MARSHALLIAN DEMAND FUNCTION AND THE ADJUSTMENT OF COMPETITIVE MARKETS

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ABSTRACT

Leon Walras (1874) was the first author who derives the demand function from an utility function, which should be maximized under a budgetary restriction. It is only a piece in a complete model of competitive equilibrium model. That is the 'modern approach' of the theory of demand. Alfred Marshall followed suit but avoided to settle the issue as a constrained maximization problem. This way, he provided a more handy and realistic tool for solving the question of the adjustment of competitive markets.

Marshall built its demand theory based on two assumptions: 1) the individual assigns a different utility function to each good consumes; 2) the marginal utility of money is constant. That makes it easy to build demand functions because the Marshallian utility functions are not 'perfect' representations of the individual preferences, in contrast with that of the modern economic theory.

Besides the fact that the Marshallian demand function neither depends on income nor on the prices of the other goods, an important difference remains, which is stressed in this paper: the MDF, in contrast with the Walrasian one, reflects the individual *marginal valuation*, or societal marginal valuation, if speaking in aggregate terms, of every additional unit of good ÔxÕ. The ordinary demand function (Walrasian) only gives information about demanded quantities along the whole range of prices, provided that all the units are paid at the same price.

I

In this paper I intend a double aim. First, to summarize the Marshallian demand theory and establish the analogies and differences with the 'modern approach', rooted in Walrasian thought. Second, to discuss the relationship

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between the Marshallian demand function (MDF) and the author's ideas on the adjustment of competitive markets. Despite the fact that it is a rather simple and primitive function, it is a pivotal tool for Marshall to build a more realistic and complex approach (than the Walrasian one).

II

In the Mathematical Appendix of *Principles* (PE: 838-9, note II), Marshall establishes the following equilibrium condition for the consumption of good 'x':

$$du/dx = d\mu/dm \ . \ dp/dx$$

which says that the marginal utility of any good x (du/dx) must be equal to marginal utility of money (dμ/dm) per the measure of the individual maximum willingness to pay for an additional unit of 'x' (dp/dx). Marshall calls it "demand price". It is (tacitly) the derivative of the function F(x) which gauges the maximum amount of money that the person would be willing to give for every amount of x.

Nobody will pay an amount of money for a unit of x that implies a utility loss higher than that which is gained from that commodity. In other words, the maximum amount that a consumer would be willing to pay for an additional unit of x will be a quantity such that the utility that is lost in the giving of this amount of money (dμ/dm . dp/dx) will be equal to the utility that will be received instead (du/dx).

Marshall supposes that there is diminishing marginal utility in goods consumption, while that of money is a constant, μ. This is an assumption to which he was driven by his determination to derive welfare consequences from marginal utilities as revealed by prices. Changing the notation, we can express this in a equation:

$$U'(x) = \mu . p$$

as well, in the form of $U'(x)/\mu = p$, and, even more simply, $f(x) = p$.

The obtained function $f(x) = U'(x)/\mu$ is the MDF, that which indicated to us the *marginal valuation* of the successive units of x. Obviously, the integral of this expression will give us the function of the total valuation of F (x), which measures directly the most that the subject is willing to pay for each quantity of x.

If we write the function f (x) = p in the inverse form, $x = f^{-1}(p)$, we can interpret it as a demand function in the ordinary sense of the word. That is, a function that indicates to us the quantity that the subject wants to buy at each price. The demand function (DF), the same in one form as the other, has a negative slope, in accordance with the assumptions of Marshall, that U'' (x) < 0.

The DF obtained in this way does not depend on the income of the person, nor the prices of the other goods. Marshall supposes that the availability of the other goods, even whether or not they will be more or less expensive, is reflected in the form of U (x). The availability of what Marshall calls "rival commodities" forces the subjective valuation of a single item of good 'x' to change; or, in other words, forces the marginal utilities or "demand prices" to change.

> The demand prices in our list are those at which various quantities of a thing can be sold in a market *during a given time and under given conditions*. If the conditions vary in any respect it will be required to change the prices; and this has to be constantly done when the desire for anything is materially altered by a variation of custom ... or by the invention of a new one (PE: 100).

This reflects the peculiarity of Marshall's utility curves. In fact, it shouldn't be considered as showing individual preferences in the common sense of this word, because the supplies of "rival commodities" and its prices effect the preferences. It may be more reasonable to speak of them as 'subjective valuation functions'; the valuation being measured in terms of 'units of utility' which, later on, can be translated to monetary units through the parameter μ.

Therefore, the individual level of income, that appears explicitly in MDF, would be reflected, to Marshall, in the value assigned to parameter μ.

> The richer a man becomes the less is the marginal utility of money to him; every increase in his resources increases the price which he is

> willing to pay for any given benefit. And in the same way every diminution of his resources increases the marginal utility of money to him, and diminishes the price he is willing to pay for any benefit (PE: 96).

The richer the individual, the lower this parameter (book III, ch. III, and note VI of Mathematical Appendix in *Principles*).

This is the most contested aspect of the whole Marshallian demand analysis. It is quite reasonable to assign a lower value to μ for those consumers who are the richest (even if we don't take into account the utility's interpersonal comparisons). Nevertheless, what is not so reasonable is to suppose that μ doesn't change when the size of total expenditure in any good changes.

For instance, when price goes down, the amount of x increases, and total expenditure will change, depending on the elasticity of the function $U'(x)/\mu$. If total expenditure is now higher or lower, it seems reasonable to expect the money's marginal utility to change. This is particularly true if we stick to the Marshallian viewpoint: the *spent money's* marginal utility—which must be the same along all lines of expenditure—is equal to the income marginal utility.

He recognizes the difficulty, but tries to avoid it, introducing a rather polemical supposition: "these changes of consumer's income ... may be neglected, on the assumption, which underlies our whole reasoning, that his expenditure on any one thing, as, for instance, tea, is only a small part of his whole expenditure" (PE: 842).

That means that spending in 'x' doesn't alter in a meaningful way the total income and, therefore, its marginal utility could be taken as a constant. If we accept this supposition, the Marshallian analysis only would be useful in a small group of commodities of little importance.

To avoid this obstacle, we might resort to another kind of hypothesis, arbitrary and restrictive as well, although Marshall doesn't do this. One possibility would be, for example, to suppose that the elasticity of $U'(x) / x$ is equal to the unity. This implies that the expenditure in X is always the same. Because of this, the total expenditure can be considered constant and this permits the marginal utility of the income to be invariable in regard to the changes of p and of x . This implies, though, that the x demand has to be a equilateral hyperbola, which is, obviously, a supposition too restrictive.

Another alternative would be to suppose a utility function of the type $U = U_1(x_1) + ... + U_1(x_n) + \mu.m$, in which the marginal utility constant, μ, is assigned, to the monetary amount *retained*, m. If the budget restriction of the person is $p_1 x_1 + ... + p_n x_{n+m} = R$, where R is the available income, the obtained demand functions will be of the type $x_i = D_i(p_i)$, $i = 1, ... , n$, and will not depend neither on the income nor on the prices of the other goods.

Placing the budget restriction in the utility function and fulfilling the conditions of the first order to obtain the maximum, demand functions are obtained directly, having similar forms to those of Marshall.

In the above mentioned case, when p_i varies, the amount of expenditure in x is altered, but effects only the monetary amount retained, that becomes larger or smaller, without changes to the expenditures on the other commodities. The marginal utility of money spent remains always the same as the retained, and is, as we know, constant and equal to μ. The arbitrary thing about this way of looking at it is that a marginal utility constant is assigned to the amount retained. At the start, there is no reason for this, particularly if we think that the retained money represents the utility of the goods that are hoped to be bought with that same money in the future and that each good generates a diminishing marginal utility.

III

Now, let us compare the Marshallian demand curve with that of the modern microeconomics theory, based on Walrasian ideas.

Besides the already noted differences (in form, at least, the Marshallian demand function neither depends on income nor on the prices of the other goods), an important difference remains, which will be stressed in this paper: the MDF, in contrast with the Walrasian one, reflects the individual *marginal valuation*, or societal marginal valuation, if speaking in aggregate terms, of every additional unit of good 'x'. The ordinary demand function (Walrasian) only gives information about demanded quantities along the whole range of prices, provided that all the units are paid at the same price.

We must remember that the WDF is obtained through a utility maximization program, in which the prices and the income appear as parameters. It is not legitimate in this case to write the function $x = D(p)$ in the inverse form, $p = D^{-1}(x)$, and interpret it as a function of marginal valuation. The MDF, however, admits, both interpretations.

Diagram 1 illustrates this point by showing the individual Walrasian demand curve. Let's suppose that for a sufficiently high price, such as p_0 only one unit of the commodity is acquired. Let us suppose that this transaction has already been done and now there is a possibility of acquiring an additional quantity at price p_1. Obviously, as the first unit was acquired at a higher price, this is going to have an income effect on later units.

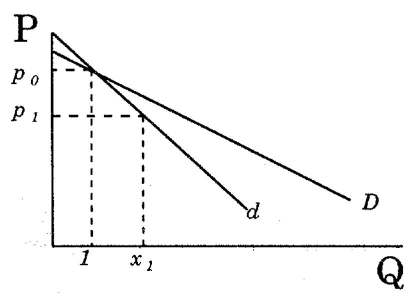

Diagram - I:

This effect will have to be *compensated* for (with an increment of the monetary income, or with variations in the prices of the related goods) to maintain the utility level constant; in this way the effect of the income is eliminated (changing from D to d). At least that compensation is produced, the new buys, graphically, will tend to a more inclined curve, like the discontinuous line, d.

Marshall, though, does not ignore the influence of the income and of the prices of the other goods in the demand function, he only hides them behind the utility function. This doesn't present unsolvable problems. What is left without a solution is the *compensation* or *adjustment*, because the change in the price of the considered good (x_i) makes the person more rich/ poor in relative terms; and, possibly, that would change the marginal utility of the good in question, which would therefore imply new adjustments in the demand of the said good. All of this is going to influence the successive decisions and the particular utility function of the individual.

The main advantage of Marshallian utility functions (Molinero & Santiago, 1998: 55) rests in the fact that they are not *perfect* images of the individual preferences, in contrast with that of the *modern* economic theory (Walrasian). Marshall assigns utility to a certain commodity along time, so that its utility function has to be revised when one parameter changes (price of other goods, time).

To construct a curve reflecting the individual marginal valuations, we should remove the below described 'income affects' (the prices of other goods and the individual income) as the price goes down, in such a way that his/her utility remains constant. This way we could build a *compensated* demand curve (with constant purchasing power), which would resemble the MDF. This is the meaning that would have to be given to the interpretation that M. Friedman made of the MDF in his article in 1949. Although the term *compensated demand* has taken a long time to be accepted into economic literature (Gary Becker called it *pure demand curve*), now it seems to be fully integrated.

The advantage that the *Marshallian* demand function has over the *ordinary* functions (Walrasians) is that it admits two different interpretations. One way, it can be interpreted in the conventional sense as any ordinary function of demand: graphically, in the form of horizontal arrows coming from the y axis; and written in the form of $x = d(p)$ it tells us the quantities of goods that the person is willing to acquire at each price (Molinero & Santiago, 1998: 58).

However, if we write it in the form, $p = u'(x)/v$, it also tells us which is the maximum quantity of money that the subject would be willing to pay for each successive unit of the commodity, as a collection of vertical lines with the base in the x axis. This second interpretation cannot be maintained, though, in a general way, in the case of ordinary demand functions of modern microeconomics theory.

Any demand function can be written in the inverse form, with the price as the dependent variable and the quantity as the independent variable. In the ordinary functions, though, the inverse form has to be interpreted carefully. The only thing that these functions tell us is which has to be the price, if we want the subject to acquire a determinate quantity at a uniform price (Molinero & Santiago, 1998: 59). It is fundamental that we assume that all the units are sold at the same price, in order that the ordinary demand function has meaning. In the case of the Marshallian, though, this last part isn't necessary, the function tells us how much money the person is willing to pay for each successive unit, and these values are maintained even though the prices do not remain constant.

IV

We will now analyze the mechanism of competitive market adjustment as was described by Marshall in Chapter III of his book V of *Principles.* Marshall's argument rests on rather reasonable assumptions, the most prominent assumption being to use the demand function in the specific Marshallian sense of marginal valuation.

When the actual microeconomics theory raises the question of how to determine the price of equilibrium in a competitive market, the answer is always elusive. In reality no precise answer exists. The price is established, like everyone knows, at the intersection of the supply and the demand, but not in any part it is explained how that result is obtained. If both the suppliers and the demandants are strictly price takers, who, then, determines the price? There is, of course, the Walrasian metaphor of the auctioneer (or the *tantamount* process conceived by him). But no one will accept this as an explanation of the process of adjustment. It is only a formal resource, that serves precisely to elude the necessity of an explanation.

What would happen if we do away with the auctioneer? At the start, everyone would be paralyzed and left without criteria de action. In reality that which paralyzes here is the model that doesn't permit the agents to take initiatives with regard to the prices. In the Marshallian schema, however, the individuals are somewhat more versatile.

To Marshall, in the 'market-day' (the needed time for prices to be fixed) supply is a fixed quantity, that has been determined in accordance with some old parameters.

> Market values are governed by the relation of demand to stocks actually in the market ... But the current supply is in itself partly due to the action of producers in the past; and this action has been determined on as the result of a comparison of the prices which they expect to get for their goods with the expenses to which they will be put in producing them (PE: 372).

Furthermore, in such a specific 'market-day', a demand reflecting the individual marginal valuations emerges, which guides the bargaining between buyers and sellers. Marshall doesn't have a theory about those transactions and, of course, doesn't explain the need for an ultimate equilibrium price. He is simply assuming a more or less automatic mechanism to reach the equilibrium price.

However, it has to be recognized that the Marshallian consumers are not passive agents that are limited to observing and adapting to the prices. In reality they are active agents that negotiate from certain criteria that gives them their demand functions. Marshall *supposes* that, as a result of a large number of isolated negotiations which take place in the 'market-day', it is possible to establish a uniform price; that this price 'clears' the market; and that it reflects the valuation of the marginal consumer, the one with the lowest willingness to pay.

Marshall recognizes the possibility of what afterwards was called 'false transactions'. That is, transactions carried out at different prices to that of equilibrium. Taking an illustration from a corn-market in a country town (book V, ch. II, s. 2), Marshall says that those operations could be performed in too high or low prices.

> It is not indeed necessary for our argument that any dealers should have a thorough knowledge of the circumstances of the market. Many of the buyers may perhaps underrate the willingness of the sellers to sell, with the effect that for some time the price rules at the highest level at which any buyers can be found ... In the same way if the sellers had underrated the willingness of the buyers to pay a high

price, some of them might begin to sell at the lowest price they would take (PE: 334).

The described situation might occur in the presence of impatient buyers that don't receive fairly accurate information regarding the ultimate price; or sellers which don't come to discover what the consumers really are willing to pay and decide to sell too early. Nevertheless, to Marshall, those transactions leave the equilibrium final price unaffected, the one which "clears the market". He adds, "we tacitly assumed that the sum which purchasers were willing to pay ... would not be affected by the question whether the earlier bargains had been made at a high or a low rate" (PE: 334). That makes sense: even with price discrimination, the MDF holds unchanged.

So, according to the Marshallian program, current supply and demand in the 'market-day' establishes an final equilibrium (closing) price that 'clears' the market. Also, everyday supply is constant, as a result of decisions taken in a former period of time and inspired by the observed prices in previous 'market-days'.

In short, we can think that decisions of production are taken based on the prevalent prices during the previous 'market-day'. In searching, as he did, for a criterion to maximize profits, he established the criterion by setting marginal revenue equal to marginal cost. Here, one must think that an observed earlier price is referred to, and it appears natural that this price would be the last one observed; even though it also could be the expected price for the future in a earlier moment. In any case, this price also will be based, correctly, in past observations. Therefore, can be said that, for Marshall, today's price is the result of today's demand, and the supply of yesterday. Once today's price is known, —and has 'cleared' today's market—the producers can make production decisions for tomorrow; and the process will be repeated again and again until it arrives, if it will arrive, at an equilibrium.

A simple way of representing this process of Marshallian adjustment would be the following: given the supply of x at the moment t, the price will be determined, for that moment, through the MDF $p_t = f(x_t)$; on the other hand, the supply varies from one moment to another according to the function $x_t = g(p_{t-1})$, that should have a positive tendency (slope). Even though a point of intersection between the two functions exists, there is no guarantee that this point will be reached, especially if the said functions are not linear. In fact, both the prices and the quantities can follow pathways more or less complex, depending on the form of the functions f and g, as well as the "initial conditions" of the process (the dynamic properties of a model like this are investigated in the work of Leontieff).

In diagrams 2 and 3 two processes of this type are shown. In the first one a stable cycle appears, and in the second one an explosive process, that can establish itself late or early within a more or less complex cycle.

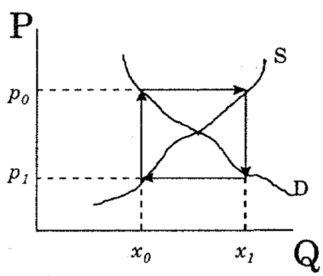

Diagram II

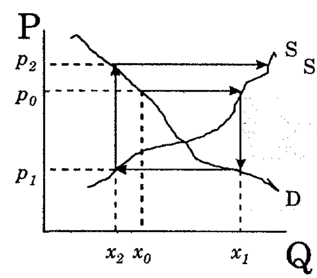

Diagram III

This type of adjustments are certainly more realistic than those accomplished by the auctioneer. The problem is that they are also much more complex. The Walrasian auctioneer can arrive at an equilibrium, if he proceeds in accordance with certain rules (raise the prices when there are excesses of demand; reduce them when there are excesses of supply; proceed always in a gradual mode, that is, without abrupt variations of prices). In Marshallian conditions, though, equilibrium can be arise more though a pure casualty. The normal would be a 'cobweb' result, more or less complex cycles, or perhaps chaos; particularly if the supply and demand functions are not linear.

It was not without good reason that, as soon as the idea of marginal value was available, Marshall put aside these possibilities and preferred the supposition that equilibrium would be automatically attained.

> When therefore the amount produced (in a unit of time) is such that the demand price is greater than the supply price, then sellers receive more than is sufficient to make it worth their while to bring goods to

> market to that amount; and there is at work an active force tending to increase the amount brought forward for sale. On the other hand, when the amount produced is such that the demand price is less than the supply price, sellers receive less than is sufficient to make it worth their while to bring goods to market on that scale ... and there is an active force at work tending to diminish the amount brought forward for sale. When the demand price is equal to the supply price, the amount produced has no tendency either to be increased or to be diminished; it is in equilibrium (PE: 345).

This is no more than a simplifying supposition, though, because the possibility to fall back on the idea of equilibrium simplifies substantially all types of analysis in economics; and Marshall surely didn't want to complicate life with an analysis in which it isn't known where it will arrive. The first work regarding the theoretical implication of the temporary gaps in the adjustments between supply and demand appears that it should be attributed to W. Leontieff that in 1934 published an article in *Zeitschrift für National-Ökonomie*, where he developed a model inspired by the ideas of Marshall. This article appears later in his book of 1966.

In the end, the investigation in this type of complex dynamics can bring forth positive outcomes in some cases, for example, how the currency markets and stock markets can function. In this field, like so many others, Marshall's ideas open a new path, which is the one that today is starting to be explored, what is called, *chaos*, with regard to it's application to the economy.

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