



Testing the Fisher hypothesis in the G-7 countries using I(d) techniques



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ABSTRACT

This paper revisits the Fisher hypothesis concerning the determination of real rates by estimating fractional integration and cointegration models for nominal interest rates and expected inflation in the G7 countries. Two sets of results are obtained under the alternative assumptions of white noise and Bloomfield (1973) autocorrelated errors respectively. The univariate analysis suggests that the differencing parameter is higher than 1 for most series in the former case, whilst the unit root null cannot be rejected for the majority of them in the latter case. The multivariate results imply that there exists a positive relationship, linking nominal interest rates to inflation; however, there is no evidence of the full adjustment of the former to the latter required by the Fisher hypothesis.

1. Introduction

The long-run behaviour of interest rates is normally analysed using the so-called Fisher relationship (see Fisher, 1930) linking nominal rates and expected inflation and requiring a one-to-one adjustment of the former to the latter, in the absence of which permanent shocks to either inflation or nominal rates have permanent effects on real rates themselves, which is inconsistent with standard models of intertemporal asset pricing as well as superneutrality (to the extent that inflation is a monetary phenomenon), both requiring stationarity of real interest rates. Neoclassical models of dynamic growth also have the property that consumption growth and real rates should both be constant in the steady state.

However, the Fisher relationship is often not supported by the empirical evidence. Many studies, going back to Mishkin (1992), find that the slope coefficient in a regression of inflation against nominal rates is significantly different from one, and the real rates exhibit a unit root. This “paradox” (see Carmichael and Stebbing, 1983) might be due to various reasons, such as not measuring accurately inflationary expectations (see, e.g. Woodward, 1992), overlooking taxation (see Darby, 1975), using short rather than long rates (see Gilbert and Yeoward, 1994), not distinguishing between short- and long-run Fisher effects (see Mishkin, 1992), and finally differences in estimation procedures; in particular, Crowder and Hoffman (1996) and Caporale and Pittis (2004) both showed that using the estimators with the best small sample properties and appropriately computed empirical critical values (rather than the asymptotical ones) produces evidence more supportive of the Fisher effect. All such studies, though, analyse the relationship between interest rates and inflation on the basis of the classic dichotomy between I(0) and I(1) variables, which imposes rather restrictive assumptions on the stochastic behaviour of the variables of interest.

By contrast, in the present paper we examine the Fisher effect in the G-7 by adopting a fractional integration/cointegration

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framework that does not restrict the order of integration d to be 0 or 1 and allows it instead to take any real value; this is clearly a much more general specification allowing for a richer dynamic structure. In particular, consider the following model:

$$R_t = \alpha + \beta \pi_{t+1} + \varepsilon_t, \quad (1)$$

where R_t is the nominal interest rate, π_t is the inflation rate (under the implicit assumption of rational expectations, i.e., $\pi_{t+1} = \pi_t^e$), where π_t^e is expected inflation, and ε_t is an I(0) error term, defined as a covariance stationary process with a spectral density function that is positive and finite at all frequencies. In this context, if both R_t and π_t are I(0) variables, standard regression methods can be applied. On the other hand, if they are I(1), a cointegration approach becomes necessary. Unlike in the classical case, we allow these variables to be I(d) with $0 < d < 1$ or $d > 1$. First we estimate \tilde{d}_R and \tilde{d}_π , i.e., their respective orders of integration, then we make the series stationary I(0) by differencing them to obtain $\tilde{R}_t = (1 - L)^{\tilde{d}_R} R_t$ and $\tilde{\pi}_t = (1 - L)^{\tilde{d}_\pi} \pi_{t+1}$, where L stands for the lag operator, i.e., $Lz_t = z_{t-1}$, and \tilde{d}_R and \tilde{d}_π are the estimated orders of integration of the two variables. Then, one can estimate the following regression:

$$\tilde{R}_t = \alpha + \beta \tilde{\pi}_t + \varepsilon_t, \quad t = 1, 2, \quad (2)$$

where the null hypothesis in (2) is:

$$H_0: \beta = 1$$

and both variables are I(0) as a result of taking differences as appropriate. We also examine the cases where \tilde{d}_R and \tilde{d}_π above are restricted to be either 1 or 0 as in the classical integration and cointegration frameworks for testing the Fisher hypothesis, but also test the order of integration of the errors in (2) to allow for a greater degree of generality as explained in Section 3.

The layout of the paper is the following. Section 2 briefly reviews the literature on the Fisher effect, and more generally on the relationship between nominal interest rates and expected inflation rates. Section 3 outlines the empirical methodology. Section 4 presents the main empirical results. Section 5 offers some concluding remarks.

2. The Fisher effect: a brief literature review

Early studies analysed the Fisher effect without considering stationarity issues. These include Fama (1975), Nelson and Schwert (1977) and Garbade and Wachtel (1978). Gilbert and Yeoward (1994) argued that papers using short rates are not informative about the Fisher effect unless short and long rates are strongly correlated; examples are the papers by Summers (1983) and Barsky (1987).

The following generation of studies took a cointegration approach (see, e.g., Mishkin, 1992; Evans and Lewis, 1995; Wallace and Warner, 1993). Engsted (1995) looked at the spread between the long-term (multi-period) interest rate and the one-period inflation estimating a VAR model and found considerable cross-country differences.

Crowder and Wohar (1999) analysed both taxable US Treasury and tax exempt municipal bond interest rates and found a “Darby effect”, i.e. evidence that rational agents require nominal rates to adjust in response to movements in “tax-adjusted” expected inflation. Crowder and Hoffman (1996) showed that the choice of estimator is crucial, an issue further investigated by Caporale and Pittis (2004), who provided extensive evidence that if the estimators with the best small sample properties are used and statistical tests are carried out with appropriate empirical critical values the data are supportive of the Fisher effect.

More recently, fractional integration and fractional cointegration techniques have been used to analyse the long memory properties of inflation and interest rates. For example, Shea (1991) investigated the consequences of long memory in interest rates for tests of the expectations hypothesis of the term structure. Phillips (1998) found stationarity but with a high degree of dependence in US interest rates. Tsay (2000) modelled interest rates as AutoRegressive Fractionally Integrated Moving Average (ARFIMA) processes and concluded that the ex-post real interest rate can be well described as a fractionally integrated process. Further evidence of long-memory behaviour in interest rates is provided by Barkoulas and Baum (1997), Meade and Maier (2003), Gil-Alana (2004a,b), Couchman et al. (2006), Gil-Alana and Moreno (2012), Haug (2014), Apergis et al. (2015), Abbritti et al. (2016), etc. As for inflation rates, evidence of long memory has been reported in many papers including Hassler (1993), Delgado and Robinson (1994), Hassler and Wolters (1995), Baillie et al. (1996), Baum et al. (1999), Hyung et al. (2006), Kumar and Okimoto (2007), etc. Lardic and Mignon (2003) found some evidence for the Fisher hypothesis in the G7 countries using semi-parametric I(d) techniques based on log-periodogram regressions; by contrast, no evidence of cointegration between short-term interest rates and inflation was found by Ghazali and Ramlee (2003) by estimating fully parameterised AutoRegressive Fractionally Integrated Moving Average (ARFIMA) models for the same set of countries. Kasman et al. (2006) examined the Fisher relationship with fractional cointegration techniques in 33 developed and developing countries. They found no evidence of cointegration when using classical methods (i.e., Johansen, 1996); however, they found fractional cointegration by using the Geweke and Porter-Hudak (GPH, 1983) log-periodogram approach on the estimated errors from the cointegrating relationship. Similar conclusions were reached in the case of Turkey by Kiran (2013) and for Nigeria by Etuk et al. (2014). The present paper differs from the earlier studies using fractional integration/cointegration methods to test the Fisher hypothesis since, instead of using semi-parametric or fully parametric methods, we adopt a testing procedure for the differencing parameter that relies on non-parametric disturbances.

3. Empirical methodology

3.1. Univariate analysis

As mentioned before in the present study we apply fractional integration methods allowing both nominal interest rates and inflation to be I(d), where d can be a fractional value. The starting point is the estimation of the differencing parameter d in:

$$(1 - L)^{d_i} x_t = v_t, \quad t = 1, 2, \dots, \tag{3}$$

where d_i is the order of integration for inflation (d_π) and interest rates (d_R), v_t is an I(0) process as defined above, and in order to allow for deterministic terms x_t are assumed to be the errors in the model:

$$y_t = \gamma + \delta t + x_t, \quad t = 1, 2, \dots, \tag{4}$$

where y_t is the original series (in our case, interest rates or inflation) and γ and δ stand respectively for the intercept and a linear time trend. For the differencing parameter d we use a testing approach suggested by Robinson (1994) which is based on the Lagrange Multiplier (LM) principle and that, unlike other methods, does not require stationarity and allows d to take any real value. More specifically, it tests the null hypothesis,

$$H_0 : (d_i =) d = d_0,$$

in (3) where x_t can be the errors in a regression model of form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, \dots, \tag{5}$$

where β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of deterministic terms or weakly exogenous terms, and d_0 can be any real value, including both the stationary ($d_0 < 0.5$) and non-stationary ($d_0 \geq 0.5$) cases. This method is based on the Whittle function in the frequency domain (Dahlhaus, 1989) and has several advantages with respect to other methods. Firstly, as just mentioned, it remains valid even for non-stationary processes, and therefore unlike other methods it does not require preliminary differencing (see, e.g., Phillips and Shimotsu, 2005; Lobato and Velasco, 2007; Abadir et al., 2007; etc.): secondly, it has a standard normal limit distribution, unlike most unit root methods where critical values have to be computed numerically case by case; in addition, this limit distribution holds independently of the inclusion of weakly exogenous or deterministic terms in the model, again unlike the unit root case (see, Schmidt and Phillips, 1992); finally, this method is the most efficient one in the Pitman sense against local departures from the null, in the sense that its limit distribution is a standard normal with a non-centrality parameter that is optimal under Gaussianity of the error term.¹ In Table 1–3 we display the value of d_0 for which H_0 cannot be rejected (the lowest statistic in absolute value), using a range of values of d_0 from -1 to 2 with 0.01 increments. In addition, we display the confidence intervals based on this method, noting that classical confidence intervals (including bootstrap type intervals) might be biased in the case of long memory processes, even for large sample sizes (see de Peretti, 2003). Note, however, that the method employed is based on inverting an asymptotic test (Davidson and MacKinnon, 1993), and hence a better performance could be obtained by using inverted bootstrap tests (see, e.g., de Peretti and Siani, 2002, Conti et al., 2008; Arteché and Orbe, 2016; etc.).^{2,3}

Note that under the null hypothesis, the model in (3) and (4) becomes:

$$y_t = \gamma + \delta t + x_t; \quad (1 - L)^{d_0} x_t = u_t; \quad t = 1, 2, \dots, \tag{6}$$

which can be written as a single equation:

$$\tilde{y}_t = \gamma \tilde{1}_t + \delta \tilde{t}_t + u_t; \quad t = 1, 2, \dots, \tag{7}$$

where $\tilde{y}_t = (1 - L)^{d_0} y_t$; $\tilde{1}_t = (1 - L)^{d_0} 1$, and $\tilde{t}_t = (1 - L)^{d_0} t$, and since u_t is I(0) by construction, standard t-tests can be carried out for γ and δ in (7).⁴

3.2. Multivariate analysis

To analyse the relationship between the two variables we first run a regression of the following form:

$$(1 - L)^{d_R} R_t = \alpha + \beta (1 - L)^{d_\pi} \pi_t + u_t, \quad t = 1, 2, \tag{8}$$

¹ See Robinson (1994) for more details. The Fortran codes used for the empirical analysis are available from the authors upon request.
² As mentioned in the conclusions, this issue is left for future work.
³ Alternative methods also based on the likelihood function may be employed. For instance, Lobato and Velasco (2007) proposed a Wald test which is based on a very similar set-up, though clearly it requires a consistent estimate of d, thus being computationally more expensive.
⁴ Note that in case of $d_0 = 1$ the time trend becomes a constant and for $d_0 < 1$ it disappears in the long run.

Table 1
Estimates of d_π and 95% confidence bands for the inflation series.

	No deterministic terms	An intercept	A linear time trend
i) Uncorrelated (white noise) errors			
CANADA	0.95 (0.82, 1.12)	0.97 (0.83, 1.15)	0.97 (0.83, 1.15)
FRANCE	1.15 (1.04, 1.29)	1.25 (1.13, 1.41)	1.25 (1.13, 1.41)
GREAT BRITAIN	1.19 (1.07, 1.35)	1.19 (1.06, 1.35)	1.19 (1.06, 1.35)
GERMANY	1.04 (0.93, 1.22)	0.96 (0.86, 1.09)	0.96 (0.86, 1.09)
ITALY	1.20 (1.09, 1.34)	1.25 (1.15, 1.38)	1.25 (1.15, 1.38)
JAPAN	1.18 (1.05, 1.36)	1.18 (1.06, 1.36)	1.18 (1.06, 1.36)
U.S.A.	1.23 (1.07, 1.44)	1.40 (1.20, 1.67)	1.40 (1.20, 1.67)
ii) Autocorrelated (Bloomfield) errors			
CANADA	0.77 (0.51, 1.09)	0.73 (0.43, 1.11)	0.74 (0.43, 1.11)
FRANCE	1.11 (0.87, 1.46)	1.18 (0.88, 1.52)	1.19 (0.88, 1.54)
GREAT BRITAIN	1.12 (0.89, 1.45)	1.12 (0.86, 1.54)	1.12 (0.88, 1.54)
GERMANY	1.12 (0.90, 1.46)	1.18 (0.93, 1.56)	1.18 (0.93, 1.56)
ITALY	1.24 (1.00, 1.54)	1.40 (1.08, 1.77)	1.40 (1.08, 1.79)
JAPAN	0.99 (0.76, 1.30)	0.99 (0.76, 1.31)	0.99 (0.76, 1.31)
U.S.A.	0.83 (0.57, 1.15)	0.69 (0.46, 1.06)	0.70 (0.42, 1.06)

In bold the most adequate specification according to the t-values for the deterministic terms. In parenthesis the 95% confidence bands of the non-rejection values using [Robinson's \(1994\)](#) tests.

$$(1 - L)^{d_u} u_t = \xi_t, \quad t = 1, 2, \quad (9)$$

estimating simultaneously α , β and d_u , for different cases. In particular, we examine three cases: i) imposing a priori $d_R = d_\pi = 1$ in (8); ii) imposing a priori $d_R = d_\pi = 0$ in (8); (note that these are the most standard cases examined in the literature), and iii) using for d_R and d_π the estimated values obtained previously using the univariate methods, first without any restrictions and then setting d_u in (8) equal to 0. Note that in the case of the estimated values for d_r and d_π in equation (8) there is no obvious economic interpretation for the fractionally differenced interest rate and inflation rate since these are numerical artifacts to produce I(0) variables. We assume the I(0) error term ξ_t in (8) to be in turn uncorrelated (white noise) and autocorrelated, in the latter case using the exponential spectral model of [Bloomfield \(1973\)](#). This is a non-parametric model, in the sense that it is only implicitly defined in terms of its spectral density function, which is given by:

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} \exp\left(2 \sum_{r=1}^m \tau_r \cos(\lambda r)\right), \quad (10)$$

where σ^2 is the variance of the error term and m indicates the last of the Fourier frequencies which is associated with the short-run dynamics. [Bloomfield \(1973\)](#) showed that the spectral density function of a highly parameterised ARMA process can be well approximated by (10); this model produces autocorrelations decaying exponentially as in the AR(MA) case and is stationary for its entire range of values. Moreover, it has been shown that it fits extremely well in the context of fractional integration ([Gil-Alana, 2004c, 2008](#)) and avoid the problem of misspecification inherent in the ARMA modelling approach that might produce an inconsistent estimate of d . As mentioned before, the methodology employed is based on [Robinson \(1994\)](#) since his procedure allows to include (weakly) exogenous variables in the regression model without affecting the limiting behaviour.

4. Empirical results

The series used for the analysis are the long-term (10 year) government bond yield for the nominal interest rate and the first differences of the log CPI series for inflation in the G7 countries, monthly, for the time period January 2006–December 2016 (including all goods); therefore we have 132 observations, which might be a short span to detect long memory effects; nevertheless, several Monte Carlo experiments conducted in [Robinson \(1994\)](#) showed that his tests perform relatively well in finite samples, and various empirical applications based on this method (see, e.g., [Gil-Alana and Robinson, 1997](#)) were conducted with even fewer observations.⁵ The data source is the St. Louis Federal Reserve Bank database, and plots of the two series are displayed in [Fig. 1](#). Nominal interest rates appear to decline in all countries, and inflation is generally lower in the second half of subsample, Japan being the single exception. Despite the monthly frequency of the data, seasonality did not appear to be an issue. Seasonal (monthly) dummies were found to be insignificant, and similarly in seasonal autoregressions the coefficients were very close to 0 in all cases.

⁵ When using the critical values computed in [Gil-Alana \(2000\)](#) for finite samples, the results were almost identical to those reported here based on the asymptotic values.

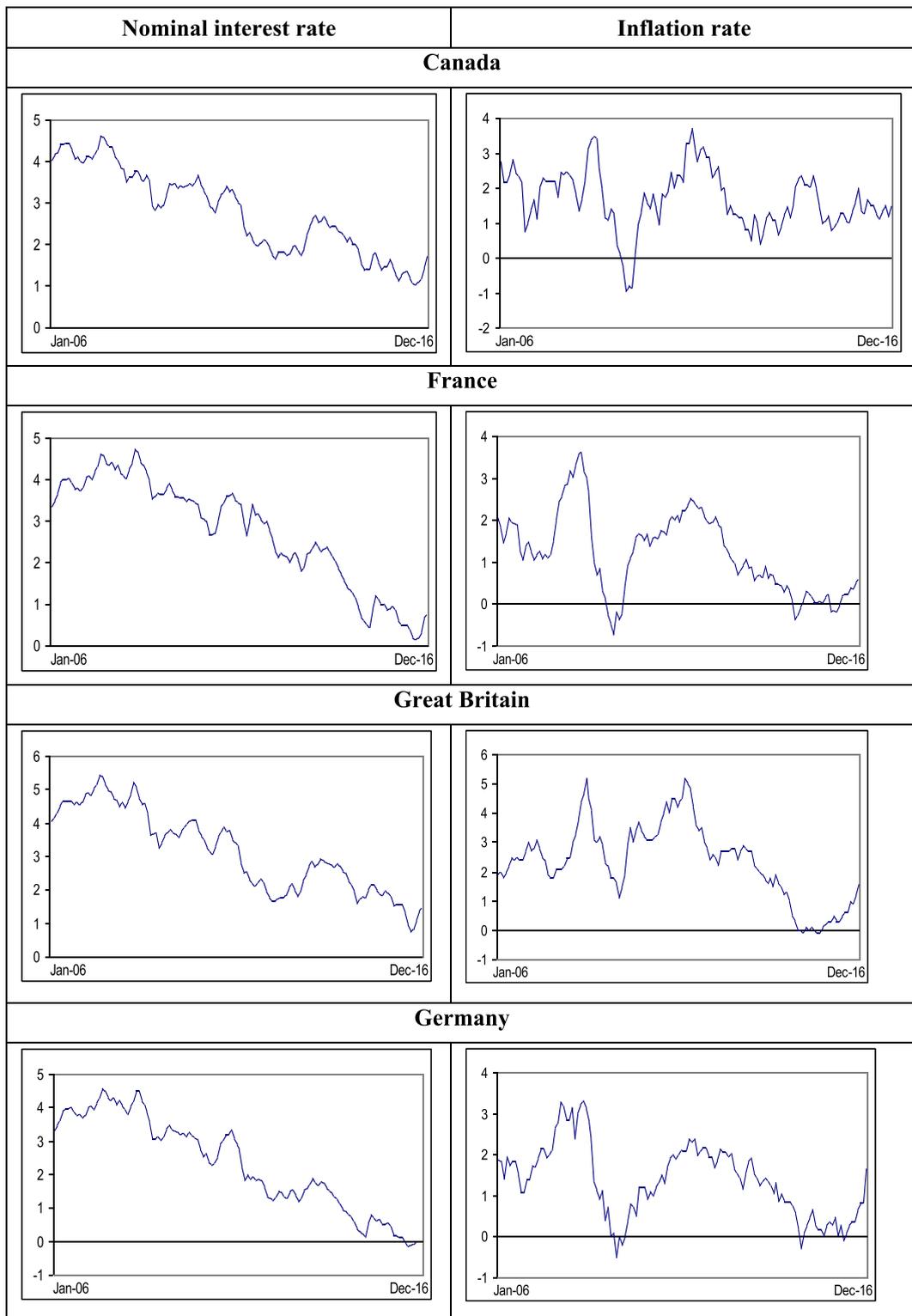


Fig. 1. Time series plots.

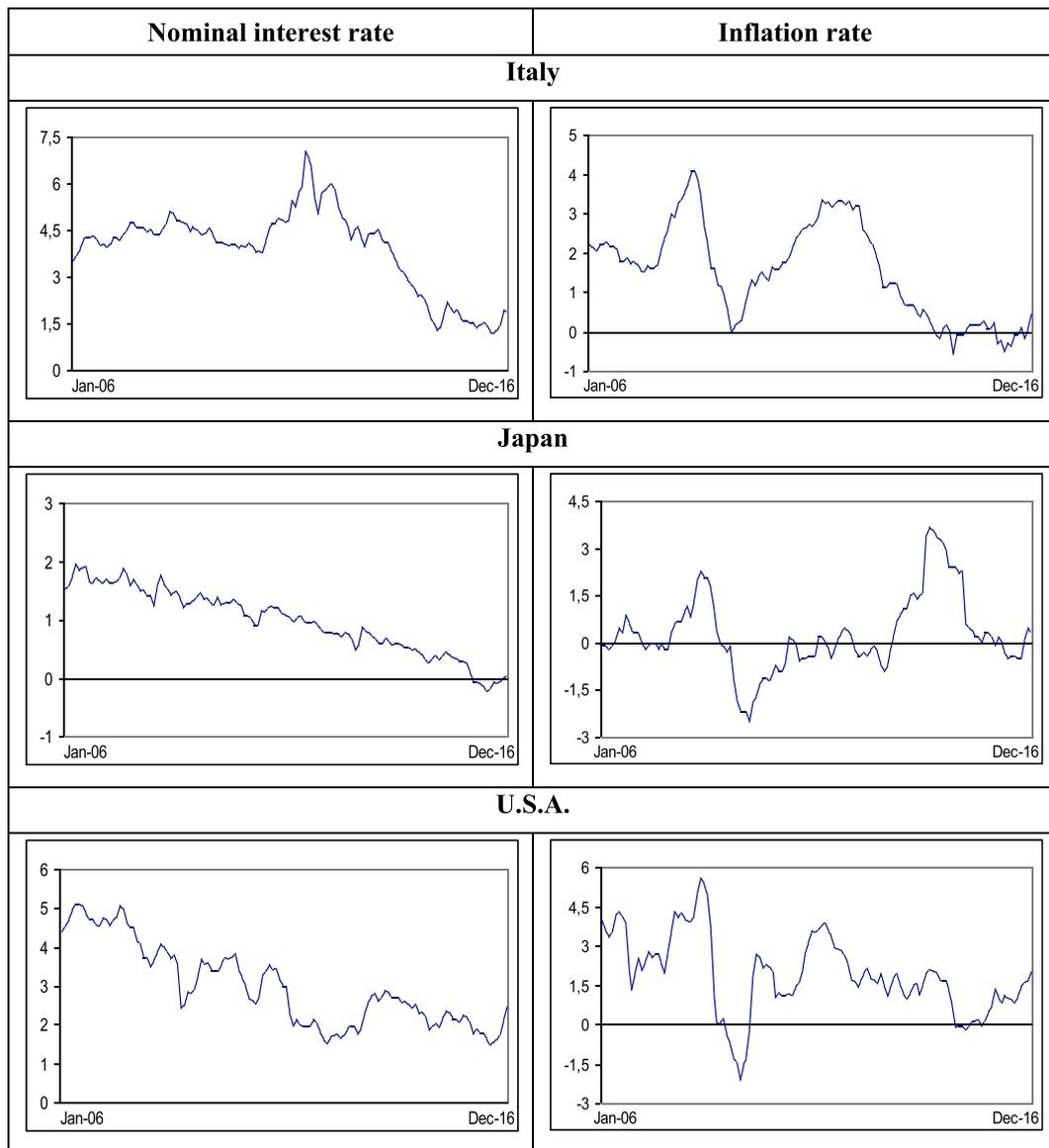


Fig. 1. . (continued).

4.1. Univariate results

We start with the univariate analysis. Tables 1 and 2 display the estimates of d (along with the 95% bands corresponding to their non-rejection values using Robinson's (1994) method), for inflation and interest rates respectively. Following the standard parameterisation for unit root models (Schmidt and Phillips, 1992) we include a linear time trend in the model, and consider the three cases of i) no deterministic terms (i.e., $\gamma = \delta = 0$ a priori in (4)), ii) an intercept (i.e., γ unknown and $\delta = 0$ a priori) and iii) an intercept with a linear time trend (both γ and δ unknown and estimated from the data).⁶ The estimated model is the one given by equation (6) with uncorrelated and Bloomfield (1973) errors (u_t) in turn. In the case of inflation rates, a time trend is required only for the US with auto-correlated errors, and no deterministic terms are required for Japan. As mentioned before, the deterministic trends were selected on the basis of the t-values for the coefficients of the differenced model, first with a specification with an intercept and a time trend, then with an intercept only, and finally with no deterministic terms. Non-linear deterministic trends of the form proposed in Cuestas and Gil-Alana (2016) were rejected in all cases.

⁶ Note that, given the arguments provided in the previous section (eq. (6)) the time trend disappears in the unit root case ($d = 1$), and if u_t is a white noise, y_t becomes a random walk with a drift.

Table 2
Estimates of d_R and 95% confidence bands for the interest rate series.

	No deterministic terms	An intercept	A linear time trend
i) Uncorrelated (white noise) errors			
CANADA	1.11 (0.98, 1.26)	1.20 (1.03, 1.42)	1.20 (1.02, 1.42)
FRANCE	1.12 (1.01, 1.27)	1.23 (1.06, 1.46)	1.23 (1.06, 1.46)
GREAT BRITAIN	1.09 (0.98, 1.24)	1.29 (1.12, 1.51)	1.29 (1.12, 1.51)
GERMANY	1.10 (0.99, 1.26)	1.25 (1.09, 1.48)	1.25 (1.09, 1.47)
ITALY	1.10 (0.99, 1.25)	1.14 (1.02, 1.35)	1.15 (1.02, 1.34)
JAPAN	1.03 (0.92, 1.18)	0.84 (0.74, 1.03)	0.78 (0.56, 1.03)
U.S.A.	1.12 (1.00, 1.27)	1.17 (0.99, 1.38)	1.17 (0.99, 1.38)
ii) Autocorrelated (Bloomfield) errors			
CANADA	1.02 (0.83, 1.32)	0.78 (0.63, 1.13)	0.72 (0.41, 1.13)
FRANCE	1.09 (0.90, 1.38)	0.81 (0.70, 1.06)	0.73 (0.49, 1.06)
GREAT BRITAIN	1.07 (0.88, 1.42)	0.82 (0.63, 1.13)	0.77 (0.50, 1.13)
GERMANY	1.03 (0.85, 1.34)	0.85 (0.72, 1.18)	0.80 (0.53, 1.17)
ITALY	1.11 (0.92, 1.37)	0.93 (0.79, 1.16)	0.92 (0.78, 1.14)
JAPAN	1.00 (0.81, 1.31)	0.66 (0.58, 0.78)	0.20 (0.03, 0.48)
U.S.A.	1.05 (0.84, 1.38)	0.73 (0.58, 1.12)	0.71 (0.49, 1.12)

In bold the most adequate specification according to the t-values for the deterministic terms. In parenthesis the 95% confidence bands of the non-rejection values using [Robinson's \(1994\)](#) tests.

In addition to the 95% confidence intervals of the non-rejection values of d_0 obtained using the tests of [Robinson \(1994\)](#) we also report the value of d_0 with the lowest statistic in absolute value, this being an approximation to the Whittle estimate d in the frequency domain which is the optimised function with the tests of [Robinson \(1994\)](#). The values of d under the assumption of white noise errors are significantly higher than 1, and evidence of a unit root (i.e., $d = 1$) is only obtained in the cases of Canada and Germany. With autocorrelation in u_t , evidence of a unit root is found in all cases except for Italy where the estimated value of d is much higher than 1.

Moving on to interest rates, a time trend seems to be appropriate for Japan with uncorrelated errors, and in all cases except Italy with autocorrelated ones. With a white noise process for u_t , the estimated values of d are much higher than 1 in all cases except for Japan and the US where the unit root null cannot be rejected. Under the assumption of autocorrelation, we obtain evidence of unit roots in all cases except for Japan, where the series seems to be stationary (i.e. with $d < 0.5$).

[Table 3](#) summarises the estimates of d with autocorrelation for u_t , in (5), a more appropriate assumption given the results of the diagnostic tests carried out on the residuals (not reported).⁷ The unit root null is almost never rejected. The only exceptions are the inflation rate in Italy, with an order of integration much higher than 1, and interest rates in Japan, with a value significantly smaller than 1. However, in the case of Italy the confidence bands for the two variables overlap suggesting that the estimated values are not significantly different.

4.2. Multivariate results

[Tables 4–6](#) show the estimated coefficients from equations (8) and (9) under three different assumptions, specifically, in [Table 4](#) d_R and d_π are both set equal to 1; in [Table 5](#), despite the evidence from [Table 3](#), both variables are assumed to be $I(0)$, and therefore $d_R = d_\pi = 0$, while in [Table 6](#) the estimated values from [Table 3](#) are used for d_R and d_π , implying in all these cases that the values of \tilde{R}_t and $\tilde{\pi}_t$ are observed, and then avoiding the issue of unbalanced cointegration as stated in [Sun and Phillips \(2004\)](#).

In [Table 4](#), we impose $d_R = d_\pi = 1$ - this is a common assumption in the empirical literature that usually treats interest rates and inflation as being non-stationary $I(1)$. Therefore, both variables in the regression model, \tilde{R}_t and $\tilde{\pi}_t$ are expected to be $I(0)$. In the case of

Table 3
Estimates of the orders of integration for each series.

	Inflation rates (d_π)	Interest rates (d_R)
CANADA	0.73 (0.43, 1.11)	0.72 (0.41, 1.13)
FRANCE	1.18 (0.88, 1.52)	0.73 (0.49, 1.06)
GREAT BRITAIN	1.12 (0.86, 1.54)	0.77 (0.50, 1.13)
GERMANY	1.18 (0.93, 1.56)	0.80 (0.53, 1.17)
ITALY	1.40 (1.08, 1.77)	0.93 (0.79, 1.16)
JAPAN	0.99 (0.76, 1.30)	0.20 (0.03, 0.48)
U.S.A.	0.70 (0.42, 1.06)	0.71 (0.49, 1.12)

In bold, evidence of unit roots at the 5% level.

⁷ We basically conduct tests of no autocorrelation in the set-up with white noise errors. If this hypothesis is rejected, we choose the non-parametric autocorrelated model of [Bloomfield \(1973\)](#).

Table 4Estimates of d_u , α and β with $d_R = d_\pi = 1$.

	d_u (95% conf. band)	α (t-value)	β (t-value)
i) Uncorrelated (white noise) errors			
CANADA	0.19 (0.03, 0.40)	-0.008 (-0.28)	0.04 (1.48)
FRANCE	0.20 (0.05, 0.42)	-0.005 (-0.16)	0.12 (1.99)
GREAT BRITAIN	0.26 (0.10, 0.48)	-0.006 (-0.14)	0.14 (2.95)
GERMANY	0.24 (0.06, 0.46)	-0.01 (-0.26)	0.03 (0.77)
ITALY	0.13 (0.02, 0.35)	-0.006 (-0.16)	0.06 (0.65)
JAPAN	-0.18 (-0.33, 0.05)	-0.01 (-3.39)	-0.004 (-0.22)
JAPAN (*)	-0.19 (-0.34, 0.03)	-0.01 (-3.16)	-0.002 (-0.64)
U.S.A.	0.14 (-0.01, 0.34)	-0.006 (-0.20)	0.09 (2.54)
ii) Autocorrelated (Bloomfield) errors			
CANADA	-0.24 (-0.49, 0.16)	-0.02 (-4.54)	0.06 (2.13)
FRANCE	-0.19 (-0.35, 0.12)	-0.02 (-3.48)	0.14 (2.94)
GREAT BRITAIN	-0.17 (-0.37, 0.12)	-0.02 (-3.18)	0.16 (3.85)
GERMANY	-0.16 (-0.35, 0.19)	-0.02 (-3.90)	0.09 (2.13)
ITALY	-0.11 (-0.28, 0.12)	-0.01 (-0.89)	0.19 (2.22)
JAPAN	-0.54 (-0.67, -0.29)	-0.01 (-16.09)	0.007 (0.52)
JAPAN (*)	-0.55 (-0.69, -0.30)	-0.01 (-13.80)	-0.003 (-1.98)
U.S.A.	-0.19 (-0.44, 0.20)	-0.01 (-2.20)	0.09 (3.11)

In bold, statistically significant positive coefficients at the 5% level.

Table 5Estimates of d_u , α and β with $d_R = d_\pi = 0$.

	d_u (95% conf. band)	α (t-value)	β (t-value)
i) Uncorrelated (white noise) errors			
CANADA	1.19 (1.02, 1.41)	3.86 (22.33)	0.04 (1.49)
FRANCE	1.20 (1.04, 1.43)	3.04 (14.93)	0.12 (2.01)
GREAT BRITAIN	1.26 (1.10, 1.48)	3.77 (20.42)	0.14 (2.96)
GERMANY	1.17 (1.02, 1.37)	3.16 (17.61)	0.05 (1.12)
ITALY	1.13 (0.97, 1.35)	3.36 (10.44)	0.06 (0.66)
JAPAN	0.84 (0.73, 1.04)	1.56 (16.82)	-0.00 (-0.30)
U.S.A.	1.13 (0.98, 1.34)	4.01 (15.88)	0.09 (2.57)
ii) Autocorrelated (Bloomfield) errors			
CANADA	0.82 (0.66, 1.18)	3.85 (23.44)	0.06 (2.19)
FRANCE	0.83 (0.71, 1.10)	3.07 (16.69)	0.16 (3.19)
GREAT BRITAIN	0.85 (0.68, 1.12)	3.78 (21.14)	0.16 (3.91)
GERMANY	0.88 (0.74, 1.19)	3.16 (17.97)	0.09 (2.13)
ITALY	0.88 (0.72, 1.14)	3.13 (10.92)	0.20 (2.44)
JAPAN	0.65 (0.57, 0.78)	1.50 (19.51)	-0.003 (-0.17)
U.S.A.	0.79 (0.60, 1.16)	4.05 (17.92)	0.10 (3.22)

In bold, statistically significant positive coefficients at the 5% level.

Table 6Estimates of d_u , α and β with d_R and d_π freely estimated from the data.

	d_u (95% conf. band)	α (t-value)	β (t-value)
i) Uncorrelated (white noise) errors			
CANADA	0.37 (0.26, 0.93)	0.77 (4.40)	0.35 (6.36)
FRANCE	0.42 (0.32, 0.99)	0.52 (4.03)	0.51 (7.32)
GREAT BRITAIN	0.30 (0.21, 0.45)	0.26 (2.55)	0.54 (6.74)
GERMANY	0.28 (0.18, 0.42)	0.18 (2.07)	0.32 (4.71)
ITALY	0.22 (0.11, 0.38)	0.10 (1.28)	0.55 (6.02)
JAPAN	0.61 (0.54, 0.83)	1.15 (12.84)	-0.01 (-0.58)
U.S.A.	0.46 (0.32, 0.95)	0.74 (3.89)	0.39 (7.93)
ii) Autocorrelated (Bloomfield) errors			
CANADA	0.25 (0.08, 0.61)	0.23 (2.52)	0.38 (6.72)
FRANCE	0.32 (0.19, 0.55)	0.28 (3.08)	0.53 (7.42)
GREAT BRITAIN	0.27 (0.10, 0.52)	0.21 (2.35)	0.54 (6.84)
GERMANY	0.23 (0.09, 0.47)	0.12 (2.34)	0.34 (4.79)
ITALY	0.15 (-0.01, 0.39)	0.05 (1.00)	0.56 (6.06)
JAPAN	0.51 (0.45, 0.60)	0.92 (13.46)	-0.01 (-0.49)
U.S.A.	0.31 (0.11, 0.50)	0.29 (2.62)	0.39 (8.25)

In bold, statistically significant positive coefficients at the 5% level.

Japan, given the stationarity of interest rates implied by Table 3, we also display the results with $d_R = 0$ and $d_\pi = 1$. It can be seen that with uncorrelated errors (now ξ_t in (9)) (Table 4i), the estimated value of d_u is significantly positive for all the countries except Japan and USA, while β is only significant for France, Great Britain and the USA, whilst with autocorrelation in ε_t , the estimated value of d_u is smaller than in the previous case, and the $I(0)$ hypothesis for the error cannot be rejected for any country except Japan and the US, and the estimated value of β is significantly positive in all cases except Japan. According to these regression results, changes in interest rates and changes in expected inflation are positively correlated in all cases except for Japan.

In Table 5 we assume that $d_R = d_\pi = 0$, i.e. we run the regression model using the original data without taking differences, as in the Fisher relationship. In the autocorrelation case, we cannot reject the null that the estimated errors are $I(1)$ with the only exception of Japan, where d_u is found to be smaller than 1. Note that in this case, given the $I(1)$ evidence provided by the univariate analysis, an estimated value of d_u smaller than 1 implies fractional cointegration, whilst non-fractional cointegration holds if $d_u = 0$. Once more, all the estimated values of β are significantly positive except in the case of Japan, though they are rather small. However, the finding on nonstationarity (or $I(1)$ behaviour) of the errors in this model implies that the relations are spurious and thus invalid.

In Table 6 we take differences using the estimated values of d reported in Table 3. Without correlation in the error term, the estimated value of d_u is found to be positive in all cases, ranging from 0.22 (Italy) to 0.61 (Japan), whilst with autocorrelated errors the $I(0)$ hypothesis cannot be rejected for Italy, d being positive in the remaining cases and ranging from 0.23 (Germany) to 0.51 (Japan). The estimates of β are very similar in the two cases of white noise and autocorrelated errors, and the values are substantially higher than previously and positive, except for Japan.

Finally, in Table 7, we compare the estimates of β with those obtained when imposing $d_u = 0$ in (9). They are generally higher than in the previous cases, especially when $d_R = d_\pi = 0$. In fact, in the $I(0)$ case, even the hypothesis $\beta = 1$ cannot be rejected for some countries (France, Germany and Italy). However, it should be noted that this specification is incorrect since the null hypothesis of $d = 0$ is rejected in favour of $d > 0$ in all the cases shown in Table 5.

Overall, the evidence based on our preferred model (Table 6ii) suggests that there exists a positive relationship between the (filtered) nominal interest rates and inflation, since the β coefficient is positive; however, it is statistically different from 1 (more precisely, it is smaller).⁸ Therefore, we do not find evidence of the full adjustment neither of nominal rates to inflation required by the Fisher hypothesis nor with the filtered data. The specificity of the results for Japan might reflect the well-known liquidity trap characterising this country, since its monetary policy was based on a very low interest rate as far back as 1995, which was followed by a zero interest rate since 1999, and Quantitative Easing (QE) since 2001, well before all the other countries where QE was introduced in response to the global financial crisis of 2007–8. It should be noted that the period analysed includes Quantitative Easing (QE); this increases inflation, but its impact on real rates is not clear since risk premia might decrease and therefore real rates could fall as well as rise. It also includes frequent shifts in inflation which might produce spurious evidence against the Fisher relationship; these disappear when they are taken explicitly into account in a Markov-switching framework (see Evans and Lewis, 1995). Modelling such switches is beyond the scope of the present study, and therefore our results should be taken with caution.

Table 7

Estimates of α and β in equations (7) and (8) with d imposed to be equal to 0.

	$d_R = d_\pi = 1$		$d_R = d_\pi = 0$		d_R and d_π estimated	
	α	β	α	β	α	β
CANADA	-0.01 (-1.30)	0.05 (1.75)	2.20 (33.49)	0.33 (9.57)	0.04 (1.66)	0.42 (7.30)
FRANCE	-0.01 (-1.31)	0.13 (2.44)	1.71 (34.14)	0.84* (25.92)	0.04 (1.89)	0.58 (6.90)
GR. BRITAIN	-0.01 (-1.38)	0.15 (3.47)	2.14 (29.40)	0.42 (15.62)	0.04 (1.70)	0.60 (6.90)
GERMANY	-0.02 (-1.84)	0.06 (1.49)	1.12 (13.62)	0.87* (17.29)	0.01 (0.60)	0.39 (4.92)
ITALY	-0.01 (-0.52)	0.12 (1.49)	2.50 (49.38)	0.91* (35.24)	0.02 (0.75)	0.59 (5.94)
JAPAN	-0.01 (-1.40)	-0.01 (-0.54)	1.00 (52.26)	-0.09 (-5.95)	0.37 (20.29)	0.02 (0.52)
U.S.A.	-0.01 (-0.74)	0.09 (2.81)	2.41 (51.07)	0.30 (5.67)	0.06 (2.13)	0.37 (8.64)

In bold, significant positive coefficients at the 5% level. *: Evidence of $\beta = 1$ at the 5% level. The values in parenthesis are the t-values of the corresponding estimated coefficients.

⁸ Note, however, that some biases may exist, coming from the d -estimates, or alternatively from the estimation in (8). With this in mind we carry out various diagnostic tests on the residuals, including tests of no serial correlation (Durbin, 1970, Godfrey, 1978a,b) and homoscedasticity, Koenker, 1981); these support the fractional specification.

5. Conclusions

This paper revisits the Fisher hypothesis in the G7 countries using regression models that include long memory and fractional integration and therefore are more general than the standard ones based on integer degrees of differentiation, and thus on the classical $I(0)/I(1)$ dichotomy. Two sets of results are produced under the alternative assumptions of uncorrelated and autocorrelated errors.

The univariate analysis suggests that the differencing parameter is higher than 1 for most series in the case with uncorrelated errors, whilst the unit root null cannot be rejected for the majority of them under a very general form of non-parametric autocorrelation. The multivariate results imply that there exists a positive relationship, linking nominal interest rates to inflation; however, the Fisher hypothesis is rejected since there is no full adjustment of the former to the latter. Country-specific factors such as the monetary policy rules adopted might account for differences across countries. For instance, as already mentioned, the contractionary monetary policy during the economic slump going back to 1995 might be the reason for the different findings in the case of Japan. As for economies such as Germany, France and Italy, although they are all members of the same monetary union, domestic factors have clearly continued to play an important role in driving interest rates.

However, one should be cautious with the results presented in this study: the sample period examined (January 2006–December 2016) is likely to include structural breaks reflecting the general turbulence between 2007 and 2012 (the sub-prime mortgages and European debts crises); breaks or regime-switching models are in fact very much related with fractional integration (Diebold and Inoue, 2001; Granger and Hyung, 2004). Future work will examine such issues.

It will also address the problem described in Sun and Phillips (2004), namely that of the data quality when an ex-post variable is used as a proxy of an ex-ante variable. Because the former is a realized measure of the unobserved latter variable incorporating unexpected short run shocks, the integration orders are biased downward. Three solutions are possible: finding a better proxy of the ex-ante variable, estimating the bias using asymptotic formulas, and modelling the short memory perturbation in the joint estimation of the integration orders; the third of these approaches appears to be the most tractable one. The issue of whether or not individuals are fully rational (and therefore it is legitimate to use an ex-post realization of the CPI as a proxy of the ex-ante expected inflation rate) will also be analysed in future papers.

Alternative explanations for our findings such as the decline in the marginal return of capital, tax effects, the failure to account for reverse causality, and uncertainty on the monetary policy are also very interesting avenues for future research. Finally, a deeper understanding of inflation and interest rate dynamics could be gained by testing for the existence of an international Fisher effect (i.e., the idea that economies with higher interest rates will experience a depreciation of their currency).

Conflicts of interest

No conflict of interest.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.inteco.2019.07.002>.

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