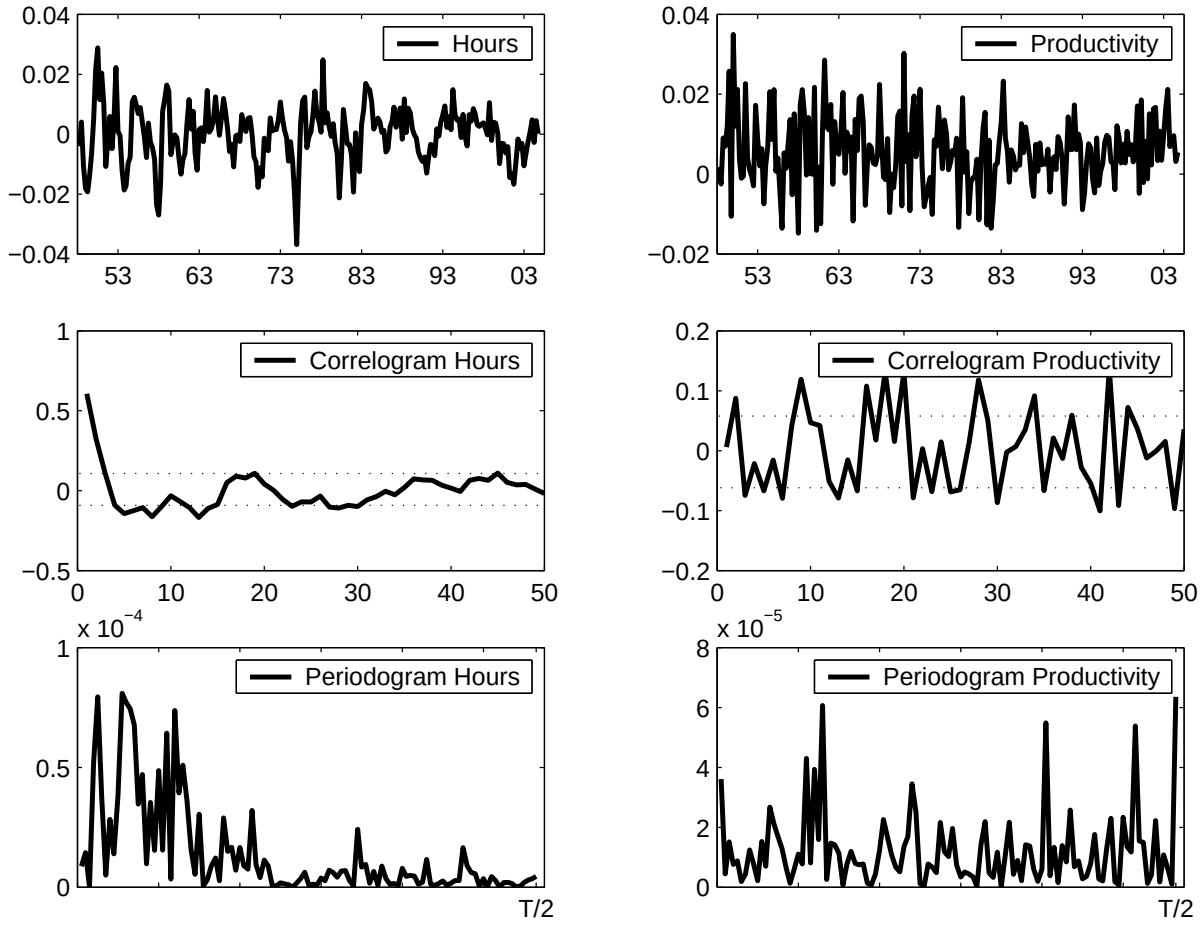
This figure shows the impulse response functions of hours per capita worked to a technology shock. Units are in percentages. The top panel shows the responses of the level and first differences of hours to a technology shock using the Galí and Rabanal (2004) (GR) specification with data for non-farm businesses. The bottom panel shows the analogous responses under the Christiano, Eichenbaum, and Vigfusson (2003) (CEV) specification with data for total businesses.

Figure 2: Graphs, Correlograms and Periodograms of Galí and Rabanal (2004) data in Levels



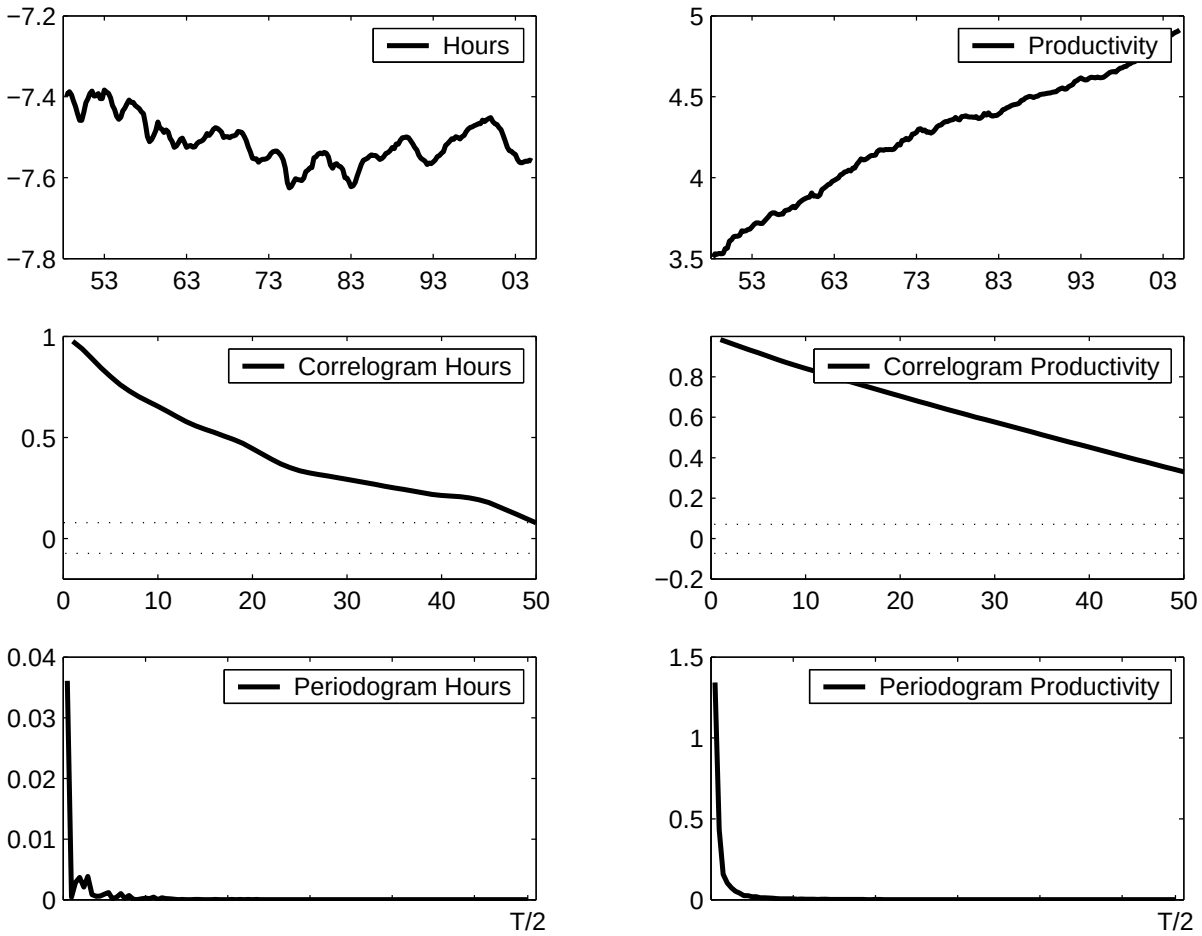
This figure presents the graphs of the series used by Galí and Rabanal (2004) in levels. The variables are expressed in natural logarithms. It also shows the correlograms and the periodograms of both the labor productivity and hours series. The dotted lines in the correlograms represent the 95% confidence bands for the null of no autocorrelation. They are computed as $\pm\frac{1}{\sqrt{T}}$, where T is the sample size. The periodograms are computed based on the discrete frequencies: $\lambda_j = \frac{2\pi j}{T}, j = 1 \ldots \frac{T}{2}$.

Figure 3: Graphs, Correlograms and Periodograms of Galí and Rabanal (2004) data in First Differences



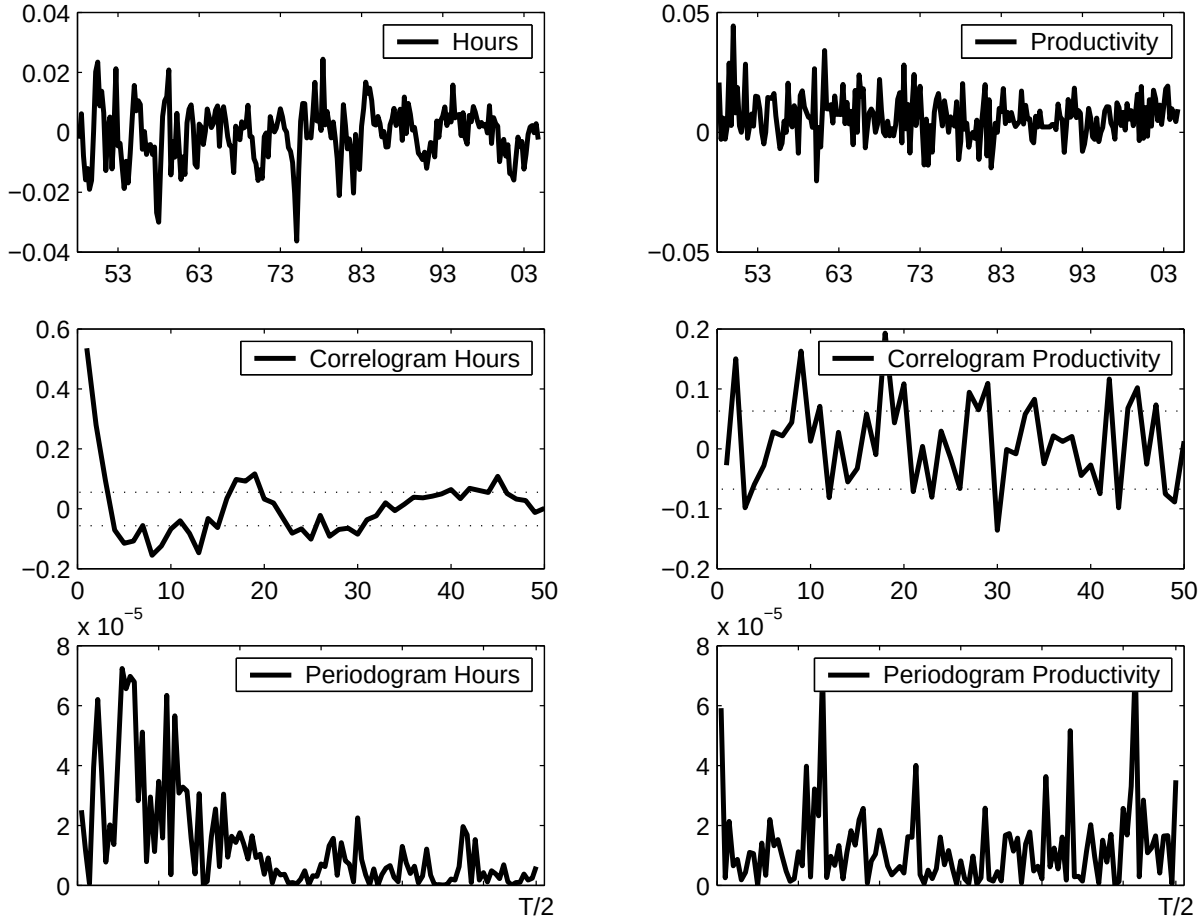
This figure presents the graphs of the series used by Galí and Rabanal (2004) in first differences of the natural logarithms. It also shows the correlograms and the periodograms of both the productivity and hours series. The dotted lines in the correlograms represent the 95% confidence bands for the null of no autocorrelation. They are computed as $\pm\frac{1}{\sqrt{T}}$, where T is the sample size. The periodograms are computed based on the discrete frequencies: $\lambda_j = \frac{2\pi j}{T}, j = 1 \ldots \frac{T}{2}$.

Figure 4: Graphs, Correlograms and Periodograms of Christiano, Eichenbaum and Vigfusson (2003) data in Levels



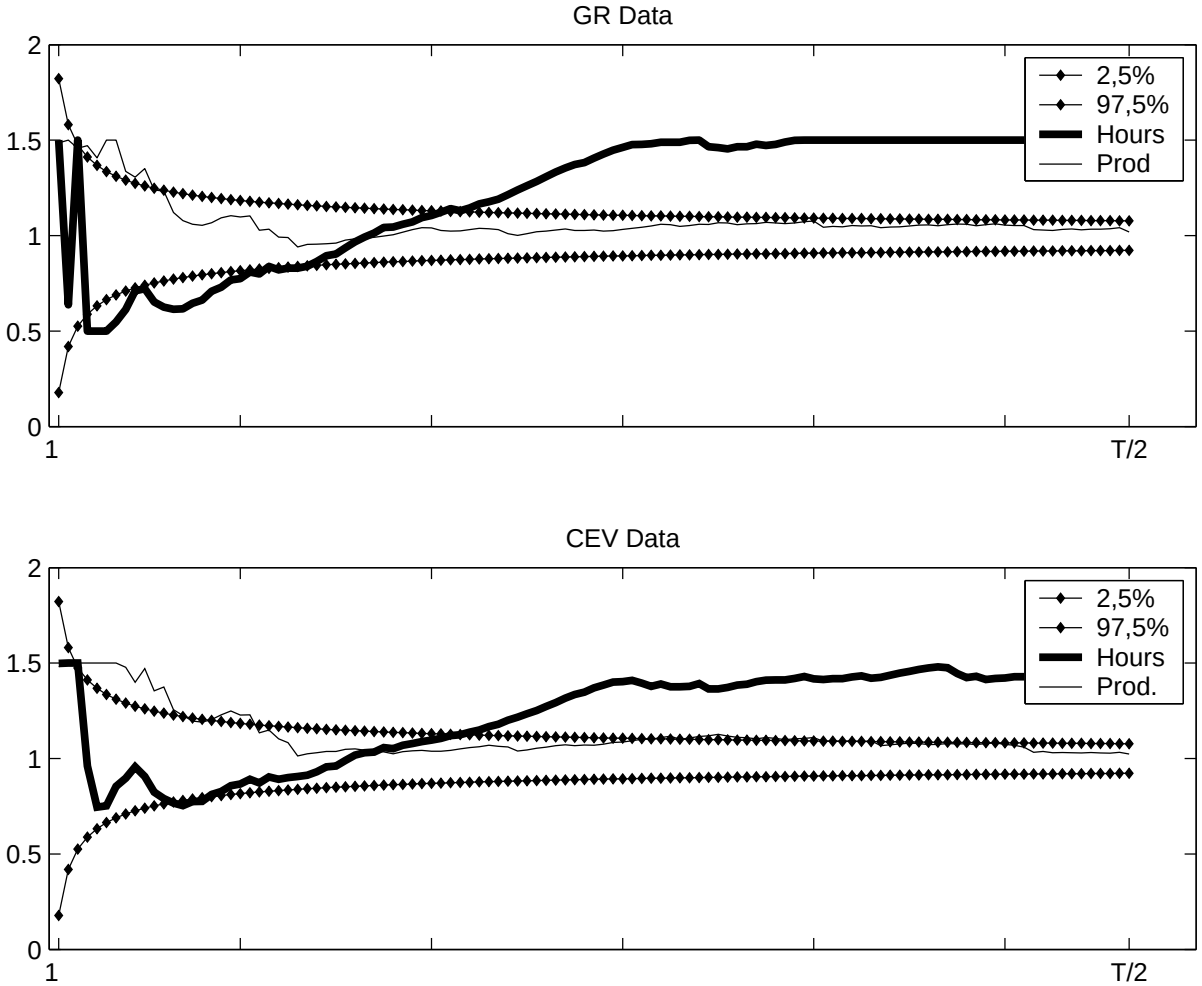
This figure presents the graphs of the series used by Christiano, Eichenbaum, and Vigfusson (2003) in levels. The variables are expressed in natural logarithms. It also shows the correlograms and the periodograms of both the productivity and hours series. The dotted lines in the correlograms represent the 95% confidence bands for the null of no autocorrelation. They are computed as $\pm\frac{1}{\sqrt{T}}$, where T is the sample size. The periodograms are computed based on the discrete frequencies: $\lambda_j = \frac{2\pi j}{T}, j = 1 \ldots \frac{T}{2}$.

Figure 5: Graphs, Correlograms and Periodograms of Christiano, Eichenbaum and Vigfusson (2003) data in First Differences



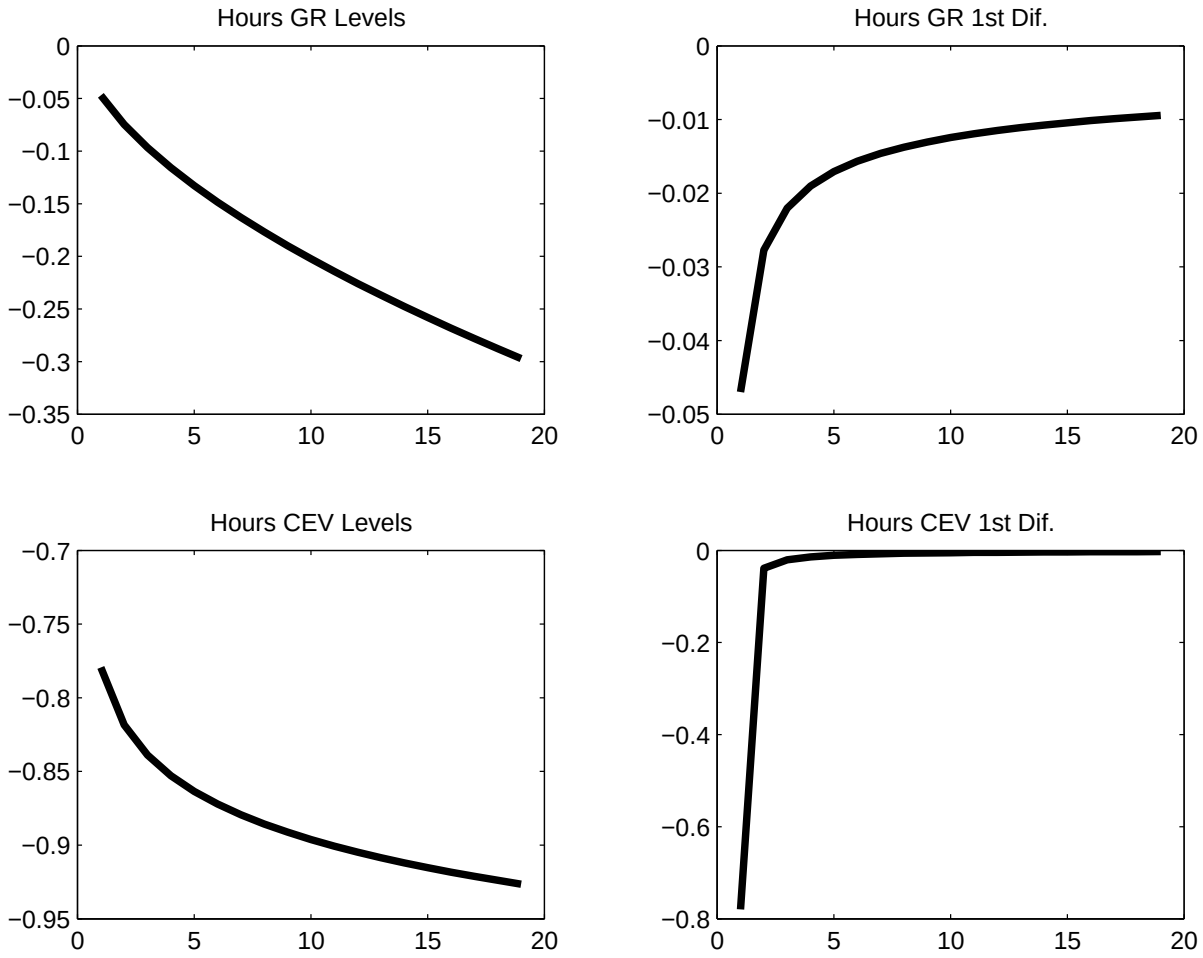
This figure presents the graphs of the series used by Christiano, Eichenbaum, and Vigfusson (2003) in first differences of the natural logarithms. It also shows the correlograms and the periodograms of both the productivity and hours series. The dotted lines in the correlograms represent the 95% confidence bands for the null of no autocorrelation. They are computed as $\pm\frac{1}{\sqrt{T}}$, where T is the sample size. The periodograms are computed based on the discrete frequencies: $\lambda_j = \frac{2\pi j}{T}, j = 1 \ldots \frac{T}{2}$.

Figure 6: Fractional Orders of Integration using the Local Whittle Semi-Parametric Estimates of d (Robinson (1995))



This figure shows the fractional orders of integration of productivity and hours worked for the data specifications in Galí and Rabanal (2004) and Christiano, Eichenbaum, and Vigfusson (2003). The orders of integration are computed according to the model proposed by Robinson (1995). The horizontal axis identifies the amplitude of the bandwidth, which goes from 1 to $\frac{T}{2}$, where T is the sample size. The vertical axis identifies the order of integration (d). 95% confidence intervals of the null for $d = 1$ appear in diamonds. The top panel corresponds to the series used by Galí and Rabanal (2004) (GR). The bottom panel corresponds to the series used by Christiano, Eichenbaum, and Vigfusson (2003) (CEV).

Figure 7: Dynamic Response of Hours Worked to a Technology Shock: White Noise Residuals

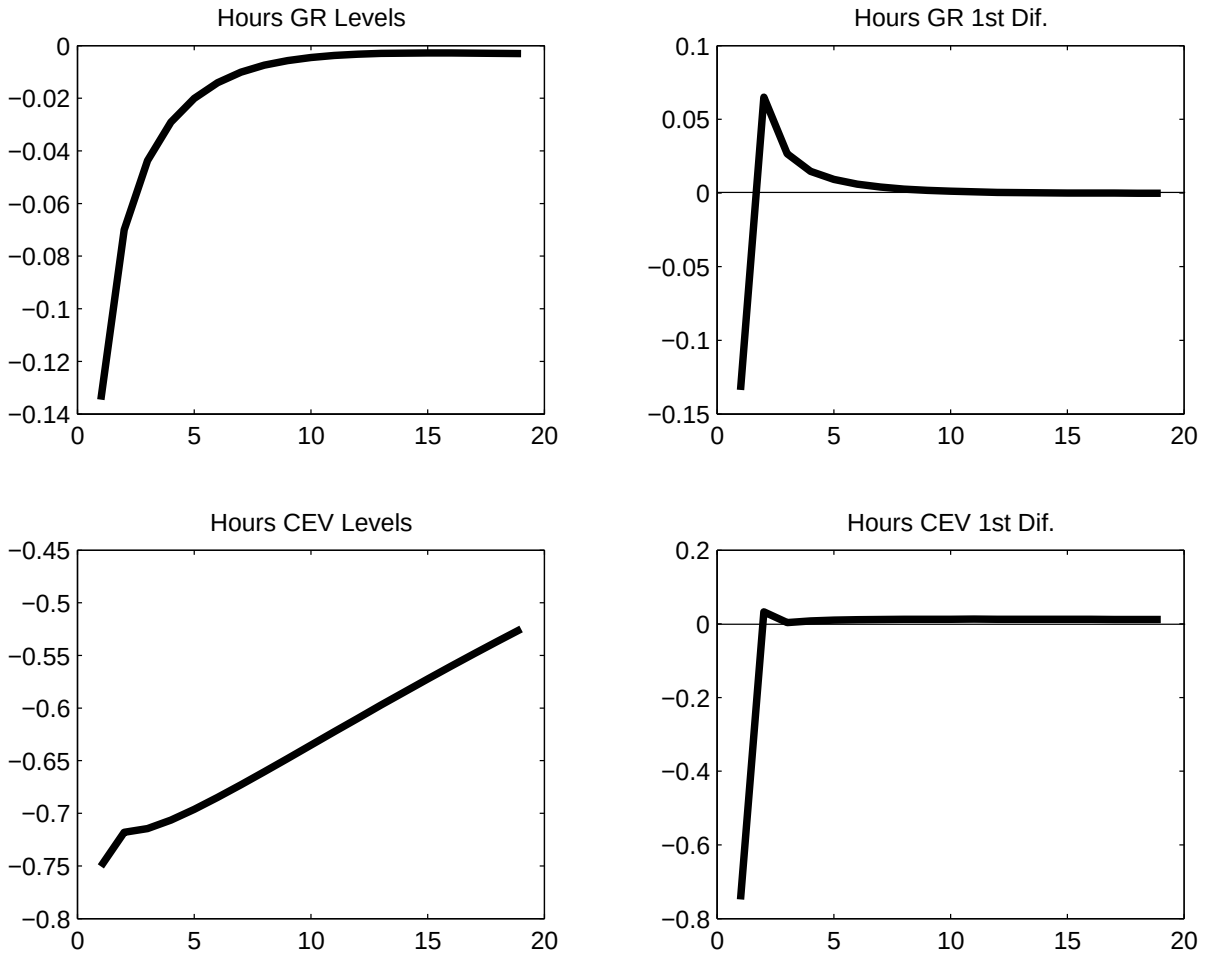


This figure shows the dynamic response of per capita hours worked to a structural technology shock in the following model:

$$ADY_t = \nu_t$$

where Y_t is a 2×1 vector including productivity and hours worked and ν_t follows a white noise process. Units are in percentages. The left panels show the responses of hours in levels whereas the right panels show the responses of hours in first differences. GR stands for the Galí and Rabanal (2004) dataset whereas CEV stands for the Christiano, Eichenbaum, and Vigfusson (2003) dataset.

Figure 8: Dynamic Response of Hours Worked to a Technology Shock: VAR(1) Residuals



This figure shows the response of per capita hours worked to a structural technology shock in the following model:

$$
\begin{aligned}
ADY_t &= \nu_t \\
\nu_t &= G\nu_{t-1} + \varepsilon_t
\end{aligned}
$$

where Y_t is a 2×1 vector including productivity and hours, ν_t follows a VAR(1) law of motion and ε_t follows a white noise process. Units are in percentages. The left panels show the responses of hours in levels whereas the right panels show the responses of hours in first differences. GR stands for the Galí and Rabanal (2004) dataset whereas CEV stands for the Christiano, Eichenbaum, and Vigfusson (2003) dataset.