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The Persistence of Earnings per Share

Luis A. Gil-Alana
Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

Rolando Pelaez
University of Houston-Downtown
Houston, USA
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Luis A. Gil-Alana and Rolando Pelaez
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ABSTRACT
This paper employs various empirical tests in order to measure the persistence of shocks to EPS for the S&P 500 index. Within the I(0)/I(1) paradigm the empirical evidence rejects the I(1) specification, supporting instead a trend-stationary representation. When fractional orders of integration are considered, the results indicate that the series is long memory (d > 0) and mean reverting (d < 1). The responses decay slowly to zero, albeit 50 quarters after an initial shock the responses remain significantly different from zero. Likewise, the variance ratio evidence suggests that the effect of a shock persists over time spans characteristic of the business cycle.

Luis A. Gil-Alana
Universidad de Navarra
Depto. Métodos Cuantitativos
Campus Universitario
31080 Pamplona
alana@unav.es

Rolando Pelaez
University of Houston-Downtown
College of Business
Houston, TX 77 002
USA
pelaezr@uhd.edu
A large body of literature in empirical finance and economics investigates the persistence of time series. The persistence of innovations to accounting earnings per share, EPS, has important implications for equity valuation. However, it has been a neglected subject; hence, this paper attempts to fill an empty box. If fluctuations in EPS consist mainly of temporary deviations from a steadily growing equilibrium level, a shock in the current quarter would not affect long-range forecasts of EPS. In this case, stock price changes should reflect mainly changes in the discount rate. However, EPS surprises often induce large changes in equity prices, suggesting that market participants attribute persistence to EPS changes.

If shocks persist, positive shocks will initially lower the price-earnings ratio creating the impression that stocks are cheap relative to earnings. Eventually, as those shocks dissipate and negative shocks intervene, an elevated P/E ratio will give rise to the view that stocks are overvalued. Thus, shocks may generate boom-bust cycles with important implications for the dynamics of stock prices. This paper attempts to assess the persistence of innovations to the quarterly EPS of the S&P 500 index. The plan of the work is as follows. Section I describes the data. Section II presents results of various parametric, semi-parametric and non-parametric tests of persistence, including the classical unit root tests, ARFIMA models, and variance ratio tests. Section III concludes.

I. The Data

Typically, firms report quarterly results between one to three months after the reference quarter. Often, one or more major firms reporting better than expected EPS may fuel a
broad-based stock market rally.\footnote{For example, on January 22-23, 2007, Corning, Cingular, Sun Microsystems, and Yahoo, among others, reported higher-than-expected EPS for 2006:Q4. Fueled by EPS optimism and in spite of higher oil prices, on January 23, 2007 the DJIA, NASDAQ, and S&P 500 rose sharply. It is worth noting that the DJIA closed at a record level that day; never mind that none of the firms reporting sanguine results were components of the DJIA.} One paradigm holds that the market sets equity prices using a forward-looking present value model. Therefore, imputing traders with backward-looking behavior is inconsistent with theory. One explanation that links the previous quarter’s EPS and current stock price changes is that traders attribute persistence to past innovations in EPS. Persistence means that the effect of a shock carries for a long time into the future.

We focus on EPS instead of dividends because of data availability. Eschewing the academic debate over the relevance of EPS versus dividends in equity valuation, it is worth noting that a commonly used valuation method ignores dividends in favor of the price-earnings ratio.\footnote{Williams (1938) argued that common stock has value only because it pays dividends. However, Nobel laureates Miller and Modigliani (1961) posited that the dividend decision does not affect stock prices, and that EPS are the proper basis for establishing the market value of equity. They developed a stock valuation model in which price is the present value of rationally expected EPS adjusted for the firm’s capital investments. Sharpe, Alexander, and Bailey (1999) present a similar model in which the market value of a firm’s equity is independent of the dividend decision. However, the dividend decision is relevant since under some conditions the Miller-Modigliani EPS discount model is equivalent to the dividend discount} The inception date for a quarterly EPS series for the S&P 500 index is 1935:Q1. In contrast, there is no comparable dividend series; a quarterly dividend series for the S&P 500 begins only in 1988:Q1. Another dividend series that Standard & Poor’s has reported since 1935 is a four-quarter moving sum, which precludes determining the actual level of dividends in a given quarter until 1988:Q1.

Standard & Poor’s reports several measures of earnings. Accounting, or as reported EPS, is not an accurate measure because it includes transitory factors, what economists call noise that sporadically distort the picture. A better measure, operating EPS, focuses more sharply on the earnings from a firm’s principal operations by
excluding goodwill impairment charges and gains/losses from asset sales. The inception date of the operating EPS series for the S&P 500 is 1988:Q1. In 2001, Standard & Poor’s introduced another measure of performance called core EPS. Core EPS differs from accounting EPS because it excludes goodwill impairment charges, gains/losses from asset sales, pension gains, litigation or insurance settlements and proceeds, and reversal of prior-year charges and provisions.

As measures of performance, operating EPS, and core EPS, are more useful than accounting EPS; however, their short histories limit their usefulness in empirical work. Accordingly, this paper focuses on the quarterly accounting or as reported EPS series. The data source is the Security Price Index Record published by Standard & Poor’s Statistical Service. The data source is the Security Price Index Record published by Standard & Poor’s Statistical Service. Several important empirical papers have used this series. Shiller (1981, 1990), and Campbell and Shiller (1988), show that accounting earnings help to predict the present value of future dividends, thus having relevance for the valuation of common stock. Campbell and Shiller (1998), and Shiller (2000) also use in empirical work the accounting EPS series for the S&P 500 index.

The log EPS series shown in Figure 1 exhibits what, at first sight, seems like a deterministic trend; however, large intermittent shocks buffet the series as in 1938:Q1 and 2002:Q4, among others. The most salient feature is the spectacular drop from 2000:Q4 through 2002:Q4. Several factors contributed to this drop, including the recession of 2001 and a wave of corporate scandals leading to the passage of the Sarbanes-Oxley Act in

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model (Shiller, 1981). In the end, the dividend decision matters because dividends and capital gains are taxed differently (Ang, Blackwell, and Megginson, 1991).
April 2002 that in turn, ushered a surge of probity.\(^3\) However, perhaps the most important factor relates to accounting rule changes regarding the treatment of goodwill that took effect in 2002. To an examination of this issue we presently turn.

Firms depreciate tangible assets periodically to reflect their declining value. Until December 2001, they also amortized intangibles, such as goodwill, regularly. Goodwill is the excess that the acquiring firm pays above the book value of the target firm. However, unlike tangibles, whose value typically decreases gradually, the value of goodwill may drop abruptly as during 2001-02 with the collapse of the equity price bubble. Similarly, a company may find that it overpaid for a brand name, patents, or expected synergies that simply failed to materialize ex post.

In December 2001, Rule 142 of the Financial Accounting Standards Board (FASB) changed the accounting treatment of goodwill. It specified that companies would no longer amortize goodwill regularly against EPS; instead, companies may leave goodwill on their balance sheets indefinitely provided that it does not become impaired. Goodwill impairment occurs if changing business circumstances bring about a decrease in the fair market value of intangible assets. In this case, firms must write down the vanishing value of intangibles to reflect their fair market pricing. The goodwill impairment charge affects contemporaneous EPS adversely, but has a favorable effect on future EPS as the drag of amortization ceases. Hundreds of companies wrote down goodwill in 2002; this event depressed accounting EPS for the S&P 500 that year. In the first quarter of 2002, AOL Time Warner wrote down $54 billion of goodwill. Overall, in 2002 AOL Time Warner wrote off nearly $100 billion of goodwill. AOL Time Warner’s

\(^3\) The major scandals involved Enron, Global Crossing, ImClone, Adelphia Communications, Arthur Andersen, Citigroup, Health South, Merrill Lynch, Qwest Communications, Rite Aid, Tyco International,
spectacular charge lowered EPS for the S&P 500 index by nearly $5.00 per share in the fourth quarter.  

II. Empirical Measures of Persistence

Empirical work in this area falls into four categories: (1) classical tests for unit roots; (2) measures of the fractional differencing parameter from ARFIMA models; (3) non-parametric measures based on the variance ratios proposed by Cochrane (1988), Campbell and Mankiw (1987) and Campbell, Lo, and MacKinlay (1997); and (4)  


4 It is worth remembering that on January 10, 2000, America Online (AOL) announced plans to acquire Time Warner Inc. for roughly $182 billion, in what at the time was the largest merger in history. The merger won accolades as made in heaven, and as the embodiment of the new paradigm. The merger planning and analysis occurred at the height of the dot.com bubble; the NASDAQ COMP stock index peaked three months later on March 10, 2000. In the wake of the technology sector meltdown, the merger became a Titanic financial wreck.
parametric measures based on ARMA models. This section presents test results for all of
the above. Overall, the results reject the unit root null. Evidently, shocks to the log of EPS
dissipate as the series returns to its long-run trend over spans characteristic of the business
cycle.

IIa. The Classical Tests for a Unit Root/Persistence

The literature on testing for a unit root is enormous. After the initial surge of interest,
econometricians realized that the informational value of these tests is limited. For
example, the differencing parameter may range along a continuum, instead of hovering on
the knife-edge of (0,1). Moreover, these tests have low power in borderline cases.
Specifically, the following tests: ADF (Dickey and Fuller, 1979), PP (Phillips and Perron,
1988), KPSS (Kwiatkowski, Phillips, Schmidt and Shin, 1992), are biased if the
alternatives are close to the unit root circle, or if they are of a fractional form (see e.g.,

Nevertheless, surprises in EPS often have spectacular effects on equity prices. If
EPS contain a unit root, exaggerated responses to earnings surprises are justified because
an innovation in the current quarter changes forecasts forever. Therefore, it is worth
testing the log of EPS for a unit root. Several authors have reported that the time series
properties of firms’ earnings are consistent with a random walk with drift (see, e.g., Ball
Brown (1993) also concludes that the random walk with drift model is a reasonable
characterization of firms’ annual earnings. Unlike these authors, we focus on aggregate
earnings.
First, we sample the period 1935:Q1 – 2006:Q3 and test the unit root null against the trend stationary alternative. The augmented Dickey-Fuller (ADF) test has been widely used; the ADF test statistic, -4.93 is below the 1% critical level (-3.98).\(^5\) Schmidt and Phillips (1992), and Elliott, Rothenberg, and Stock (1996) have developed more powerful tests of the unit root null than the Dickey-Fuller tests. The Schmidt and Phillips (SP) Lagrange Multiplier test allows for a trend under both the null and the alternative, without introducing irrelevant parameters as in the Dickey-Fuller tests. The SP test statistic, -6.89, is also below the 1% critical level (-3.59). The Elliott, Rothenberg, and Stock (1996) modified ADF test, the so-called DF-GLS test, also dominates the ADF test (Maddala and Kim 1998). With four autoregressive lags, the DF-GLS test statistic, -4.89, is well below the 1% critical value (-3.48). All three tests strongly reject the unit root for the log of EPS.

An alternative is to test the trend stationary null as proposed by Kwiatkowski, Phillips, Schmidt, and Shin (KPSS, 1992). The KPSS test rejects the null if the test statistic, \(\eta_c\), exceeds the critical values. With the lag truncation parameter ranging from 1 to 8, none of the test statistics exceeds the 1.0% critical value, 0.216, e.g., \(\eta_1(1) = 0.138; \eta_1(2) = 0.10; \eta_6(3) = 0.083; \eta_6(4) = 0.07; \eta_6(5) = 0.063; \eta_6(6) = 0.057; \eta_6(7) = 0.053;\) and \(\eta_6(8) = 0.05\). The lag truncation parameter, shown in parentheses, corrects for error autocorrelation. This test provides evidence supporting the view that log EPS does not contain a unit root, and suggesting that the series is well characterized by a trend stationary representation.

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\(^5\) The number of autoregressive lags (4) in the ADF test equation was selected using the Bayes Information
Illb. The Fractional Differencing Parameter

Following the work of Granger (1980), Granger and Joyeaux (1980), and Hosking (1981), a rapidly growing body of literature has emerged on long-memory or fractionally integrated ARFIMA processes. Robinson (1994a, 2003), Beran (1994), and Baillie (1996) are excellent surveys of the literature. A process $y_t$ is integrated of order $d$ if,

$$
(1 - L)^d y_t = u_t, \quad t = 1, 2, \ldots, \tag{1}
$$

$$
y_t = 0, \quad t \leq 0,
$$

where, $u_t$ is an I(0) process, defined as a covariance stationary process with spectral density function that is positive and finite, and $L$ is the backward shift operator.$^6$ In the event that $d$ is not an integer, the series $y_t$ requires fractional differencing in order to obtain a possibly stationary ARMA series. ARIMA(p,d,q) models in which $d$ is a positive integer are special cases of the general process in (1).

Prior to estimating ARFIMA models, it is useful to inspect the time-series plots, correlograms and periodograms shown in Figure 2. The first-differenced series appears stationary. Volatility clustering is evident at the beginning and end of the sample period; some outliers are also visible. The first 50 sample autocorrelations of the level series decay very slowly indicating non-stationarity, and the correlogram of the first differences points to a seasonal pattern that invites modeling. Finally, the periodogram of the level series presents a large value at the smallest frequency, suggesting that the series is I(d) with $d > 0$, while the periodogram of the first differences indicates a value close to 0 at the smallest frequency, indicating that the series is now over-differenced. Thus, the log of EPS might be I(d) with $d$ constrained between 0 and 1. Note that if a series is I(d) with $d$

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$^6$ The condition $y_t = 0, t \leq 0$ is required for the Type II definition of fractional integration. For an alternative definition (Type I), see Marinucci and Robinson (1999).
> 0, the spectral density is unbounded at the lowest frequency, \((f(0) = \infty)\), and, if it is over-differenced, then \(f(0) = 0\). The periodogram should then mimic that behavior.

### Figure 2: Level and differenced series, correlograms and periodograms

<table>
<thead>
<tr>
<th>Log of EPS: LEPS</th>
<th>(1 – L) LEPS</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlogram of LEPS</th>
<th>Correlogram of (1 – L) LEPS</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Periodogram of LEPS</th>
<th>Periodogram of (1 – L) LEPS</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

The large sample standard error under the null hypothesis of no autocorrelation is \(1/\sqrt{T}\) or roughly 0.059. The periodograms were computed based on the discrete Fourier frequencies \(\lambda_j = 2\pi j/T\).
Robinson’s (1994b) parametric approach does not require preliminary differencing; it allows us to test any real value \( d \) encompassing stationary and nonstationary hypotheses. We use the following formulation to test the null hypothesis in (4) for any real value \( d_o \),

\[
y_i = \beta' z_i + x_i, \tag{2}
\]
\[
(1 - L)^d x_i = u_i, \tag{3}
\]
\[
H_o: \ d = d_o, \tag{4}
\]

Henceforth \( y_i \) is the log of EPS, \( \beta \) is a \((k \times 1)\) vector of unknown parameters; \( z_i \) is a \((k \times 1)\) vector of deterministic terms, and \( u_i \) is assumed to be I(0). In specifying the functional form of \( u_i \) we consider three cases: white noise; AR(1); and seasonal AR(1). Higher AR orders yielded essentially the same results.

We must also specify the deterministic components of \( z_i \) in equation (2). In the simplest case there are no regressors (i.e., \( z_i = 0 \)); next we include an intercept, \( (z_i = 1) \), and lastly, an intercept and a linear time trend such that \( z_i = (1, t)^T \). We test \( H_o \) in (4) for \( d_o \) values ranging from 0 to 2 in 0.001 increments. Table 1 shows the test results; the numbers in bold are the maximum likelihood estimates of \( d \) obtained with the Whittle function. Table 1 also shows the 95% confidence bands for the non-rejections of \( d_o \) using Robinson’s (1994b) parametric approach.
Table 1: Confidence bands for non-rejection of $d_0$

<table>
<thead>
<tr>
<th>Case</th>
<th>No det. components</th>
<th>An intercept</th>
<th>A linear time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>White noise</td>
<td>[0.681 (0.726)]</td>
<td>[0.583 (0.619)]</td>
<td>[0.491 (0.561)]</td>
</tr>
<tr>
<td>AR (1)</td>
<td>[0.739 (0.862)]</td>
<td>[0.636 (0.728)]</td>
<td>[0.527 (0.704)]</td>
</tr>
<tr>
<td>Seasonal AR(1)</td>
<td>[0.658 (0.731)]</td>
<td>[0.569 (0.626)]</td>
<td>[0.490 (0.584)]</td>
</tr>
</tbody>
</table>

It is worth noting that the null hypothesis of $d = 0$ is rejected in all cases. The unit root null ($d = 1$) is also rejected with the exception of the simple AR(1) model without deterministic terms. Evidently, the series is I(d) with the order of integration constrained between 0 and 1. Regardless of the type of disturbance term considered, the order of integration decreases, as additional deterministic terms are included.\(^7\) Table 2 shows that the trend coefficients are highly significant for the three cases. In this light, and in view of the correlograms and periodograms in Figure 2, we select a model with a deterministic time trend and a seasonal AR for the disturbance term. For this specification, the seasonal AR coefficient in Table 2 is 0.3579, and the associated order of integration of the series in Table 1 is 0.584. These values are consistent with the plots in Figure 2, implying over-differentiation, along with a seasonal AR pattern. Based on these results, a plausible model for this series consists of (5) – (7) below:

\[
y_t = -1.8669 + 0.0096t + x_t \\
(1 - L)^{0.584} x_t = u_t \\
u_t = 0.3579u_{t-4} + \varepsilon_t
\]

\(^7\) Robinson and Iacome (2005) examine the interaction of long memory and deterministic trends.
Table 2: Estimated coefficients in models with an intercept and linear trend

<table>
<thead>
<tr>
<th>Model for $u_t$</th>
<th>Constant</th>
<th>Time Trend</th>
<th>AR coeff.</th>
<th>Seas AR coeff</th>
</tr>
</thead>
<tbody>
<tr>
<td>White noise</td>
<td>-1.8884 (-12.713)</td>
<td>0.0098 (7.736)</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>AR (1)</td>
<td>-1.7847 (-10.258)</td>
<td>0.0084 (3.423)</td>
<td>-0.1972</td>
<td>-----</td>
</tr>
<tr>
<td>Seas. AR(1)</td>
<td>-1.8669 (-12.206)</td>
<td>0.0096 (6.850)</td>
<td>-----</td>
<td>0.3579</td>
</tr>
</tbody>
</table>

(t-values in parentheses)

In order to ensure that the series is mean reverting ($d < 1$), we also estimate the model using Robinson’s (1995) semi-parametric method. Figure 3 shows the estimation results using both the original data, and the detrended series.\(^8\) The horizontal and vertical axes show the bandwidth numbers and the estimated values of $d$ (the thin line), respectively. The two thick lines plot the 95% confidence intervals for the unit root null hypothesis. For practically all bandwidths, the estimated values of $d$ fall below the 95% confidence intervals. This buttresses the idea that log EPS exhibits substantially less persistence than a random walk. Finally, Figure 4 displays the impulse response function based on the model in (5) – (7). The values slowly revert to zero; yet, even 50 quarters after the initial shock the responses are significantly different from zero.

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\(^8\) In this case, prior to estimation, the original data was first differenced and subsequently 1.0 was added to the estimated value of $d$. Phillips and Shimotsu (2005) offer a refinement of this procedure, albeit at a cost since estimation requires additional user-chosen values.
IIc. Variance Ratio Tests

This section reports test results of alternative measures of persistence and trend reversion proposed by Campbell and Mankiw (1987), Cochrane (1988), and Campbell, Lo, and MacKinlay (1997). The measures are closely related, yet differ with respect to corrections for degrees of freedom and heteroscedasticity. In the present case, the three versions of the variance ratio test reject the random walk null.

Cochrane’s (1988) measure of the variance ratio (VR) is a robust alternative to the classical unit root tests. Lo and MacKinlay (1988) argue that the VR is more powerful than the Dickey-Fuller unit root test. Perron (1989) shows that in the presence of a sudden change
in the level of a series, the classical unit root tests are biased toward non-rejection of the null. This problem does not arise with the VR statistic. Perron (1993) argued that when a time series contains a break in the level of its deterministic trend, the variance ratio is a robust measure of the spectral density at the zero frequency. Unlike the classical unit root tests that only provide accept or reject information, the VR also provides additional information about the persistence of a series. This is important because market participants would like to assess the impact of a shock on the long-term forecast of EPS. Long-term forecasts of a random walk vary one-for-one with shocks at each date, but long-term forecasts of a trend-stationary series are unaffected.

The VR of a series \( y_t \) may be expressed as,

\[
VR_k = \frac{\text{var}(y_t - y_{t-k})}{k \text{var}(y_t - y_{t-1})},
\]

where \( k \) is the length of the differencing interval. Cochrane (1988) also shows that \( VR_k \) summarizes the autocorrelation of a series,

\[
VR_k = [1 + 2 \sum_{j=1}^{k} \left( \frac{k - j}{k} \right)] \cdot \rho_j
\]

where \( \rho_j \) is the \( j \)th order autocorrelation of the differenced series, i.e.,

\[
\rho_j = \frac{\text{cov}(\Delta y_t, \Delta y_{t-j})}{\text{var}(\Delta y_t)}.
\]

The limit of \( VR_k \) as \( k \) goes to infinity is the scaled spectral density of \( \Delta y_t \) at the zero frequency (Cochrane 1988). If \( y_t \) is a random walk, the variance of its \( k \)-differences will increase linearly with \( k \), i.e., the variance of the annual changes will be four times the variance of the quarterly changes. Thus, for a random walk, the expected value of \( VR_k \) is unitary. Rejection of the unit root null requires a \( VR_k \) significantly different from one at any
differencing interval. If $y_t$ is trend-stationary, $VR_k$ approaches zero as $k$ increases. Intuitively, in this case, equation (9) contains both positive and negative autocorrelations, but the negative autocorrelations dominate in the long term. Figure 5 shows the variance ratio for log EPS, plus/minus one asymptotic standard deviation. All $VR_k$ are significantly lower than unity. The variance ratio decreases as the differencing interval increases, dropping below 0.2 for $k$ greater than 20 quarters, and settling to a value of 0.04 in 60 quarters. These results reject the unit root null, and are consistent with $y_t$ being a trend-stationary process.

Campbell and Mankiw (1987) propose a measure of persistence related to Cochrane’s variance ratio. Assuming that the first-difference of the logged series is a stationary ARMA process, the change in log EPS ($\Delta y_t$) may be expressed as,

$$ \phi(L)\Delta y_t = \theta(L)\epsilon_t, $$

(11)

where,

$$ \phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p, $$

(12)

and,

$$ \theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q. $$

(13)

---

As the differencing interval increases relative to the sample size, the variance ratio exhibits a downward bias, requiring an adjustment for degrees of freedom. $VR_k$ was computed with a degrees of freedom correction, $T/(T-k)$, following Campbell and Mankiw (1987).
Rearranging (11) we arrive at the moving average representation (or impulse response function) for \( \Delta y_t \):

\[
\Delta y_t = \phi(L)^{-1}\theta(L)e_i
\]  

(14)

equivalently,

\[
\Delta y_t = A(L)e_i .
\]  

(15)

If the growth rate of EPS (\( \Delta y_t \)) is stationary, \( A_i \) approaches zero as \( i \) approaches infinity i.e., stationarity of the differenced series implies that an innovation does not change the long term forecast of the growth rate (Campbell and Mankiw, 1987). Inverting the difference operator \( 1 - L \), the moving average representation for the level of \( y_t \) becomes:

\[
y_t = (1-L)^{-1}A(L)e_i
\]  

(16)

\[
y_t = B(L)e_i
\]  

(17)
where,

\[ B_t = \sum_{j=0}^{t} A_j. \]  

(18)

A shock in period \( t \) affects the growth rate in period \( t + j \) by an amount \( A_j \). However, the final impact of a shock in period \( t \) on the level variable is the infinite sum of the moving average coefficients for the differenced process, which Campbell and Mankiw write as \( A(I) \). The value of \( A(I) \) measures the persistence of shocks to \( y_t \), i.e. the change in the long-term forecast of \( y_t \) due to an innovation. Equations (16) – (18) keep open the possibility that the level of log EPS is stationary around a deterministic linear trend, in which case the expected value of \( A(I) \) is zero. For a unit root process, \( A(I) \) will remain close to unity at all differencing intervals. Campbell and Mankiw provide two ways to estimate \( A(I) \): a non-parametric approach that is closely related to Cochrane’s variance ratio, and alternatively, a parametric ARMA approach. Each has adherents and critics, but there is no consensus as to which one is best. We report first results based on their non-parametric approach. Cochrane’s variance ratio provides a lower bound on \( A(I) \). Defining \( R^2 \) as the fraction of the variance that is predictable from the history of \( \Delta y_t \), Campbell and Mankiw show that,

\[ A(I) = \sqrt{\frac{VR}{1-R^2}}. \]  

(19)

The more highly predictable is \( \Delta y_t \), the greater is the difference between the two measures of persistence. Figure 3 shows that the non-parametric estimate of \( A(I) \) is lower than unity at all window sizes, indicating once more that EPS exhibits substantially less persistence than a random walk.
Campbell, Lo, and MacKinlay (1997) propose another variance-ratio test of the random walk null. Under the null, the first-differences of the level series are uncorrelated at all leads and lags, and the variance ratio is unity. Because the volatilities of financial series tend to change over time, they compute the asymptotic distribution of the test statistic under heteroscedasticity. Table 3 shows the test results. Again, the variance ratios are significantly different from unity and reject the random walk null at the 5% level of significance.
Table 3: Variance Ratio of log EPS and Heteroscedasticity-consistent P-values*
1935(1) - 2006(3), T=287

<table>
<thead>
<tr>
<th>k (Quarters)</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>VR</td>
<td>.24</td>
<td>.38</td>
<td>.38</td>
<td>.32</td>
</tr>
<tr>
<td>P-val</td>
<td>.02</td>
<td>.02</td>
<td>.03</td>
<td>.03</td>
</tr>
<tr>
<td></td>
<td>.04</td>
<td>.03</td>
<td>.04</td>
<td>.04</td>
</tr>
</tbody>
</table>

*The P-values are the probabilities of obtaining the estimated variance ratios under the null.

IId. The Evidence from ARMA Models

We estimate the ARMA model in (11) for the differenced log EPS series and obtain the impulse response function for the level of the series. Initially, we consider all possible ARMA models with AR and MA values ranging from 0 through 6. This yields a total of 49 models from the simplest ARMA(0,0) to the most complex, the ARMA(6,6). The ARMA(4,4) model yielded the lowest Bayes Information Criterion (BIC) and was selected on that basis.\(^\text{10}\) Table 4 shows the BIC statistics for the 49 models.

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\(^{10}\) The Bayes Information Criterion (BIC) imposes a greater penalty for each additional regressor than the Akaike Information Criterion (AIC), and thus selects a more parsimonious model than the latter.
Table 4: BIC Statistics: ARMA Models of (1 – L)LEPS

<table>
<thead>
<tr>
<th>AR</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-90.48</td>
<td>-135.95</td>
<td>-130.31</td>
<td>-134.09</td>
<td>-146.72</td>
<td>-147.87</td>
<td>-143.15</td>
</tr>
<tr>
<td>0</td>
<td>-133.01</td>
<td>-148.55</td>
<td>-155.15</td>
<td>-163.18</td>
<td>-157.55</td>
<td>-156.72</td>
<td>-153.22</td>
</tr>
<tr>
<td>1</td>
<td>-127.38</td>
<td>-148.51</td>
<td>-139.63</td>
<td>-157.54</td>
<td>-164.42</td>
<td>-160.14</td>
<td>-164.00</td>
</tr>
<tr>
<td>2</td>
<td>-159.05</td>
<td>-167.49</td>
<td>-163.42</td>
<td>-165.95</td>
<td>-162.67</td>
<td>-146.15</td>
<td>-162.42</td>
</tr>
<tr>
<td>3</td>
<td>-166.21</td>
<td>-162.28</td>
<td>-156.64</td>
<td>-160.13</td>
<td>-178.08*</td>
<td>-153.17</td>
<td>-139.15</td>
</tr>
<tr>
<td>4</td>
<td>-162.99</td>
<td>-160.91</td>
<td>-165.86</td>
<td>-162.67</td>
<td>-172.75</td>
<td>-169.03</td>
<td>-164.62</td>
</tr>
<tr>
<td>5</td>
<td>-158.60</td>
<td>-155.93</td>
<td>-167.79</td>
<td>-162.47</td>
<td>-162.03</td>
<td>-152.62</td>
<td>-163.95</td>
</tr>
</tbody>
</table>

Table 5 shows the estimation results for the selected model. All coefficients are statistically significant at conventional levels. The cumulative impulse response function for the selected model, shown in Figure 7, decreases rapidly - after about 10 quarters the effect of a one standard deviation shock on the level variable (log EPS) has dissipated. This contrasts with the greater persistence shown earlier in Figures 4 – 6.

Table 5. Maximum Likelihood Estimation of ARMA(4,4)
Dependent Variable: (1-L)LEPS. Quarterly Data: 1936:02 - 2006:03. T = 282.
R-Bar² = 0.35, Log Likelihood = 111.71240, Durbin-Watson Stat. = 1.966.
Q(36-8)= 19.4350, Significance Level of Q = 0.8842.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff.</th>
<th>T-Stat</th>
<th>Signif.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CONST.</td>
<td>0.015410794</td>
<td>22.88910</td>
<td>0.00000000</td>
</tr>
<tr>
<td>2. AR{1}</td>
<td>-0.128666842</td>
<td>-2.37679</td>
<td>0.01815368</td>
</tr>
</tbody>
</table>

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3. AR{2}  -0.203967852  -4.38227  0.00001677
4. AR{3}  -0.146550800  -2.58733  0.01018961
5. AR{4}  0.764820395   18.36100  0.00000000
6. MA{1}  -0.246623967   -3.59094  0.00039061
7. MA{2}  0.162079073    2.10993  0.03577456
8. MA{3}  -0.161582451   -2.23011  0.02655349
9. MA{4}  -0.730870475   -8.88617  0.00000000

Figure 7. Accumulated Response to a 1 Standard Deviation Shock +/- 2 S.E.

III. Conclusions

Underscoring the importance of the subject, a large body of literature in empirical finance and economics investigates the persistence of time series. This paper investigates the persistence of accounting earnings. This is worth doing because accounting earnings have informational value for future dividends; play an important role in equity valuation, and the paper fills a vacuum in the literature. Further, to the extent that shocks to EPS persist, they may help to generate boom-bust cycles in equity values. Thus, it is important to measure the persistence of shocks to EPS.

Within the I(0)/I(1) paradigm the empirical evidence obtained from a large number of tests rejects the I(1) specification, supporting instead a trend-stationary representation. When
fractional orders of integration are considered, the results indicate that the series is long memory ($d > 0$) and mean reverting ($d < 1$). The responses decay slowly to zero, albeit 50 quarters after an initial shock the responses remain significantly different from zero. Likewise, the variance ratio evidence suggests that the effect of a shock persists over time spans characteristic of the business cycle. The evidence from parametric ARMA models hints that shocks die out more rapidly, in about 10 quarters, but this evidence is less robust due to problems with the estimation of these models when the inverse MA roots are close to unity. In the present case, one of the inverse MA roots is 0.99. Moreover, the ARMA process is over-differenced.

Future research should consider the interaction between outliers and measured persistence. In addition, the interaction between structural breaks and persistence deserves attention. Although fractional integration and structural breaks are related issues (Diebold and Inoue, 2001, Granger and Hyung, 2004), theoretical models relating both concepts are still not fully developed. Work in this direction is now in progress.
References


