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ABSTRACT

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Key Words: Market definition, Antitrust Analysis, Oligopolistic Competition, Horizontal Product Differentiation
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1. Introduction

The exercise of market definition in merger control is intended to identify the products that form the relevant market and to calculate the market share of each competing product. This is a preliminary task in the process of evaluating the anticompetitive effects that a merger is likely to cause in the relevant market. It therefore has an intermediate role in the analysis of mergers.

The *hypothetical monopolist* test introduced in the 1982 Department of Justice and Federal Trade Commission Horizontal Merger Guidelines in the United States has become predominant in the definition of the relevant market.[1] The rationale for this test is that the relevant market should include all those products and the geographic area such that, if they were all owned by a single firm, this firm would enjoy a significant degree of market power. This means that products other than those forming the relevant market did not impose a significant competitive constraint. We thus observe that market power is at the core of market definition analysis.

In this work I review the approaches to market definition that are ruled in two jurisdictions: in the United States of America and in the European Community.[2,3] The U.S. jurisdiction states the hypothetical monopolist test as the test that must be performed when defining a relevant market, whereas in the European Commission Notice for market definition the emphasis is put in the concept of *substitutability* among products and areas, and no test is followed. Nevertheless, a version of the hypothetical monopolist test is regarded as one way of making the determination of which products and areas should be included.

Another difference between the two approaches is the role attributed to supply-side substitution. In the U.S. competition law, supply-side substitutes are not included in the definition of the relevant market, but are included as *market participants* in the calculation of market shares. In the E.C. competition law, supply-side substitutes are included in the definition of the relevant market if substitution takes place quickly and easily without the need of incurring significant sunk costs. If those conditions (feasibility and immediacy) are not satisfied, supply-side substitutes are only considered later on in the assessment stage and no role is attributed to them in the calculation of market shares.

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[1] See U.S. Department of Justice and the Federal Trade Commission (1997) and G.J. Werden (2003).

[2] Hereafter I will often use the initials U.S. and E.C. to refer to those jurisdictions.

[3] The term "European Community" (and not "European Union") is used as the competition law analysed applies to mergers by European firms that have a Community dimension, and not to those mergers than have only a national dimension in countries of the European Union.

The aim of this paper is to set up a formal framework for market definition analysis in an industry with horizontally differentiated goods. There are two alternative approaches to test for the relevant market: one is the price-increase approach and the other is the profit-maximisation approach. The price-increase approach analyses whether the firms in the candidate market would increase their joint profit if they raised their prices by a small amount. The profit-maximisation approach, by contrast, analyses whether the joint profit-maximising behaviour of firms in the candidate market would lead to a price increase of at least a small but significant amount.[4,5]

I consider three scenarios under each approach. I show that the European hypothetical monopolist test is based on the price-increase approach analysing responses in both price and product specification by the other firms. By contrast, tests valid under the 1992 revision of the U.S. Horizontal Merger Guidelines must be done using the profit-maximisation approach without allowing for reactions of firms outside the candidate market. The European hypothetical monopolist test tends to yield wider markets than the U.S. test. Subsequent revisions of the U.S. Horizontal Merger Guidelines have modified the valid scenario for market definition. The 1984 revision turns the original test based on the price-increase approach to the profit-maximisation approach and the 1992 revision imposes tighter conditions to its previous version.[6]

The remainder of this document is organised as follows: in Section 2, I review the market definition approaches followed in competition policy in the European Community and the United States. In Section 3, I review the different tests of market delineation proposed by economists. In Section 4, I develop a formal analysis that provides the framework for the validity of market definition tests under different policies. Conclusions are presented in Section 5.

[4] The candidate market is arbitrarily chosen by the investigator. A good approach is to take the products of the merging firms as the original candidate market and perform the test. If those goods do not constitute a relevant market, the investigator should include the closest rival good(s) and repeat the analysis until the set of products are proved to be a relevant market.

[5] Both the market definition guidelines and the practice followed in the two jurisdictions analysed suggest an appropriate value for the small and significant increase in price in the range of 5-10%.

[6] Therefore, tests that were valid under the 1982 Guidelines are not valid under the 1984 revision and those valid under the 1984 revision may not be valid under the 1992 revision.

2. Market Definition in Competition Policy

The definition of the relevant market is a task performed in most cases of horizontal mergers, anti-competitive agreements and abuses of market power investigated by the competition authorities. Market definition is the exercise of determining the boundaries of the market in terms of product and geographic area. In the current work, I focus on the analysis of market definition when it is aimed to be used in merger control.[7]

There is a wide consensus among competition authorities, legal experts and economists that anti-competitive effects from mergers are more likely to arise when the merging firms have high market shares. The rationale for this is that, without close substitutes for a product, the firm that markets it has the ability to exercise market power. Firms enjoying market power are able to charge prices above the competitive level and may have lower incentives to preserve the quality of the goods, thus reducing consumer welfare. This problem is less likely to arise if a significant number of consumers are able to switch their purchases towards other products. The higher the number of competing products, the lower the market share of merging firms.

The ability of a firm to profitably increase prices above competitive levels is constrained by the actions of three market participants: *demand-side* competitors, *supply-side* competitors and *potential entrants*. Demand-side competitors are the producers and/or sellers of goods that are demand-side substitutes for the product analysed. Demand-side substitutes are those goods that are currently produced and marketed and towards which the consumers might switch if the price of its rival good increased significantly. Supply-side substitutes are the goods that are not currently produced by firms, but the latter possess the assets, technology and knowledge to produce them and they would do this if the hypothetical price increase made it profitable. The difference between supply-side substitution and potential entry is that supply-side competitors possess the assets that are necessary to produce and market the goods without incurring significant sunk costs, and production would be ready to start within less than one year after the price increase took place. Potential entrants do not possess the necessary assets and/or need more than one year to bring their products to market. Moreover, whereas the competitive constraint imposed by supply-side substitutes has a significant impact on both pre-entry and post-entry prices, potential entry only affects post-entry prices.

[7] For the analysis of market definition in cases of monopoly and joint dominance, the reader is referred to Office of Fair Trading (1998), paragraphs 5.4. to 5.6. and Office of Fair Trading (2001).

In order to understand the analysis of market definition and the way the hypothetical monopolist test has become a predominant tool, I will review the different guidelines and approaches to market definition in competition law in the U.S. and in the E.C.. There are three main differences between the two approaches: First, supply-side substitutes are included in the definition of the relevant market in the E.C. but not in the U.S.; second, the hypothetical monopolist test is the sole test for defining the relevant market in the U.S. whereas in the E.C. it is suggested as one way of performing the search for substitutable products; and, finally, the hypothetical monopolist test suggested in the European Commission Notice is based on the price-increase approach whereas the current hypothetical monopolist test valid in the U.S. is based on the profit-maximisation approach.

2.1. Market definition in the United States Competition Law

In the United States of America, the 1982 Department of Justice Horizontal Merger Guidelines and their successive revisions have defined the hypothetical monopolist test as the sole test for market delineation in antitrust analysis.[8] The 1982 Merger Guidelines declared:

> A market consists of a group of products and an associated geographic area such that (in the absence of new entry) a hypothetical, unregulated firm that made all the sales of those products in that area could increase its profits through a small but significant and non-transitory increase in price (above prevailing or likely future levels).[9]

One point that is worth mentioning is that it is the goods, rather than the firms, which are included in the definition of the relevant market. A second observation suggests that the market definition under the 1982 Horizontal Merger Guidelines relies on the price-increase approach explained in the introduction above.

The 1984 revision of the merger guidelines (as it is explained by G.J. Werden (2003)) moved its language to emphasize that the test should be performed by analysing whether the profit-maximising behaviour of the hypothetical monopolist would imply at least a small but significant and non-transitory increase in price, rather than by analysing whether a small but significant and non-transitory increase in price would increase the hypothetical

[8] A good revision of the history of the hypothetical monopolist paradigm and of the refinements introduced by the successive revisions of the 1982 Horizontal Merger Guidelines can be found in G.J. Werden (2003).

[9] The hypothetical monopolist test is also known as the SSNIP test, where SSNIP stands for the initials of small but significant and non-transitory increase in price. Hereafter I may use the initials SSNIP both to refer to this test and to such a kind of price increase by firms in a candidate relevant market.

monopolist's profit.[10] We can therefore affirm that the 1984 revision turns the analysis towards the profit-maximisation approach.

The 1992 revision of the guidelines introduced an assumption about the behaviour of rivals.[11] The revision imposes the condition that the terms of sale of all other products must be assumed to held constant. This has two main implications for the behaviour of firms outside the candidate market: first, they are assumed not to react to the price increase by changing their prices; and, second, they are assumed not to react to the price increase by producing other goods nor changing their geographic locations.[12]

The constant-price assumption implies that the goal of the Antitrust Agency is to evaluate the *actual* competitive constraint imposed by rival firms and not the *eventual* competitive constraint that the merged entity would face once the merger has taken place.

The constant-production/location assumption implies that only demand-side substitution is taken into account in the definition of the relevant market.

Let us now analyse the role attributed to supply-side substitutes in U.S. merger control. As I have just mentioned, the relevant market is defined taking into account only demand-side substitution but not supply-side substitution:

> Market definition focuses solely on demand substitution factors – i.e., possible consumer responses. Supply substitution factors – i.e., possible production responses – are considered elsewhere in the Guidelines in the identification of firms that participate in the relevant market and the analysis of entry.[13]

After completing this review of the treatment of market definition in the U.S. competition law, we can highlight three facts of the American approach: first, the relevant market must be defined using the hypothetical monopolist test; second, firms producing supply-side substitutes are considered market participants and taken into account in the calculation of

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[10] The 1984 revision to the Horizontal Merger Guidelines reads: "A market is defined as a product or group of products and a geographic area in which it is sold such that a hypothetical, profit maximising firm, not subject to regulation, that was the only present and future seller of those products in that area would impose a 'small but significant and non-transitory increase in price' above prevailing or likely future levels".

[11] The 1992 revision of the Horizontal Merger Guidelines reads: "A market is defined as a product or group of products and a geographic area in which it is produced or sold such that a hypothetical profit maximising firm, not subject to price regulation, that was the only present and future producer or seller of those products in that area likely would impose at least a 'small but significant and non-transitory' increase in price, assuming the terms of sale of all other products are held constant. A relevant market is a group of products and a geographic area that is no bigger than necessary to satisfy this test."

[12] From the 1992 revision on, the U.S. Horizontal Merger Guidelines are jointly issued by the Department of Justice and the Federal Trade Commission.

[13] U.S. Department of Justice and the Federal Trade Commission (1997), p. 4. The Guidelines also describe the conditions that firms producing supply-side substitutes must satisfy in order to be considered as participants in the relevant market.

market shares; finally, whereas the original 1982 Horizontal Merger Guidelines relied on the price-increase approach of the hypothetical monopolist test, the current market definition test is based on the profit-maximisation approach.[14]

2.2. Market definition in the European Community Competition Law

The 1997 European Commission Notice on the definition of the relevant market for the purposes of Community competition law states that:

> A relevant product market comprises all those products and/or services which are regarded as interchangeable or substitutable by the consumer, by reason of the products' characteristics, their prices and their intended use. […] The relevant geographic market comprises the area in which the undertakings concerned are involved in the supply and demand of products or services, in which the conditions of competition are sufficiently homogeneous and which can be distinguished from neighbouring areas because the conditions of competition are appreciably different in those areas. The relevant market within which to assess a given competition issue is therefore established by the combination of the product and geographic markets.[15]

A first observation we can make from this definition of the relevant market is that the European Commission approach to market definition does not define a test that has to be followed, but it rather relies upon the concept of *substitutability*. It however postulates that:

> One way of making this determination can be viewed, as a thought experiment, postulating a hypothetical small, non-transitory change in relative prices and evaluating the likely reactions of customers to that increase.

We can thus observe that the SSNIP test suggested in the 1997 European Commission Notice is based on the price-increase approach. The Notice further specifies that an adequate price increase is in the range of 5-10%.

The Notice also mentions other types of quantitative tests that can be used to define the relevant market:

> There are a number of quantitative tests that have been designed for the purpose of delineating markets. These tests consist of various econometric and statistical approaches: estimates of elasticities and cross-price elasticities for the demand of a

[14] G.J. Werden (2002b) develops a market delineation algorithm based on the hypothetical monopolist test described by the 1992 revision of the U.S. Horizontal Merger Guidelines.

[15] European Commission (1997), p. 2.

> product, tests based on similarity of price movements over time, the analysis of causality between price series and similarity of price levels and/or convergence.[16]

As I have just commented, the Commission's market definition relies upon the concept of *substitution* by consumers. As it is also stated that market definition should include "all those products and/or services which are regarded as interchangeable or substitutable by the consumer", this definition allows for both demand-side and supply-side substitutes to be included in the relevant market as it does not specify if those products and services need to be already produced.

This is actually corroborated later on in the notice where it affirms:

> Supply-side substitution may also be taken into account when defining markets in those situations in which its effects are equivalent to those of demand substitution in terms of effectiveness and immediacy. This requires that suppliers must be able to switch production to the relevant products and market them in the short term and without incurring significant additional costs or risks in response to small and permanent changes in relative prices.

If those conditions are not met, supply-side substitutes would only be taken into account at the assessment stage of competition analysis as well as potential entrants and no role would be given to them in the calculation of market shares.

The consideration of supply-side substitutes in the definition of the relevant markets can only be used to widen the relevant market defined on demand-side grounds but can never be used to narrow it.

Summarising, the definition of the relevant market in European Community competition law is built upon the concept of substitutability of goods by consumers and it does not rely on any analytical test. Supply-side substitutes are included in the definition of the relevant market when substitution takes place with immediacy and without incurring significant sunk costs. The hypothetical monopolist test suggested as one way of performing the market definition analysis is based on the price-increase approach.

I will summarise in next section the different tests proposed by economists for antitrust market definition. Most of those tests are valid for the definition of *economic markets* but fail to test for the relevant markets for merger control (often referred to as *antitrust markets*).

[16] European Commission (1997), p. 2.

3. Tests of Market Delineation

The interest on the empirical delineation of markets was at its peak in the mid-eighties coinciding with the publication of the 1982 U.S. Horizontal Merger Guidelines and its 1984 revision. The most common feature of those works was the search for the relevant geographic market for a homogeneous good. The dominant approach for defining the geographic boundaries of a market had trusted in shipment data for long. That approach did not take into account the prices of the goods, which had become the core of antitrust analysis, intended to identify whether firms enjoy market power, i.e. whether they can charge prices considerably above competitive levels. Market definition tests then became predominantly tests of price data. Some relevant critiques stressing the incompatibility of the markets defined using price data alone with the antitrust markets defined by the Guidelines turned the attention of economists towards tests of demand functions.

I will divide this section into four subsections: first, devoted to tests using shipment data; second, tests of time series of prices; third, tests of demand functions; and a last one to draw conclusions on the empirical tests of market definition.[17]

3.1. Tests of shipment data

The most widely used test for delineating geographic markets, due to the easiness to get the data and to implement it, was for decades the Elzinga-Hogarty test (after K. Elzinga and T. Hogarty (1972, 1973)). This test uses aggregate inflows and outflows of consumers to determine market boundaries. Geographic market boundaries are expanded until both flows are below a cut-off level. The two criteria to define a relevant geographic market according to the Elzinga-Hogarty test are: "little in from outside" – LIFO – and "little out from inside" – LOFI –.

The Elzinga-Hogarty test was used for defining relevant markets in some of the investigations of mergers between hospitals in the United States in past decades. The U.S. antitrust agencies approved some merger cases where the total number of patients who travelled from and to a local area for medical care was higher than ten percent of the area population. They understood that the figure implied that people inside the area saw outside hospitals as good substitutes for the inside hospitals and vice versa.

[17] A good revision of the different tests used in competition analysis can be found in Office of Fair Trading (1999).

This approach has been strongly criticised. G. J. Werden (1981 and 1990) and C.S. Capps, D. Dranove, S. Greenstein and M. Satterthwaite (2001 and 2002) show the weakness of the Elzinga-Hogarty test in the analysis of competition among hospitals. Moreover, the last two papers show that the approach is wrong if patients are heterogeneous in their travel preferences. They actually prove empirically that, even in suburban areas with high outflows of consumers, some hospital mergers could lead to significant price increases.

3.2. Tests of time series of prices

The tests of market delineation proposed by economists have typically focused on the classical definition of an *economic market* due to A. Marshall (1920), who defined a market as an area in which "prices of the same goods tend to equality with due allowance for transportation costs".[18]

There is a vast stream of literature on market definition using tests of time series of prices. The most relevant are tests of price correlations (G. Stigler and R. Sherwin (1985)), tests of price reaction to shocks (I. Horowitz (1981)), tests of causality and exogeneity (N.D. Uri, J. Howell and E.J. Rifkin (1985), N.D. Uri and E.J. Rifkin (1985) and M.E. Slade (1986), among others)) and stationarity tests (M. Forni (2004)).

According to G.J. Werden and L.M. Froeb (1993), the tests just enumerated are valid to define an *economic market* but not to define an *antitrust market* in the fashion of the U.S. Horizontal Merger Guidelines. Such an antitrust market definition should imply that firms producing the goods in the relevant market would jointly enjoy some degree of market power. Markets defined using time series of prices do not necessarily imply the ability of the hypothetical monopolist to profitably increase prices.

Whereas those tests using time series of prices are not compatible with the U.S. antitrust market definition, they are valid tests to provide evidence for the definition of a relevant market under the European Community legislation, that is based on the concept of substitutability.[19]

Tests of time series of prices for market delineation use information on prices only. This is the reason why there is a vast stream of literature on this methodology as price data are much more often available to the researcher than quantity data (that are necessary to estimate elasticities of demand).

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[18] Those transportation costs are also referred to as *arbitrage costs*. When products are homogeneous, two areas are in the same geographic market for that good if there is arbitrage between those areas.

[19] See European Commission (1997), p. 7. See also Office of Fair Trading (1998), paragraph 3.6 for the United Kingdom.

3.3. Tests of demand functions

As market definition is primarily performed under the concept of demand-side substitution, basic consumer theory would suggest the use of cross-price elasticities of demand to test whether two products are substitutes of each other so that they should be included in the same relevant market.[20,21] As G.J. Werden (1998) points out, when "using cross elasticities to delineate markets, the question posed is whether one given product is in the same market with another" so it would not be valid under the U.S. Horizontal Merger Guidelines focused on the evaluation of the pricing behaviour entailed by the joint profit maximisation of the hypothetical monopolist, even though he reports many cases where the U.S. antitrust agencies have trusted in cross-price elasticities of demand to define relevant markets. It would again be valid as evidence for the delineation of the relevant market in the European Community.[22]

A different approach to face the exercise of market definition for antitrust purposes is the use of own-price elasticities of the residual demand curve facing a firm. This approach is based on the idea that, when products are differentiated, the elasticity of the residual demand of a firm provides information about the existence of relevant substitutes for a good and therefore about the degree of market power enjoyed by the firm. The methodology was developed by J.B. Baker and T.F. Bresnahan (1988) to estimate the degree of market power in an industry with differentiated products (the beer industry in the United States). D.T. Scheffman and P.T. Spiller (1987) apply this procedure to define the relevant geographic market for a homogeneous good (unleaded gasoline in the Eastern United States). The residual demand faced by a firm describes the relationship between that firm's price and quantity taking into account the supply response of all other firms.[23] Once

[20] J.S. Bain (1952), p. 25-26, 50-53.

[21] The cross elasticity of demand of good x with respect to good y, $\eta_{x,y}$ is the relative change in the quantity sold of good x when the price of y is changed: $\eta_{x,y} = (\Delta q_x / q_x)/(\Delta p_y / p_y)$.

[22] Competition authorities in the European Community and in the United Kingdom pay great attention to the pattern of consumer substitution between products. Products that have exhibited a significant degree of substitution during a period of time will yield positive values of cross-price elasticities of demand.

[23] According to G.J. Werden (1998), the condition included in the 1992 revision of the U.S. Horizontal Merger Guidelines that the terms of sale of all other firms must remain constant invalidates the use of residual demand elasticities as this method accounts for the supply responses of rivals when substitutes are inelastically supplied. This approach was actually valid under the 1982 and 1984 versions.

the residual demand of a firm has been estimated and the value of the own-price elasticity obtained, we can compare that value with the one implied by a SSNIP.[24]

Another technique that has been widely used in market delineation is the analysis of critical elasticity and critical loss.[25] The main focus of this analysis is the own-price elasticity of the hypothetical monopolist. Economic theory teaches that the profit-maximising price of a monopolist is determined by the monopolist's own-price elasticity of demand.[26,27] The critical elasticity and critical loss analysis compares the prevailing price with the price that maximises the hypothetical monopolist's profit. The critical elasticity is the maximum elasticity of demand a hypothetical profit-maximising monopolist could face at pre-merger prices to be able to profitably increase its price by a small amount. The critical loss is the maximum reduction in output a hypothetical monopolist can tolerate in order for the increase in price to be profitable. One of the problems of this test is that the monopolist is only hypothetical, so it is not possible to calculate directly the elasticity of demand at the monopoly price. The elasticity of demand at the monopoly price usually exceeds that at pre-merger prices. Critiques of the critical elasticity and critical loss analysis can be found in G.J. Werden (1998, 2002a) and D.P. O'Brien and A.L. Wickelgren (2003).

Recent years have witnessed a significant increase in the availability of accurate retail sales data and in computational developments that have favoured the emergence of simulation works to estimate the likely effects of mergers.[28] This makes the analysis of market definition less relevant as the effects of a merger in terms of price increase and consumer surplus can be directly computed. However, this methodology can also be applied to market definition. By estimating the demand system of the industry, we can calculate the effect on profits of an increase in the price of the firms that form the candidate relevant market, and also calculate the increase in price that their joint profit-maximising behaviour would entail. Works under the simulation approach use estimates of both own- and cross-price elasticities of demand. Two noteworthy contributions are those by R. Brenkers and F.

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[24] See also D.R. Kamerschen (1994) and R. Ekelund, G. Ford and J. Jackson (1999) for other works under this approach.

[25] The tests under this approach follow F.I. Johnson (1989) and B.C. Harris and J.J. Simons (1989).

[26] This is a different approach from the one just mentioned. The focus of the previous one was on the residual demand of a firm (that is the industry demand minus the supply of all other firms in the market). The focus of critical loss and critical elasticity analysis is on the monopolist's demand.

[27] As G.J. Werden (1998) points out (p. 387-388), this analysis respects the condition introduced in the 1992 revision of the Horizontal Merger Guidelines that the terms of sale of all other products must be held constant.

[28] See D. Hosken, D.P. O'Brien, D.T. Scheffman and M. Vita (2002) and R.J. Epstein and D.L. Rubinfield (2004).

Verboven (2006) and M. Ivaldi and S. Lörincz (2008). The former analyses market definition in the car market, where competition takes place at two levels: manufacturer level and retail level. Brenkers and Verboven estimate the demand system for firms in each of the two levels and compute the joint profit of manufacturers in the candidate relevant market before and after a small increase in wholesale car price. Their methodology is compatible with the European SSNIP test when no supply-side substitution is feasible.[29] The work by M. Ivaldi and S. Lörincz (2008) compares the initial equilibrium situation with that which would arise if the firms in the candidate market behaved as a profit-maximising cartel. Henceforth, Ivaldi and Lörincz present their test as "a full equilibrium relevant market test", in which price responses by firms outside the candidate market are analysed. Therefore, this test is not compatible with the SSNIP test suggested in the 1997 European Commission's Notice, that is a non-equilibrium test.[30] The test is also incompatible with the 1992 U.S. Horizontal Merger Guidelines, as it takes into account the price reactions by firms outside the candidate market.

3.4. Conclusions on market definition tests

As we have observed, there is a wide range of opinions about the conditions that an empirical test must satisfy in order to correctly define the relevant market under antitrust legislations. Most of the tests proposed by economists have proved to be invalid for this purpose as they do not satisfy some of the conditions imposed by legislations.

In order to shed some light on the way a valid market definition test under the 1992 revision of the U.S. Horizontal Merger Guidelines must be performed, G.J. Werden (2002b) develops a mathematic algorithm of the hypothetical monopolist test.

The next section aims to complement Werden's effort by theoretically illustrating market definition analysis and the features of the hypothetical monopolist test under the E.C. legislation and under the 1982 U.S. Horizontal Merger Guidelines and their revisions.

[29] R. Brenkers and F. Verboven (2006) justify the use of demand-side substitution alone with the following argument: "Developing and marketing new cars typically involves substantial investment costs and significant time delays". In contrast to Brenkers and Verboven's view of supply-side substitution (that is a very widely extended view), supply-side substitution doesn't necessarily imply to build a completely new product. It is just necessary to do a slight modification in an existing product, when that modification is feasible with the existing assets and can take place in the short run.

[30] The authors report evidence that the European SSNIP test may lead to misleading conclusions about the relevant market.

4. Theoretical Setup for Market Definition Analysis

The goal of this section is to theoretically illustrate the conditions that a market definition test must satisfy in order to be compatible with the hypothetical monopolist's tests described in both the U.S. and the E.C. guidelines. As theoretical foundation in the analysis, the approach to competition with horizontally differentiated products is used.

The analysis of competition and market structure with horizontally differentiated products has attracted the interest of economists due to two main implications of product differentiation: first, firms do not care about the decrease in rivals' output caused by an increase in its own output; and, second, firms do not appropriate the whole surplus that the introduction of their products generates. As a consequence, social and private (firms') incentives will differ. The economic analysis of horizontal product differentiation has had two main objectives: first, to predict the degree of product differentiation in a market; and second, to find whether there is too much or too little product diversity, according to social desirability.

H. Hotelling (1929) pioneered the approach to horizontal product differentiation analysis. Consumers are modelled to be heterogeneous in their preferences for the differentiated goods, defined by their characteristics.[31] The original Hotelling model is a two-stage game with two firms choosing locations in the first stage and prices in the second. Since there is no perfect equilibrium, the equilibrium level of product differentiation cannot be predicted.

Economists have approached this non-existence problem by relaxing some of the assumptions used by Hotelling. C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) solved this problem by assuming a quadratic transportation cost rather than the linear cost suggested by Hotelling. They found that firms in equilibrium maximise product differentiation.[32] By relaxing other assumptions of the model, economists have tended to find intermediate levels of product differentiation. The most relevant cases are a non perfectly price inelastic demand (J. Hinloopen and C. van Marrewijk (1999) and K. Rath and G. Zhao (2001)), products differentiated along several dimensions (T. Tabuchi (1994), A. Ansari, N. Economides and J. Steckel (1998) and A. Irmen and J.F. Thisse (1998)) and a number of firms higher than two (N. Economides (1989) and S. Brenner (2005)). In this

[31] An alternative approach to product differentiation, with homogeneous consumers, follows A. Bowley (1924).

[32] This finding is backed by the economic logic as firms are able to relax price competition by differentiating their products, thus obtaining higher profits. See, among many others, A. Shaked and J. Sutton (1982).

work I focus on the last case as market definition analysis, using the SSNIP test for merger control, requires that the number of firms must be higher than two.

In order to identify the econometric tests that are valid under the European Commission Notice and under the U.S. Horizontal Merger Guidelines and each of their revisions, I will sketch two models of competition with horizontally differentiated products where the number of firms is higher than two, one based on N. Economides (1989) and the other based on S. Brenner (2005). I will first describe the equilibrium that is reached in each model in terms of locations, sales, prices and profits and then perform the hypothetical monopolist test by choosing two neighbouring firms and analysing whether they form a separate relevant market under different scenarios.

The first model is based on N. Economides (1989) and follows the price-increase approach previously mentioned. It analyses whether a price increase by the firms in the candidate market may be profitable so they should be taken as the relevant market. I will consider a triangular city with three firms that, in equilibrium, are equidistantly located and charge the same price.[33] When two of those firms, taken to form a separate candidate market, increase their prices by a small amount the resulting situation is a non-equilibrium setup, so tests under the price-increase approach are non-equilibrium tests. I will analyse three different scenarios: First, when the firm outside the candidate market is not allowed to react to the price increase; second, when the firm is only allowed to react by changing its price; and third, when firms are allowed to react by changing both price and location. The first and second scenarios only take into account demand-side substitution, while the third scenario considers both demand- and supply-side substitution and, under the assumptions of this exercise, yields a wider relevant market than the first two scenarios. The third scenario corresponds to the definition of relevant market referred to in the European Comission Notice, and both the first and the second scenarios may fit in the test defined by the original 1982 U.S. guidelines, as it is not specified in the guidelines whether price reactions are taken into account.

The second model follows the profit-maximisation approach as it asks whether the joint profit-maximising price of firms in the candidate market is significantly higher than the original price. It uses the model of S. Brenner (2005) that describes the equilibrium in a linear city with three to nine firms when the costs of transport are quadratic. As in the

[33] This is an illustrative version of the circular city analysed by N. Economides (1989) with the same implications as the original circular version.

previous model, the game is played in two stages with firms choosing locations in the first stage and prices in the second and equilibrium is characterised by a vector of equilibrium locations, prices, sales and profits. The hypothetical monopolist test is performed in the following way: two neighbouring firms next to an edge are taken to form the candidate market. I again analyse three different scenarios: First, when all other firms in the industry are not allowed to react by changing neither prices nor locations. The two "merging" firms price according to their second-stage reaction functions, obtained from the condition of joint profit maximisation; second, when firms are allowed to change their prices, keeping their locations constant, so the new prices charged are the second-stage equilibrium prices for all the firms in the industry; and third, when they are allowed to change both prices and locations. In the latter case, the whole two-stage game is played under the assumption of joint profit-maximising behaviour of the two firms in the candidate market. The relevant market under each scenario of this approach is delineated by comparing the changes in prices and profits of the candidate market firms with those in the original noncooperative equilibrium. The market definition test that currently rules in the United States (defined by the 1992 U.S. Horizontal Merger Guidelines) must be analysed under Scenario 1 whereas the definition in the 1984 revision may be analysed under both Scenario 1 and Scenario 2. The analysis is repeated for a model with three to nine firms. Using the same assumptions as in S. Brenner (2005), the two *merging* firms constitute a separate market under the first two scenarios. Under the third scenario, the merged entity only increases profits when the number of firms is lower than five. With five or more firms the market loss in detriment of the other firms is so high that it is not profitable for them to behave as a single entity.

4.1. Price-increase approach

I start my theoretical illustration of the market definition exercise with a model in the fashion of the circular city with quadratic transportation costs analysed in N. Economides (1989). For the sake of simplicity, I consider a triangular city, but the relevant features of the model (every firm in the market is in direct competition with the two adjacent firms and no firm has a monopoly power over any group of consumers) hold likewise. The market is depicted in Figure 1. It resembles a city that is formed by a triangular mass of buildings where there are three streets in the periphery of the city. It may also resemble a triangular island where transit can only be made on the coastline.

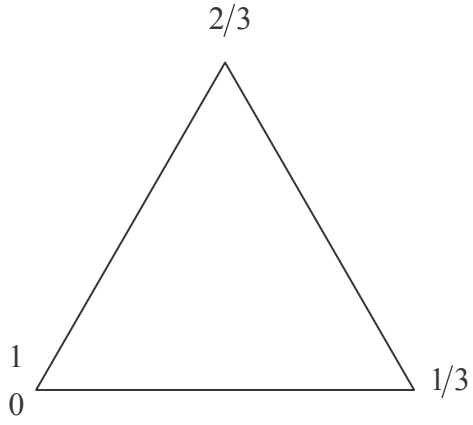

Figure 1

The length of the market (i.e. the perimeter of the triangle) is equal to 1, so each side has a length of $1/3$. Positions are defined by their distance to point 0 in a counter-clock-wise fashion. For example, if each firm were located in each corner as in Figure 2, we would say that firm 1 is at location 0 (or 1), firm 2 is at location $1/3$ and firm 3 is at location $2/3$.

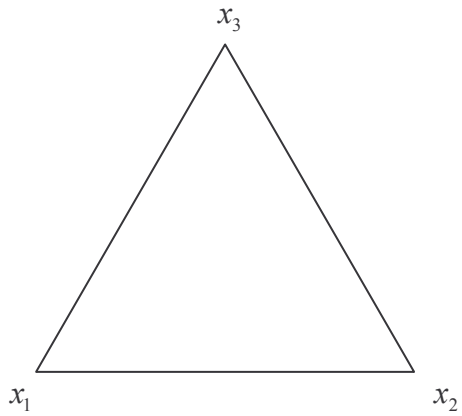

Figure 2

Let us assume that the game is played in two stages. The three firms decide in the first period where to locate, and in the second period they choose prices.[34]

Formally, in the first stage, firms simultaneously choose locations $x = (x_1, x_2, x_3)$ on the triangular city. In the second stage, firms simultaneously decide on prices $p = (p_1, p_2, p_3)$. Firms produce a homogeneous good and sell one or zero units to each consumer. Marginal costs, without loss of generality, are normalised to zero.

Consumers are uniformly distributed on the triangle of unit perimeter. The utility of the consumer located at point ω when she buys from firm j, located at x_j, is:

[34] The notations and algebra in this model are those used by S. Brenner (2005).

$$u_\omega(x_j, p_j) = k - p_j - (x_j - \omega)^2,$$

where $k > 0$ is her reservation price, which is assumed to be equal for all consumers. The existence of a reservation price implies that not all consumers in the triangular city will necessarily buy one unit of the good, making Hotelling's principle of minimum differentiation less plausible.

Consumer utility u_ω has a peak at the point where $\omega = x_j$, that occurs when consumer's location and firm's location coincide. The term $(x_j - \omega)^2$ can be interpreted as the (quadratic) disutility incurred by the consumer when she travels from her location to firm j's location. I follow S. Brenner (2005) and do not include a transportation cost factor multiplying the disutility term. Each consumer will buy one unit of the good from the firm that provides her with the highest utility, only if the reservation price is higher than the sum of price plus transportation cost.

Let us now characterise the non-cooperative perfect Nash equilibrium in pure strategies of this game. In such an equilibrium, all three firms will have a positive market share. If a firm's choice of location and price did not allow it to sell any units of the good, this firm would choose a different location and/or price in order to have positive sales, so no firm can sell zero units of the good in equilibrium.

Let us assume that the reservation price is sufficiently high so that the indifferent consumer between any two firms is strictly better off by consuming the differentiated good rather than not consuming at all. Consequently all consumers buy one unit of the good.[35]

Without loss of generality and for the sake of simplicity, I will impose the condition that firm 1 is located at the South-West corner of the triangle, at point 0 or 1.[36] The formal assumption about firms' locations is $x_1 = 0 \le x_2 \le x_3 \le x_1 = 1$.

I start solving for the equilibrium in this model by calculating the locations of the three marginal consumers (that between firm 1 and firm 2, that between firm 2 and firm 3 and that between firm 3 and firm 1). Demand of each firm is:

$$D_1(p|x) = \frac{1}{2} + \frac{x_2 - x_3}{2} + \frac{p_2 - p_1}{2x_2} + \frac{p_3 - p_1}{2(1 - x_3)}$$

[35] Thus, the equilibrium in this model is restricted to the *competitive* type of equilibrium described in S. Salop (1979) and N. Economides (1993), among others. The problem of nonexistence of *competitive* locational equilibria experienced by N. Economides (1993) is solved by means of the use of a quadratic transportation cost, as in S. Brenner (2005).

[36] By doing so I am not imposing ex-ante that all firms will be at the vertices, but rather that, if they choose equidistant locations, they will be at the vertices. This is done to make the analysis more visual.

$$D_2(p|x) = \frac{x_3}{2} + \frac{p_3 - p_2}{2(x_3 - x_2)} + \frac{p_1 - p_2}{2x_2}$$

$$D_3(p|x) = \frac{1 - x_2}{2} + \frac{p_1 - p_3}{2(1 - x_3)} + \frac{p_2 - p_3}{2(x_3 - x_2)}.$$ [37]

Maximising the second-stage profit function $\Pi_j(p|x) = p_j D_j(p|x)$ with respect to p_j leads to the following price reaction functions:

$$p_1(p|x) = \frac{x_2(1 - x_3)}{2} + \frac{(1 - x_3)p_2 - x_2 p_3}{2(1 + x_2 - x_3)}$$

$$p_2(p|x) = \frac{x_2(x_3 - x_2)}{2} + \frac{(x_3 - x_2)p_1 + x_2 p_3}{2x_3}$$

$$p_3(p|x) = \frac{(1 - x_3)(x_3 - x_2)}{2} + \frac{(x_3 - x_2)p_1 + (1 - x_3)p_2}{2(1 - x_2)}$$

The resolution of this system of equations yields the second-stage equilibrium prices $p_j^*(x)$ and quantities $q_j^*(x)$, that are reported in Appendix 1.

The solution to the first-stage profit-maximisation condition $\Pi_j(x) = p_j^*(x) q_j^*(x)$ leads to the location equilibrium $x_1^* = 0, x_2^* = 1/3, x_3^* = 2/3$, depicted in Figure 2. This corresponds to the equidistant location equilibrium found by N. Economides (1993). Equilibrium prices ($p_i^* = 1/9 \quad \forall i = 1,2,3$), outputs ($q_i^* = 1/3 \quad \forall i = 1,2,3$) and profits ($\Pi_i^* = 1/27 \quad \forall i = 1,2,3$) are displayed in Table 1(a).

Let us now analyse market definition in this equilibrium under the three scenarios I have explained in the introduction of this section.

- Scenario 1: The *hypothetically merging* firms (firm 2 and firm 3) increase their prices by a small but significant amount that I take as 5% of the equilibrium price. Firm 1 is not allowed to react to this price increase by changing any of its two strategy variables: price and location. This is illustrated in Table 1(b). The 5% increase in the price of firms 2 and 3 causes a reduction in the output of those two firms in detriment of firm 1, but the joint profit of firms 2 and 3 increases. We can thus conclude that, under this first scenario, firms 2 and 3 can be considered to be in a separate market.

[37] Note that in those calculations we do not have x_1 as we have fixed it to 0 (for the calculations of the marginal consumer between firm 1 and firm 2) and to 1 (for the calculations of the marginal consumer between firm 3 and firm 1).

	Firm 1	Firm 2	Firm 3	Firm 2+3
(a) Equilibrium				
location	0	0.3333	0.6667	-
price	0.1111	0.1111	0.1111	-
output	0.3333	0.3333	0.3333	0.6667
profit	0.0370	0.0370	0.0370	0.0741
(b) Scenario 1				
location	0	0.3333	0.6667	-
price	0.1111	0.1167	0.1167	-
output	0.3500	0.3250	0.3250	0.6500
profit	0.0389	0.0379	0.0379	0.0758
(c) Scenario 2				
location	0	0.3333	0.6667	-
price	0.1139	0.1167	0.1167	-
output	0.3417	0.3292	0.3292	0.6583
profit	0.0389	0.0384	0.0384	0.0768
(d) Scenario 3				
location	0.3333	0.3333	0.6667	-
price	0.1139	0.1167	0.1167	-
output	0.5062	0	0.4937	0.4937
profit	0.0577	0	0.0576	0.0576

Table 1

- Scenario 2: Under this scenario, firm 1 is allowed to react to the 5% increase in its rivals' price by changing its own price. This case is depicted in Table 1(c). Firm 1 maximises profits by increasing its price by 2.5%. All firms increase profits in comparison both to the equilibrium situation and to that depicted in Scenario 1. Under Scenario 2, the relevant market is formed by firms 2 and 3.

- Scenario 3: Under this scenario, firm 1 is allowed to change its location to any other point in the triangular city. It would remain at point $x_1 = 0$ (or 1) if it were not able to get higher profits at a different location. Firm 1 maximises profits when it relocates to either point $x = 1/3$ or to point $x = 2/3$ of the triangular city. This means that firm 1 will move to either firm 2's or firm 3's location. Without loss of generality, I will assume that it chooses firm 2's location ($x_1 = x_2 = 1/3$). In Table 1(d), we can see that only firm 1 and firm 3 make positive sales. As firm 1's price is lower than firm 2's price, firm 1 undercuts firm 2, the firm at whose location firm 1 has moved to. All the consumers who travel to $x = 1/3$ will buy firm 1's good as it is cheaper than firm 2's. The joint profit of firms 2 and 3 is now lower than in equilibrium, so the 5% price increase is unprofitable. The conclusion we reach from the analysis of Scenario 3 is that, if firms outside the candidate market are allowed to react to the small increase in price by changing its

location, the price increase is not profitable so the relevant market under Scenario 3 includes all three firms.

This simple exercise has illustrated the competitive role that may be played by supply-side substitutes. This example has shown an industry with three firms, each of which is in direct competition with the other two. The tight conditions imposed by some versions of the hypothetical monopolist test that only include demand-side substitutes in the definition of the relevant market may lead us to a picture of the market that may seem unrealistically narrow. Supply-side substitution can be approached by a change in the vector of product characteristics, in the Hotelling analysis by a movement along the space of available locations. Many firms can change their characteristics in order to be more competitive and might do this if its rivals' prices increased significantly (as the hypothetical test considers), doing the price increase unprofitable. The threat of doing this may preclude firms from increasing their prices so those firms are actually competitors and this can only be found when we analyse supply-side substitution in addition to demand-side substitution for market definition, leading to a wider relevant market.

4.2. Profit-maximisation approach

With two firms, the model of the linear market with quadratic transportation costs was analysed by C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) and D. Neven (1985). In equilibrium, firms choose maximum product differentiation.[38]

In this section, I focus on the model described in S. Brenner (2005) where he solves for the equilibrium prices and locations for the models with three to nine firms. Here I reproduce part of his analysis. Let us assume that consumers are evenly distributed along a linear city of length 1, as the one depicted in Figure 3 for the case of three firms.

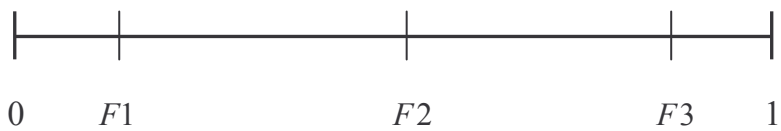

Figure 3

Transportation costs are quadratic. The utility of consumer located at point ω when she buys from firm j, located at x_j, is:

[38] In a market of length 1, firms' equilibrium locations are 0 and 1 with each firm's price equal to 0.5.

$$u_\omega(x_j, p_j) = k - p_j - (x_j - \omega)^2,$$

where $k > 0$ is her reservation price, which is assumed to be equal for all consumers. As in the non-equilibrium model of subsection 4.1, I assume that the reservation price is so high that all consumers in the linear city end up buying one unit of the good.[39]

Let us now proceed to solve for the equilibrium in this model with $n > 2$ firms. After solving for the condition of the marginal consumer between every two firms of equal utility from the consumption of either of them goods, we reach the following demand functions:

$$D_1(p|x) = \frac{p_2 - p_1}{2(x_2 - x_1)} + \frac{x_1 + x_2}{2}$$

$$D_j(p|x) = \frac{p_{j+1} - p_j}{2(x_{j+1} - x_j)} - \frac{p_j - p_{j-1}}{2(x_j - x_{j-1})} + \frac{x_{j+1} - x_{j-1}}{2}, \text{ for } 1 < j < n$$

$$D_n(p|x) = 1 - \frac{p_n - p_{n-1}}{2(x_n - x_{n-1})} - \frac{x_n + x_{n-1}}{2}.$$

Where $p = (p_1, p_2, ..., p_n)$ is the vector of firms' prices and $x = (x_1, x_2, ..., x_n)$ is the vector of firms' locations.

Maximising the profit function $\Pi_j(p|x) = p_j D_j(p|x)$ with respect to p_j leads to the price reaction functions:

$$p_1(p|x) = \frac{[p_2 + (x_1 + x_2)(x_2 - x_1)]}{2}$$

$$p_j(p|x) = \frac{(x_j - x_{j-1})}{2(x_{j+1} - x_{j-1})} p_{j+1} + \frac{(x_{j+1} - x_j)}{2(x_{j+1} - x_{j-1})} p_{j-1} + \frac{(x_{j+1} - x_j)(x_j - x_{j-1})}{2}, \text{ for } 1 < j < n$$

$$p_n(p|x) = \frac{[p_{n-1} + (2 - x_{n-1} - x_n)(x_n - x_{n-1})]}{2}.$$ [40]

This is a system of n equations with n unknowns, that yields equilibrium values of prices $p_j^*(x)$ and outputs $q_j^*(x)$, each as a function of the vector of locations, x.[41] Equilibrium locations are then computed by differentiating profit $\Pi_j(x) = p_j^*(x) q_j^*(x)$ with respect to

[39] So the equilibrium in this model is again restricted to the *competitive* type of equilibrium described in S. Salop (1979) and N. Economides (1993), among others.

[40] Here we can observe that, if any of the firms in this industry changes its price, the prices of all the firms will consequently be changed. This means that, according to the Marshallian definition of an economic market based on firms' cross effects, all the firms in the linear segment are part of the same economic market. The remainder of this analysis supports the antitrust authorities' argument that the concept of economic markets do not correspond to the concept of relevant market for antitrust purposes.

[41] The values for the three-firm and four-firm cases are reported in Appendix 3.

x_j. Let us call x^0 the vector of equilibrium locations thus reached. Then, equilibrium is described by values $\{x^0, p^0(x^0), q^0(x^0), \Pi^0(x^0)\}$.

	(a) Equilibrium				(b) Scenario 1				(c) Scenario 2				(d) Scenario 3			
With 2 firms	location	price	quantity	profit												
firm 1	0	1	0,5	0,5												
firm 2	1	1	0,5	0,5												
With 3 firms	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit
firm 1	0,125	0,2031	0,2708	0,0550	0,125	0,2031	0,5208	0,1058	0,125	0,3281	0,4375	0,1436	0	1	0,5	0,5
firm 2	0,5	0,1719	0,4583	0,0788	0,5	0,3594	0,3229	0,1160	0,5	0,4219	0,4063	0,1714	1	1	0,5	0,5
firm 3	0,875	0,2031	0,2708	0,0550	0,875	0,4766	0,1563	0,0745	0,875	0,5391	0,1563	0,0842	1	1	0	0
firm 2+3				0,1338				0,1905				0,2556				0,5
With 4 firms	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit
firm 1	0,124	0,1066	0,1959	0,0209	0,124	0,1066	0,1959	0,0209	0,124	0,1194	0,2196	0,0262	0,125	0,2031	0,2708	0,0550
firm 2	0,396	0,0717	0,3041	0,0218	0,396	0,0717	0,4679	0,0335	0,396	0,0974	0,4133	0,0403	0,5	0,1719	0,4583	0,0788
firm 3	0,604	0,0717	0,3041	0,0218	0,604	0,1398	0,2062	0,0288	0,604	0,1527	0,2371	0,0362	0,875	0,2031	0,2396	0,0487
firm 4	0,876	0,1066	0,1959	0,0209	0,876	0,2106	0,1300	0,0274	0,876	0,2234	0,1300	0,0290	1	0,2109	0,0313	0,0066
firm 3+4				0,0427				0,0562				0,0653				0,0553
With 5 firms	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit
firm 1	0,104	0,0713	0,1585	0,0113	0,104	0,0713	0,1585	0,0113	0,104	0,0735	0,1634	0,0120	0,124	0,1066	0,1959	0,0209
firm 2	0,329	0,0452	0,2327	0,0105	0,329	0,0452	0,2327	0,0105	0,329	0,0496	0,2555	0,0127	0,396	0,0717	0,3041	0,0218
firm 3	0,5	0,0372	0,2177	0,0081	0,5	0,0372	0,3472	0,0129	0,5	0,0511	0,2991	0,0153	0,604	0,0717	0,3041	0,0218
firm 4	0,671	0,0452	0,2327	0,0105	0,671	0,0895	0,1534	0,0137	0,671	0,0965	0,1738	0,0168	0,876	0,1066	0,1649	0,0176
firm 5	0,896	0,0713	0,1585	0,0113	0,896	0,1382	0,1083	0,0150	0,896	0,1452	0,1083	0,0157	1	0,1143	0,0310	0,0035
firm 4+5				0,0218				0,0287				0,0325				0,0211
With 6 firms	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit
firm 1	0,087	0,0513	0,1337	0,0069	0,087	0,0513	0,1337	0,0069	0,087	0,0518	0,1348	0,0070	0,104	0,0713	0,1585	0,0113
firm 2	0,279	0,0324	0,1947	0,0063	0,279	0,0324	0,1947	0,0063	0,279	0,0333	0,1998	0,0066	0,329	0,0452	0,2327	0,0105
firm 3	0,426	0,0253	0,1716	0,0043	0,426	0,0253	0,1716	0,0043	0,426	0,0280	0,1899	0,0053	0,5	0,0372	0,2177	0,0081
firm 4	0,574	0,0253	0,1716	0,0043	0,574	0,0253	0,2807	0,0071	0,574	0,0353	0,2392	0,0084	0,671	0,0452	0,2327	0,0105
firm 5	0,721	0,0324	0,1947	0,0063	0,721	0,0645	0,1278	0,0082	0,721	0,0695	0,1448	0,0101	0,896	0,0713	0,1325	0,0094
firm 6	0,913	0,0513	0,1337	0,0069	0,913	0,0996	0,0915	0,0091	0,913	0,1046	0,0915	0,0096	1	0,0767	0,0260	0,0020
firm 5+6				0,0132				0,0174				0,0196				0,0114
With 7 firms	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit
firm 1	0,075	0,0383	0,1155	0,0044	0,075	0,0383	0,1155	0,0044	0,075	0,0384	0,1158	0,0044	0,087	0,0513	0,1337	0,0069
firm 2	0,241	0,0242	0,1685	0,0041	0,241	0,0242	0,1685	0,0041	0,241	0,0244	0,1697	0,0041	0,279	0,0324	0,1947	0,0063
firm 3	0,368	0,0190	0,1470	0,0028	0,368	0,0190	0,1470	0,0028	0,368	0,0196	0,1512	0,0030	0,426	0,0253	0,1716	0,0043
firm 4	0,5	0,0182	0,1381	0,0025	0,5	0,0182	0,1381	0,0025	0,5	0,0203	0,1534	0,0031	0,574	0,0253	0,1716	0,0043
firm 5	0,632	0,0190	0,1470	0,0028	0,632	0,0190	0,2413	0,0046	0,632	0,0266	0,2054	0,0055	0,721	0,0324	0,1947	0,0063
firm 6	0,759	0,0242	0,1685	0,0041	0,759	0,0482	0,1107	0,0053	0,759	0,0520	0,1256	0,0065	0,913	0,0513	0,1120	0,0057
firm 7	0,925	0,0383	0,1155	0,0044	0,925	0,0744	0,0790	0,0059	0,925	0,0782	0,0790	0,0062	1	0,0551	0,0218	0,0012
firm 6+7				0,0085				0,0112				0,0127				0,0069
With 8 firms	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit
firm 1	0,066	0,0297	0,1017	0,0030	0,066	0,0297	0,1017	0,0030	0,066	0,0297	0,1018	0,0030	0,075	0,0383	0,1155	0,0044
firm 2	0,212	0,0188	0,1484	0,0028	0,212	0,0188	0,1484	0,0028	0,212	0,0189	0,1487	0,0028	0,241	0,0242	0,1685	0,0041
firm 3	0,324	0,0148	0,1295	0,0019	0,324	0,0148	0,1295	0,0019	0,324	0,0149	0,1305	0,0019	0,368	0,0190	0,1470	0,0028
firm 4	0,441	0,0141	0,1204	0,0017	0,441	0,0141	0,1204	0,0017	0,441	0,0146	0,1240	0,0018	0,5	0,0182	0,1381	0,0025
firm 5	0,559	0,0141	0,1204	0,0017	0,559	0,0141	0,1204	0,0017	0,559	0,0157	0,1339	0,0021	0,632	0,0190	0,1470	0,0028
firm 6	0,676	0,0148	0,1295	0,0019	0,676	0,0148	0,2125	0,0031	0,676	0,0207	0,1810	0,0037	0,759	0,0242	0,1685	0,0041
firm 7	0,788	0,0188	0,1484	0,0028	0,788	0,0374	0,0976	0,0037	0,788	0,0404	0,1107	0,0045	0,925	0,0383	0,0968	0,0037
firm 8	0,934	0,0297	0,1017	0,0030	0,934	0,0577	0,0695	0,0040	0,934	0,0607	0,0695	0,0042	1	0,0412	0,0188	0,0008
firm 7+8				0,0058				0,0077				0,0087				0,0045
With 9 firms	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit	location	price	quantity	profit
firm 1	0,059	0,0238	0,0909	0,0022	0,059	0,0238	0,0909	0,0022	0,059	0,0238	0,0910	0,0022	0,066	0,0297	0,1017	0,0030
firm 2	0,19	0,0150	0,1324	0,0020	0,19	0,0150	0,1324	0,0020	0,19	0,0150	0,1327	0,0020	0,212	0,0188	0,1484	0,0028
firm 3	0,29	0,0117	0,1152	0,0013	0,29	0,0117	0,1152	0,0013	0,29	0,0118	0,1160	0,0014	0,324	0,0148	0,1295	0,0019
firm 4	0,393	0,0111	0,1056	0,0012	0,393	0,0111	0,1056	0,0012	0,393	0,0114	0,1086	0,0012	0,441	0,0141	0,1204	0,0017
firm 5	0,5	0,0106	0,1116	0,0012	0,5	0,0106	0,1116	0,0012	0,5	0,0118	0,1089	0,0013	0,559	0,0141	0,1204	0,0017
firm 6	0,607	0,0111	0,1056	0,0012	0,607	0,0111	0,1056	0,0012	0,607	0,0126	0,1199	0,0015	0,676	0,0148	0,1295	0,0019
firm 7	0,71	0,0117	0,1152	0,0013	0,71	0,0117	0,1893	0,0022	0,71	0,0164	0,1618	0,0027	0,788	0,0188	0,1484	0,0028
firm 8	0,81	0,0150	0,1324	0,0020	0,81	0,0298	0,0870	0,0026	0,81	0,0322	0,0988	0,0032	0,934	0,0297	0,0852	0,0025
firm 9	0,941	0,0238	0,0909	0,0022	0,941	0,0462	0,0623	0,0029	0,941	0,0485	0,0623	0,0030	1	0,0319	0,0165	0,0005
firm 8+9				0,0042				0,0055				0,0062				0,0031

Table 2

They are the reported in Table 2(a) for the industry with two up to nine firms.[42] The equilibrium vectors of prices and quantities, $p^0(x^0)$ and $q^0(x^0)$, correspond to equilibrium locations x^0 derived under the noncooperative model of competition just described. $\Pi_0(x_0)$ is the corresponding vector of equilibrium profits.

We can observe that firms at the edges do not locate at the very ends of the market. So with more than two firms in the industry, the principle of maximum differentiation cannot be said to hold. The second main feature we observe in Table 2(a) is that equilibrium prices exhibit a U-shape pattern. Firms at the edges enjoy a kind of monopoly power over the consumers located between them and the ends. This allows them to charge higher prices than the interior firms. Equilibrium profits exhibit an inverted U-shape pattern when there are three or four firms in the industry. With more than four firms, equilibrium profits exhibit a U-shape pattern. This feature plays a relevant role for market definition as we will see below for the analysis of Scenario 3.

The setup described under Scenario 1 is not an equilibrium. The two firms in the candidate market, that I take to be firm n and firm $n-1$, now maximise their joint profit and price according to their price reaction functions derived from the condition of joint profit maximisation. For the analysis of Scenarios 2 and 3 we need to compute the new equilibrium when firms n and $n-1$ maximise their joint profit. Under Scenario 2 firms are only allowed to change prices and they price according to the new second-stage price equilibrium with locations equal to the original equilibrium locations x^0. Under Scenario 3, firms are allowed to change both prices and locations, so locations are calculated from the first-stage location equilibrium using the second-stage price reaction functions in Scenario 2.

To solve for the new equilibrium we again start using the condition for the marginal consumer between two firms who derives equal utility from the consumption of either good. This condition yields the same demand functions as in the noncooperative equilibrium:

$$D_1(p|x) = \frac{p_2 - p_1}{2(x_2 - x_1)} + \frac{x_1 + x_2}{2}$$

[42] The case of two firms is included to illustrate the tendencies of firms to relocate away from each other in the case of a three-firm industry under Scenario 3.

$$D_j(p|x) = \frac{p_{j+1} - p_j}{2(x_{j+1} - x_j)} - \frac{p_j - p_{j-1}}{2(x_j - x_{j-1})} + \frac{x_{j+1} - x_{j-1}}{2}, \text{ for } 1 < j < n$$

$$D_n(p|x) = 1 - \frac{p_n - p_{n-1}}{2(x_n - x_{n-1})} - \frac{x_n + x_{n-1}}{2}.$$

The problem a firm $j \neq \{n, n-1\}$ faces is to maximise $\Pi_j(p|x) = p_j D_j(p|x)$. The hypothetical monopolist (denoted by m) over goods n and $n-1$ aims to maximise $\Pi_m(p|x) = p_{n-1} D_{n-1}(p|x) + p_n D_n(p|x)$. These two conditions yield the following price reaction functions:

$$p_1(p|x) = \frac{[p_2 + (x_1 + x_2)(x_2 - x_1)]}{2}$$

$$p_j(p|x) = \frac{(x_j - x_{j-1})}{2(x_{j+1} - x_{j-1})} p_{j+1} + \frac{(x_{j+1} - x_j)}{2(x_{j+1} - x_{j-1})} p_{j-1} + \frac{(x_{j+1} - x_j)(x_j - x_{j-1})}{2}, \text{ for } 1 < j < n$$

$$p_{n-1}(p|x) = \frac{(x_{n-1} - x_{n-2})}{(x_n - x_{n-2})} p_n + \frac{(x_n - x_{n-1})}{2(x_n - x_{n-2})} p_{n-2} + \frac{(x_n - x_{n-1})(x_{n-1} - x_{n-2})}{2}$$

$$p_n(p|x) = p_{n-1} + \frac{(2 - x_{n-1} - x_n)(x_n - x_{n-1})}{2}.$$

The solutions of this system are the equilibrium prices $p_j^*(x)$, with corresponding equilibrium quantities $q_j^*(x)$. I will use the notation $p^1(x)$ for the vector of equilibrium prices thus derived and $q^1(x)$ for the corresponding vector of equilibrium outputs. The vector of corresponding profits is $\Pi^1(x) = p^1(x) q^1(x)$.

Let us now analyse market definition under the three specified scenarios.

- Under Scenario 1, firms n and $n-1$ charge prices according to their joint profit-maximising reaction functions. Let us denote $p_m(p_{-m}^0 | x^0) = \{p_{n-1}(p_{-m}^0 | x^0), p_n(p_{-m}^0 | x^0)\}$ the vector of prices of the *merging* firms. All firms remain in their first-stage equilibrium locations x^0 and the prices of firms other than the merging parties remain at their second-stage equilibrium levels p_{-m}^0.[43] Firms $n-1$ and n form a relevant separate market under Scenario 1 if the following three conditions hold:

[43] Note that the vector of prices of firms in the candidate market p_m does not have a superscript as it does not correspond to equilibrium prices.

(1) $$\frac{\Pi_m\left(p_m\left(p_{-m}^0 \middle| x^0\right)\right)-\left[\Pi_{n-1}^0\left(x^0\right)+\Pi_n^0\left(x^0\right)\right]}{\Pi_{n-1}^0\left(x^0\right)+\Pi_n^0\left(x^0\right)}>0$$

(2) $$\frac{p_{n-1}\left(p_{-m}^0 \middle| x^0\right)-p_{n-1}^0\left(x^0\right)}{p_{n-1}^0\left(x^0\right)}>5\%$$

(3) $$\frac{p_n\left(p_{-m}^0 \middle| x^0\right)-p_n^0\left(x^0\right)}{p_n^0\left(x^0\right)}>5\%$$

Condition (1) implies that the two merging firms must increase their joint profit, whereas conditions (2) and (3) require that each firm increases its price by at least a small but significant amount that I again take to be 5%.[44] Table 2(b) reports the values of prices, sales and profits (with unchanged locations) reached under Scenario 1. Comparing those values with the ones in Table 2(a), we can observe that in all cases for $3 \le n \le 9$, the joint profit of the merged entity has increased and the increase in price for each of the two firms is significantly higher than five percent. So we can conclude that firms n and $n-1$ constitute a separate relevant market under Scenario 1.

- Under Scenario 2, firms are allowed to react to the new structure by changing prices. The prices charged by firms are the equilibrium second-stage prices of the game with joint ownership of firms n and $n-1$ and firms located at the equilibrium locations of the original noncooperative game. Scenario 2 is thus characterised by vector $\left\{x^0, p^1\left(x^0\right), q^1\left(x^0\right), \Pi^1\left(x^0\right)\right\}$. Under Scenario 2, firms n and $n-1$ constitute a separate market if the following three conditions are met:

(4) $$\frac{\Pi_m^1\left(x^0\right)-\left[\Pi_{n-1}^0\left(x^0\right)+\Pi_n^0\left(x^0\right)\right]}{\Pi_{n-1}^0\left(x^0\right)+\Pi_n^0\left(x^0\right)}>0$$

(5) $$\frac{p_{n-1}^1\left(x^0\right)-p_{n-1}^0\left(x^0\right)}{p_{n-1}^0\left(x^0\right)}>5\%$$

(6) $$\frac{p_n^1\left(x^0\right)-p_n^0\left(x^0\right)}{p_n^0\left(x^0\right)}>5\%$$

Condition (4) implies that the *merged* entity has to earn higher profits than under noncooperative behaviour. Conditions (5) and (6) imply that each of the firms must increase its price by a small share taken to be 5%. The answer to whether firms $n-1$ and n form a separate market is found in Table 2(c). Values in that column correspond to vector

[44] M. Ivaldi and S. Lörincz (2008) consider the increase in the weighted average price of the firms in the candidate market, rather than in the price of each of them. Although my assumption may look stronger, this is not much relevant given that there are just two firms in the candidate market.

$\{x^0, p^1(x^0), q^1(x^0), \Pi^1(x^0)\}$, with firms pricing according to the new "ownership" structure at the original equilibrium locations. We observe that the profit of the merged entity, $\Pi_m^1(x^0)$, is significantly higher than the sum of profits of the two firms in the original stage, $\Pi_{n-1}^0(x^0)+\Pi_n^0(x^0)$, for all the models analysed, $3 \le n \le 9$. And in all those cases the increase in the price of both firms is significantly higher than 5%. We can thus conclude that, under Scenario 2, firms n and $n-1$ form a separate market.

- The analysis of market definition under Scenario 3 needs the computation of the equilibrium locations in the game with cartelistic behaviour by firms n and $n-1$. By maximising the first-stage equilibrium profits of that game as a function of locations, $\Pi_j^1(x) = p_j^1(x)q_j^1(x)$ and $\Pi_m^1(x) = p_{n-1}^1(x)q_{n-1}^1(x) + p_n^1 D_n^1(x)$, with respect to p_j^1 in the former and with respect to p_{n-1}^1 and p_n^1 in the latter, we reach the new equilibrium vectors $\{x^1, p^1(x^1), q^1(x^1), \Pi^1(x^1)\}$. Under Scenario 3 firms n and $n-1$ form a separate market if the following three conditions are met:

(7) $$\frac{\Pi_m^1(x^1) - [\Pi_{n-1}^0(x^0) + \Pi_n^0(x^0)]}{\Pi_{n-1}^0(x^0) + \Pi_n^0(x^0)} > 0$$

(8) $$\frac{p_{n-1}^1(x^1) - p_{n-1}^0(x^0)}{p_{n-1}^0(x^0)} > 5\%$$

(9) $$\frac{p_n^1(x^1) - p_n^0(x^0)}{p_n^0(x^0)} > 5\%$$

Condition (7) means that the joint profit of the merged entity has to be higher than the sum of profits they had when the two firms behaved noncooperatively. Conditions (8) and (9) reduce to the condition that the hypothetical monopolist must increase profits by at least a small but significant amount. This can be tested by comparing the figures in Table 2(d) with those in Table 2(a). Under this scenario, only in those cases with three and four firms, firms n and $n-1$ form a separate market.

The case of three firms is rather special as the relocation tendencies of firms are to move further apart and the model is thus reduced to the two-firm model of C. d'Aspremont, J.-J. Gabszewicz and J.-F. Thisse (1979) with firms maximising differentiation. In that only case, the profit of the merged entity is higher under Scenario 3 than under Scenarios 1 and 2. In all other cases, when firms are allowed to relocate, the merged entity earns a lower profit than with price adjustment alone. When the number of firms equals five, the joint

profit of the merged entity is lower than in the original equilibrium, so firms n and $n-1$ do not form a relevant market under Scenario 3 when $n \geq 5$. The higher the number of firms, the higher the loss of the merged entity.

As I commented above, the distribution of profits in the original equilibrium exhibits an inverted U-shape pattern when there are less than five firms. In those cases, with 3 and 4 firms, the relevant market under Scenario 3 is defined as firms $n-1$ and n only. By contrast, when equilibrium profits exhibit a U-shape pattern, for $n \geq 5$, firms n and $n-1$ cannot jointly increase profits by behaving as a single profit-maximising entity, as the price and location reactions of the other firms avoid it. So for $n \geq 5$, firms n and $n-1$ do not constitute a relevant market under Scenario 3.[45]

It should be mentioned that, as well as with the use of two end-point firms just performed, the market definition analysis under the profit-maximisation approach might have been done by analysing the joint profit-maximising behaviour of two non-end-point firms. In that case, as those two firms do not have any kind of monopoly power over any group of consumers, their ability to increase profits is further constrained by neighbouring firms, so the analysis would enforce our conclusion that relevant markets tend to be expanded when rival firms are allowed to react.

After describing and illustrating the two approaches to market definition, we can now discuss the types of analysis for market definition that are valid under the European Commission Notice and under the 1982 U.S. Horizontal Merger Guidelines and each of their successive revisions.

4.3. Application to merger guidelines in the U.S. and the E.C.

Before proceeding to the analysis of the relevant markets for merger control in the European Community and in the United States, let us briefly comment on the definition of the economic market in the exercise performed above for both the price-increase and the profit-maximisation approaches. The Marshallian economic definition of a market differs from the definition of a relevant market for antitrust purposes.[46] The economic market definition includes all those firms whose behaviour affects the demand of all other firms

[45] If we want to define the relevant market with $n > 4$, we must repeat the analysis by considering that firms $n-2$, $n-1$ and n behave as a single profit-maximising entity and take all previous steps again. This lies beyond the scope of this analysis that is aimed to illustrate market definition and the different implications of the approaches followed in different jurisdictions that we will comment on in subsection 4.3.

[46] This has been stressed in many works, such as G.J. Werden and L.M. Froeb (1993).

and/or causes changes in their behaviour, while the antitrust market definition excludes all those firms that do not impose a relevant competitive constraint to firms in the candidate market. In both the 3-firm model in the triangular city and the n-firm model in the linear city just reviewed, a price change by any of the firms has an effect on the profits of all other firms and induces changes in their prices according to their price reaction functions. So, the economic market includes all those firms located both on the triangle and all those on the linear segment, while we have seen cases where the relevant antitrust market only included two firms.

Let us now proceed to discuss the applicability of the framework described in subsections 4.1 and 4.2 to the relevant markets defined according to the policies that rule in the European Community and in the United States. Let us first identify which of the two types of substitution (demand-side and supply-side) takes place under each scenario analysed. Under the price-increase approach, Scenarios 1 and 2 only account for demand-side substitution as Scenario 1 analyses demand responses to changes in prices and Scenario 2 analyses firms' price reactions and their consequent demand responses. This is also the pattern under the profit-maximisation approach, where Scenarios 1 and 2 only describe demand-side substitution.[47] Scenario 3, under both the price-increase and profit-maximisation approaches, takes into account both demand-side and supply-side substitution as firms may react to the higher prices not only by changing their prices but also by changing the characteristics of the good they produce in order to make it a closer substitute to those of the *merging* firms.

The model in M. Ivaldi and S. Lörincz (2008) compares the original industry equilibrium with that which results if firms in the candidate market maximise their joint profit. The model allows for reaction in the prices of firms outside the candidate market but does not take into account changes in the characteristics of the products. By allowing firms to react, they claim that their model analyses supply-side substitution in addition to demand-side substitution. However, I consider that, analysing demand-side substitution under Scenarios 1 and 2 and supply-side substitution under Scenario 3, corresponds to a more realistic picture of the concept of supply-side substitution defined in the market definition guidelines in the jurisdictions analysed.[48] I would then consider that the model in M. Ivaldi

[47] In our particular exercise, the two joint-profit maximising firms form a separate market when taking into account demand-side substitution alone.

[48] U.S. Department of Justice and the Federal Trade Commission (1997), paragraph 1.32., European Commission (1997), p. 4 and Office of Fair Trading (1998), paragraph 3.13.

and S. Lörincz (2008) corresponds to the type of analysis described by Scenario 2 of the profit-maximisation approach.

Let us now proceed to set up the different policies to market definition on each of the scenarios described. A synoptic summary is provided in Table 3.

The definition of the relevant market in the European Community includes both demand- and supply-side substitutes. Therefore Scenarios 1 and 2 under both approaches are ruled out. As I have already explained in this work, the hypothetical monopolist test suggested in the European Commission Notice is based on the price-increase approach. As the European market definition relies on the concept of substitutability (as mentioned in Section 2 above) and not on any particular test, it is not totally clear whether a profit-maximisation approach considering both demand- and supply-side substitution would be valid at least as a complementary evidence. Nevertheless, I follow M. Ivaldi and S. Lörincz (2008) who consider that the European hypothetical monopolist test is a non-equilibrium test based on responses to a small increase in price.

As shown in Table 3, market definition according to the 1997 European Commission Notice will be classified under Scenario 3 of the price-increase approach. In those cases where supply-side substitution is not feasible, tests under Scenario 2 would be valid for market definition. And in those cases where prices are too rigid to be changed in the short-run, Scenario 1 of the price-increase approach would provide a valid framework.[49]

The 1982 U.S. Horizontal Merger Guidelines specified that only demand-side substitutes should be included in the relevant market.[50] This rules out the use of tests under Scenario 3 of both approaches. This means that the relevant appropriate framework for market definition under the 1982 Merger Guidelines is the price-increase approach as the test is based on the profitability of an increase in price. As there is no assumption about the price behaviour of firms outside the candidate market, we can assume that price reactions by firms outside the candidate market are not ruled out. I thus conclude that the valid framework for defining the relevant market under the 1982 U.S. guidelines is provided by Scenarios 1 and 2 of the price increase approach.

[49] The last row of Table 3 specifies whether firms form a separate market under each scenario of each approach just for the models developed in subsections 4.1 and 4.2. Therefore, those conclusions must not be taken as a general rule.

[50] With the exception of those cases where supply-side substitution is nearly universal that the market is aggregated as a matter of convenience. According to G.J. Werden (2002) this does not correspond to a market definition, so I will not consider that case in the analysis.

As reported above, the 1984 revision of the guidelines introduces a change in the market definition test specifying that the test should ask whether the hypothetical profit-maximising monopolist would increase its price rather than asking whether the price increase is profitable. This implies that the appropriate framework for market definition under the 1984 revision of the Guidelines is provided by Scenarios 1 and 2 of the profit-maximisation approach.

The 1992 revision of the Guidelines modifies the definition in the 1984 revision imposing the condition that "the terms of sale of all other products are held constant". This rules out Scenario 2 of the profit-maximisation approach as firms are not allowed to react in any way. So the only valid scenario for analysing market definition under the 1992 revision of the Horizontal Merger Guidelines is Scenario 1 of the price-maximisation approach.

	Price-increase approach			Profit-maximisation approach		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
EC Notice 1997			x			
US HMG 1982	x	x				
US HMG 1984				x	x	
US HMG 1992				x		
Separate market	yes	yes	no	yes	yes	no

Table 3

From this theoretical setup of the market definition analysis we can conclude that, if we take literally the reference to the hypothetical monopolist test in the European Commission Notice, the relevant market definition under none of the two main jurisdictions allows for an equilibrium analysis. We also observe that the definition of the relevant market under each of the U.S. guidelines revisions tends to yield similar or identical relevant markets, but the type of formal tests valid under each version are different. A third conclusion we can draw is that the relevant market according to the European definition may yield a wider definition than the one according to the U.S. definition if supply-side substitution is feasible. If supply-side substitutes in the investigations taken by the U.S. antitrust agencies are correctly identified and considered in the calculation of the relevant market, the problem of an excessive narrowness of the market is solved, otherwise too many anticompetitive concerns may be raised as a consequence of the narrowness of the market.

5. Conclusions

In this paper I have developed a formal analysis that provides the framework for the validity of market definition tests under the regulations in the two main jurisdictions. The hypothetical monopolist test is defined in the U.S. Horizontal Merger Guidelines as the sole test for market definition. That test is based on the profit-maximisation approach as it asks whether a joint profit-maximising behaviour by firms in the candidate market implies a significant increase in price. By contrast, the hypothetical monopolist test mentioned in the 1997 European Commission Notice is based on the price-increase approach as it asks whether a small but significant increase in price by firms in the candidate market is profitable for them.

Moreover, whereas the U.S. market definition only includes demand-side substitutes, the European market definition includes both demand-side and supply-side competitors and it thus may yield a wider relevant market than the latter.

In the United States, tests valid under the different revisions of the 1982 Horizontal Merger Guidelines tend to differ from each other as those revisions have been changing the way market definition must be approached. The hypothetical monopolist test under the current regulation is based on the profit-maximisation approach with no allowance for rival firms' reactions.

The aforementioned reasons imply that empirical tests of market definition must take into account the conditions that each test imposes, specially in the case of the United States where those conditions are more restricted and explicitly stated.

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Appendix 1: Equilibrium prices and quantities of the price-increase approach

Here I report the second-stage equilibrium prices and quantities as a function of first-stage locations for the three firms analysed in the market definition exercise under the price-increase approach:

$$p_1^*(x) = \frac{x_2(1-x_3)}{2} + \frac{2x_2(1-x_3)(x_3-x_2)(1+x_2-x_3)(2-x_2+2x_3-x_2x_3)}{A\times B - x_2(1-x_3)(2+x_2-x_3)^2}$$

$$p_2^*(x) = \frac{x_2(x_3-x_2)(1+x_2-x_3)[(1+x_3)A+(1-x_3)(2-x_2)(2+x_2-x_3)]}{A\times B - x_2(1-x_3)(2+x_2-x_3)^2}$$

$$p_3^*(x) = \frac{(1-x_3)(x_3-x_2)(1+x_2-x_3)[(2-x_2)B+x_2(1+x_3)(2+x_2-x_3)]}{A\times B - x_2(1-x_3)(2+x_2-x_3)^2}$$

$$q_1^*(x) = \frac{1+x_2-x_3}{4} + \frac{(x_3-x_2)(1+x_2-x_3)}{A\times B - x_2(1-x_3)(2+x_2-x_3)^2}\times$$
$$\{(1+x_3)A+(2-x_2)B-2(2-x_2+2x_3-x_2x_3)(1+x_2-x_3)+(2+x_2-x_3)[(1-x_3)(2-x_2)+x_2(1+x_3)]\}$$

$$q_2^*(x) = \frac{1+x_3}{4} + \frac{1+x_2-x_3}{2A\times B - 2x_2(1-x_3)(2+x_2-x_3)^2}\times$$
$$\{(1-x_3)(2-x_2)B-x_3(1+x_3)A-(1-x_3)[(2+x_2-x_3+(x_3-x_2)(2-x_2))-2(x_3-x_2)(2-x_2+2x_3-x_2x_3)]\}$$

$$q_3^*(x) = \frac{2-x_2}{4} + \frac{1+x_2-x_3}{2A\times B - 2x_2(1-x_3)(2+x_2-x_3)^2}\times$$
$$\{2(1+x_3)A-(1-x_2)(2-x_2)B+2x_2(x_3-x_2)(2-x_2+2x_3-x_2x_3)-x_2(2+x_2-x_3)(-1+3x_3-2x_2x_3)\}$$

$$A = 4(1-x_2)(1+x_2-x_3)-x_2(x_3-x_2)$$

$$B = 4x_3(1+x_2-x_3)-(x_3-x_2)(1-x_3)$$

Appendix 2: Equilibrium prices and quantities of the profit-maximisation approach

I will report here the equilibrium values of prices and quantities as a function of the vector of locations for the cases of a market with three and with four firms under the price-maximisation approach:

(i) 3-firm equilibrium:

$$p_1^*(x) = \frac{(x_2 - x_1)[(x_3 - x_2)(2 + x_3 - x_1) + 3(x_3 - x_1)(x_1 + x_2)]}{6(x_3 - x_1)}$$

$$p_2^*(x) = \frac{(x_2 - x_1)(x_3 - x_2)(2 + x_3 - x_1)}{3(x_3 - x_1)}$$

$$p_3^*(x) = \frac{(x_3 - x_2)[(x_2 - x_1)(2 + x_3 - x_1) + 3(x_3 - x_1)(2 - x_2 + x_3)]}{6(x_3 - x_1)}$$

$$q_1^*(x) = \frac{(x_3 - x_2)(2 + x_3 - x_1)}{12(x_3 - x_1)} + \frac{x_1 + x_2}{4}$$

$$q_2^*(x) = \frac{2 + x_3 - x_1}{6}$$

$$q_3^*(x) = \frac{(x_2 - x_1)(2 + x_3 - x_1)}{12(x_3 - x_1)} + \frac{2 - x_2 + x_3}{4}$$

(ii) 4-firm equilibrium:

$$p_1^*(x) = \frac{(x_3 - x_2)(x_2 - x_1)A}{2B} + \frac{(x_2 - x_1)(x_1 + x_2)}{2}$$

$$p_2^*(x) = \frac{(x_3 - x_2)(x_2 - x_1)A}{B}$$

$$p_3^*(x) = \frac{(x_3 - x_2)(3x_3 + x_2 - 4x_1)A}{2B} - \frac{(x_3 - x_2)(3x_3 + x_2 - x_1)}{2}$$

$$p_4^*(x) = \frac{(x_3 - x_2)(3x_3 + x_2 - 4x_1)A}{4B} + \frac{2(x_4 - x_3)(2 - x_3 - x_4) - (x_3 - x_2)(3x_3 + x_2 - x_1)}{4}$$

$$q_1^*(x) = \frac{(x_3 - x_2)A}{4B} + \frac{x_1 + x_2}{4}$$

$$q_2^*(x) = \frac{(x_3 - x_1)A}{2B}$$

$$q_3^*(x) = \frac{2 + x_4 + x_3 - x_2 - x_1}{4} + \frac{(x_3 - x_2)(2x_3 + x_2 - x_1)}{8(x_4 - x_3)}$$
$$- \frac{[(3x_3 + x_2 - 4x_1)(x_3 - x_2) + 2(3x_3 - x_2 - 2x_1)(x_4 - x_3)]A}{8(x_4 - x_3)B}$$

$$q_4^*(x) = 1 + \frac{(3x_3 + x_2 - 4x_1)(x_3 - x_2)A}{8(x_4 - x_3)B} - \frac{2 + x_3 + x_4}{4} - \frac{(x_3 - x_2)(2x_3 + x_2 - x_1)}{8(x_4 - x_3)}$$

$$A = \left[4x_4(x_3 - x_1) + 4(x_4 - x_3) + 3x_2(x_1 - x_2 - x_3) + x_1x_3 + 2x_4^2\right]$$

$$B = 3[4x_4(x_3 - x_1) - x_3(x_3 + 2x_2) + x_2(4x_1 - x_2)]$$