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High and low prices and the range in the European stock markets: A long-memory approach

Guglielmo Maria Caporale[a,*], Luis A. Gil-Alana[b,c], Carlos Poza[c]

[a] Brunel University London, UK
[b] University of Navarra, Pamplona, Spain
[c] University Francisco de Vitoria, Madrid, Spain

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ABSTRACT

This paper uses fractional integration techniques to examine the stochastic behaviour of high and low stock prices in Europe and then to test for the possible existence of long-run linkages between them by looking at the range, i.e., the difference between the two logged series. Specifically, monthly, weekly and daily data on the following five European stock market indices are analysed: DAX30 (Germany), FTSE100 (UK), CAC40 (France), FTSE MIB40 (Italy) and IBEX35 (Spain). In all cases, the order of integration of the range is lower than that of the original series, which implies the existence of a long-run equilibrium relationship between high and low prices. Further, multiple breaks are found in the high and low-price series but no breaks in the range, and the estimated fractional differencing parameter is positive in all cases, which represents evidence of long memory.

1. Introduction

In financial economics the difference between high and low intraday or daily prices is known as the range. Volatility can be expected to be higher if the range is wider. Parkinson (1980) showed that in fact the price range is a more efficient volatility estimator than alternative ones such as the return-based estimator. It is also frequently used in technical analysis by traders in financial markets (see, e.g., Taylor and Allen, 1992). However, as pointed out by Cheung et al. (2009), focusing on the range itself might be useful if one's only purpose is to obtain an efficient proxy for the underlying volatility, but it also means discarding useful information about price behaviour that can be found in its components. Therefore, in their study Cheung et al. (2009) analyse simultaneously both the range and daily high and low using daily data for various stock market indices. Since the latter two variables generally do not appear to diverge significantly over time, having found that they both exhibit unit roots by carrying out ADF (Dickey and Fuller, 1979) tests, they model their behaviour using a cointegration framework as in Johansen (1991) and Johansen and Juselius (1990) to investigate whether they are linked through a long-run equilibrium relationship, and interpreting the range as a stationary error correction term. They then show that such a model has better in-sample properties than rival ARMA specifications but does not clearly outperform them in terms of its out-of-sample properties.

Unlike Cheung et al. (2009), the present study uses fractional integration methods that are more general than the standard framework based on the I(0) versus I(1) dichotomy. According to the efficient market hypothesis (EMH), asset prices should be unpredictable and follow a random walk (see Fama, 1970), i.e. they should be integrated of order 1 or I(1). However, the choice between stationary I(0) and non-stationary I(1) processes is too restrictive for most financial series (Barunik and Dvorakova, 2015).

Diebold and Rudebusch (1991); Hassler and Wolters (1994); Lee and Schmidt (1996) and others showed that in fact unit root tests have very low power in the context of fractional integration. Therefore, our analysis allows the differencing parameter for the individual series to take fractional values.

Fiess and MacDonald (2002), Cheung (2007) and Cheung et al. (2009) all modelled high and low prices together with the range in a cointegration framework to analyse the foreign exchange and stock markets respectively. However, their studies restrict the co-integrating parameter to be unity (even though this is not imposed in Granger's (1986) seminal paper). By contrast, we analyse the behaviour of the range, i.e. the difference between the two (logged) series, by means of fractional integration tests that allow the differencing parameter d to take any real value, including fractional ones. Mean reversion of the range will imply that there exists a long-run equilibrium relationship between the two series, i.e. fractional cointegration holds, with the speed of the adjustment process towards the long-run equilibrium possibly being much slower than in the classical cointegration framework. By taking this approach we are able to establish whether or not the two series move together in the long run by carrying out univariate regressions as opposed to bivariate cointegrating ones.

We apply these methods to provide new empirical evidence for the European stock markets by analysing monthly, weekly and daily data on five European stock market indices, i.e. the DAX30 (Germany), FTSE100 (UK), CAC40 (France), FTSE MIB40 (Italy) and IBEX35 (Spain). By estimating the differencing parameter at different data frequencies we obtain information about possible differences in the degree of persistence across frequencies. This is particularly relevant to investors aiming to design appropriate trading strategies to exploit market inefficiencies, since it sheds light on the value of d at different frequencies, profit opportunities arising when this parameter is different from 1, i.e. when prices do not follow a random walk and are therefore predictable (see Caporale et al., 2019). Further, it is conceivable that both high and low-price series are trending whilst a combination of the two is stationary; cointegration analysis sheds light on this, and in its fractional extension also allows for a very slow speed of adjustment. Such knowledge can allow investors to make better predictions and to develop more profitable trading strategies. It can also inform the policy decisions of monetary authorities based on developments in financial markets.

The layout of the paper is the following: Section 2 briefly reviews some previous empirical studies; Section 3 outlines the methodology; Section 4 describes the data and discusses the main empirical findings; Section 5 offers some concluding remarks.

2. Literature review

High and low prices stand for the highest and lowest values over a fixed sampling interval. Cheung (2007) showed that their fluctuations are linked, with any underlying trend in stock prices affecting the peaks and troughs in the same manner. Specifically, he found daily highs and lows of the three main US stock price indices to be cointegrated. Data on openings, closings, and trading volume appear to provide additional explanatory power for variations in highs and lows: the augmented VECM models including the extra variables explain 40–50 % of total variation. Further, the generalized impulse response results are sensitive to whether or not the additional regressors are included in the model. Cheung et al. (2009) reported that VECM models produce more accurate forecasts of the range (the difference between highs and lows) than alternative specifications. Similar results with the same methodology were obtained by He and Wan (2009) for the high and low exchange rates of the USD against the GBP and JPY.

Afzal and Sibbertsen (2019) analysed high and low stock prices in six Asian countries (India, Korea, Malaysia, Indonesia, Pakistan and Sri Lanka) by estimating fractionally cointegrated vector error correction models (FVECM) and found that daily highs and lows have a long-run relationship; also, the autocorrelations of the range series decay at a hyperbolic rate, which suggests that volatility is non-stationary. Further, the FVECM specification has a better out-of-sample performance for the range than the heterogeneous autoregressive (HAR) and autoregressive fractionally integrated moving average (ARFIMA) models.

Al-Shboul and Anwar (2016), using daily data on five sectoral indices from 2006 to 2014, investigated the possibility of fractional integration in sectoral returns (and their volatility measures) in Jordan's Amman stock exchange (ASE). Their empirical analysis, which uses the log periodogram (LP) and the local Whittle (LW) semi-parametric fractional differencing techniques, shows that sectoral returns exhibit short memory whilst volatility is characterised by long memory; however, the latter finding can in fact be attributed to the presence of structural breaks. Further, the impulse response functions (IRF) based on an ARFIMA specification indicate that shocks to sectoral returns exhibit short run persistence, whereas shocks to volatility display long-run persistence.

Xiong et al. (2015) proposed an interval forecasting method for agricultural commodity futures prices based on a vector error correction model (VECM) and multi-output support vector regression (MSVR), which can capture nonlinearities; the adopted framework is shown to have better forecasting accuracy than rival specifications. Barunik and Dvorakova (2015) found a long-run relationship between daily high and low stock prices using a fractionally cointegrated vector autoregressive (FCVAR) model; the same framework is also used to test for long memory in their linear combination, i.e., the range; range-based volatility has in fact been shown to be a highly efficient and robust estimator of volatility (Parkinson, 1980). They analysed the Czech PX index, the German Deutscher Aktienindex (DAX), the UK's Financial Times Stock Exchange index (FTSE 100), the US Standard and Poor's (S&P) 500 and the Japanese Nihon Keizai Shimbun (NIKKEI) 225 during the 2003–2012 period, that is, before and during the financial crisis. They found that the ranges of all of the indices display long memory and are mostly in the non-stationary region, which suggests that volatility is a non-stationary process.

Chatzikonstanti and Venetis (2015) examined whether the observed long-memory behaviour of the logged range series is spurious and showed that, once breaks are accounted for, volatility persistence disappears. Their conclusion is that volatility can be adequately represented as a process with multiple breaks and a short-run component.

3. Methodology

When testing for cointegration in a bivariate system as in the present case the usual assumption in the literature is that the individual series are integrated of order 1, i.e., I(1), while there exists a linear combination of the two which is integrated of order 0, i.e., I(0). However, the original definition of cointegration in the seminal paper of Engle and Granger (1987) does not restrict the orders of integration to be 1 or 0, but allows for fractional values d for the original series, and an order of cointegration equal to d - b (with b > 0) for their linear combination. This is the approach followed in the present study, which allows for any real values, d and b, as the order of integration of the series of interest.

More specifically, a process $\{x_t, t = 0, \pm 1, \ldots\}$ is said to be integrated of order d, and denoted as I(d) if it can be represented as:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \ldots, \tag{1}$$

where L is the lag operator ($Lx_t = x_{t-1}$) and u_t is (0), defined as a covariance-stationary process with a positive and bounded spectrum. Thus, u_t can be a white noise but also a weakly autocorrelated process, for example, of the AutoRegressive Moving Average (ARMA) form.[1] When d in (1) is not integer, one can use the Binomial expansion such that:

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \ldots,$$

implying that equation (1) can be expressed as

$$x_t = dx_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \ldots + u_t$$

Thus, if d = 0, x_t is a short-memory or I(0) process (with the effects of shocks disappearing at an exponential rate if u_t is AR(MA)), while d > 0 implies long memory behaviour, so-called because of the strong degree of dependence between observations far apart in time.[2] Note also that, if d < 0.5, xt is covariance-stationary, while d ≥ 0.5 indicates that the series is non-stationary (in the sense that the variance of the partial sums increases in magnitude with d); further, if d < 1 the series is mean-reverting, with the effects of shocks disappearing in the long run, while d ≥ 1 implies lack of mean reversion, with the effects of shocks persisting forever. Thus, we can distinguish between the following cases:

a) d = 0:I(0) or short-memory behaviour.
b) 0 < d < 0.5:Covariance-stationary, long-memory (mean-reverting) behaviour.
c) 0.5 ≤ d < 1:Non-stationary though mean-reverting behaviour, with long-lasting effects of shocks.
d) d = 1:Non-stationary unit roots (lack of mean reversion).
e) d > 1:Explosive behaviour (lack of mean reversion).

Therefore, the parameter d is very important: the higher is the value of d, the higher is the degree of persistence, whilst the lower it is, the faster is the convergence process of a series to its original level after a shock.

These processes were originally proposed by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981) and justified in terms of aggregation of heterogeneous processes initially by Robinson (1978) and Granger (1980) and later by other authors such as Cioczek-George and Mandelbrot (1995); Taqqu et al. (1997); Chambers (1998); Parke (1999); Oppenheim and Viano (2004); Zaffaroni (2004), Beran et al. (2010), etc., and are widely employed in the analysis of economic and financial data.

In this study we analyse the relationship between high and low prices as well as the range, defined as the difference between the two logged series and therefore not estimated using a regression model. As a first step, we estimate the orders of integration of the series by using the Whittle function in the frequency domain (Dahlhaus, 1989) and following a testing procedure developed by Robinson (1994) that is suitable for statistical inference even in the case of non-stationary series. Using this method, we test the null hypothesis:

$$H_O: d = d_0, \tag{2}$$

in (1) for any real value d_0, where x_t denotes the errors in a regression model of the form:

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, \ldots, \tag{3}$$

where y_t stands for the observed series, and α and β are unknown coefficients, specifically an intercept and a linear trend.

In addition, since fractional integration is closely related to non-linearities and structural breaks (e.g., Breidt et al., 1998; Diebold and Inoue, 2001; Davidson and Terasvirta, 2002; Granger and Hyung, 2004; Sibbertsen, 2004; Deo et al., 2006; Baillie and Morana, 2009; Lahiani and Scaillet, 2009; Choi et al., 2010; Hwang and Shin, 2018; etc.), the method described in Gil-Alana (2008) and based on Bai and Perron (2003) for testing multiple breaks is also applied.

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[1] Thus, if u_t in (1) is ARMA(p, q), x_t is said to be a fractionally integrated ARMA, i.e., an ARFIMA(p, d, q) process.
[2] In this case (d > 0, with d < 1) the shocks disappear at a hyperbolic rate.

Table 1

Results with MONTHLY data and UNCORRELATED disturbances (Model given by Eq. (4) with white noise u_t).

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.96 (0.85, 1.11)	**1.09 (0.95, 1.29)**	1.09 (0.95, 1.29)
France	0.96 (0.85, 1.10)	**1.09 (0.93, 1.32)**	1.09 (0.93, 1.32)
Germany	0.95 (0.84, 1.10)	0.96 (0.83, 1.15)	**0.96 (0.82, 1.15)**
Italy	0.96 (0.84, 1.11)	**1.05 (0.92, 1.23)**	1.05 (0.92, 1.23)
U.K.	0.96 (0.84, 1.10)	**1.01 (0.87, 1.20)**	1.01 (0.87, 1.20)
Series: LOW	No terms	An intercept	A linear trend
Spain	0.96 (0.86, 1.11)	1.05 (0.90, 1.26)	**1.05 (0.90, 1.26)**
France	0.96 (0.85, 1.11)	**0.93 (0.78, 1.14)**	0.93 (0.78, 1.14)
Germany	0.96 (0.85, 1.11)	0.96 (0.80, 1.17)	**0.96 (0.81, 1.17)**
Italy	0.96 (0.85, 1.11)	**0.97 (0.83, 1.17)**	0.97 (0.83, 1.17)
U.K.	0.96 (0.85, 1.11)	0.86 (0.72, 1.03)	**0.86 (0.74, 1.03)**
Series: RANGE	No terms	An intercept	A linear trend
Spain	0.50 (0.38, 0.65)	0.36 (0.24, 0.53)	**0.34 (0.18, 0.55)**
France	0.51 (0.39, 0.66)	0.40 (0.28, 0.58)	**0.43 (0.28, 0.63)**
Germany	0.45 (0.34, 0.59)	0.35 (0.25, 0.48)	**0.38 (0.26, 0.53)**
Italy	0.54 (0.41, 0.70)	0.37 (0.26, 0.55)	**0.39 (0.24, 0.59)**
U.K.	0.42 (0.31, 0.55)	0.28 (0.18, 0.40)	**0.27 (0.14, 0.43)**

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values. The coefficients in bold are those of the models selected on the basis of the statistical significance of the regressors.

4. Data and empirical results

For the analysis we use the following five European stock market indices: DAX30 (Germany), FTSE100 (UK), CAC40 (France), FTSE MIB40 (Italy) and IBEX35 (Spain). High and low prices as well as the logged series are examined at a monthly, weekly and daily frequency; the range is also analysed to test for a possible long-run equilibrium relationship (the literature has shown that it is a very efficient estimator of volatility, see e.g. Garman and Klass, 1980, and Yang and Zhang, 2000). The sample period goes from the beginning of January 2009 to the end of January 2019 in the case of monthly and weekly data (121 and 522 observations, respectively) and from the beginning of January 2011 to the end January 2019 in the case of daily data (2053 observations). The data source is Thomson Reuters Eikon. The focus of the analysis is on the main European stock markets and the sample starts in 2011, this being the year when the European sovereign debt crisis started becoming a banking crisis as well, as a result of a bailout programme for Greece imposing large losses on private creditors; this development obviously affected the dynamics in the European financial markets.

The estimated model is the following:

$$y_t = \beta_0 + \beta_0 t + x_t; \quad (1 - L)^d x_t = u_t \quad t = 0, 1, ..., \tag{4}$$

where y_t denotes each of the time series under investigation in turn, i.e., high and low prices for each of the five European stock market indices, with the disturbances being assumed in turn to be a white noise or autocorrelated as in the exponential spectral model

Table 2

Results with MONTHLY data and AUTOCORRELATED disturbances (Model given by Eq. (4) with white noise u_t).

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.93 (0.74, 1.20)	**0.88 (0.65, 1.18)**	0.88 (0.64, 1.18)
France	0.93 (0.73, 1.18)	0.77 (0.59, 1.13)	**0.77 (0.53, 1.13)**
Germany	0.93 (0.72, 1.18)	0.82 (0.65, 1.15)	**0.81 (0.54, 1.15)**
Italy	0.91 (0.74, 1.17)	**0.83 (0.59, 1.11)**	0.83 (0.59, 1.11)
U.K.	0.91 (0.74, 1.17)	0.88 (0.59, 1.27)	**0.88 (0.65, 1.27)**
Series: LOW	No terms	An intercept	A linear trend
Spain	0.93 (0.74, 1.19)	**0.74 (0.49, 1.07)**	0.74 (0.50, 1.07)
France	0.92 (0.73, 1.18)	0.65 (0.51, 0.95)	**0.61 (0.33, 0.95)**
Germany	0.91 (0.73, 1.18)	0.67 (0.56, 0.97)	**0.66 (0.43, 0.97)**
Italy	0.92 (0.73, 1.17)	**0.70 (0.49, 1.03)**	0.69 (0.47, 1.03)
U.K.	0.92 (0.73, 1.18)	0.69 (0.51, 1.04)	**0.74 (0.53, 1.05)**
Series: RANGE	No terms	An intercept	A linear trend
Spain	0.51 (0.29, 0.81)	0.26 (0.11, 0.54)	**0.09 (-0.21, 0.61)**
France	0.53 (0.31, 0.82)	0.33 (0.15, 0.77)	**0.32 (0.05, 0.83)**
Germany	0.60 (0.36, 0.92)	**0.49 (0.22, 1.06)**	0.67 (0.25, 1.06)
Italy	0.45 (0.21, 0.75)	0.23 (0.06, 0.51)	**0.18 (-0.11, 0.63)**
U.K.	0.57 (0.36, 0.87)	0.33 (0.15, 0.80)	**0.37 (0.08, 0.84)**

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values. The coefficients in bold are those of the models selected on the basis of the statistical significance of the regressors.

Table 3
Results with WEEKLY data and UNCORRELATED disturbances (Model given by Eq. (4) with white noise u_t).

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.99 (0.93, 1.05)	**1.11 (1.04, 1.19)**	1.11 (1.04, 1.19)
France	0.99 (0.93, 1.05)	**1.07 (1.00, 1.15)**	1.07 (1.00, 1.15)
Germany	0.99 (0.93, 1.05)	**1.07 (1.00, 1.14)**	1.06 (1.00, 1.14)
Italy	0.99 (0.93, 1.06)	**1.16 (1.09, 1.25)**	1.16 (1.09, 1.25)
U.K.	0.99 (0.93, 1.05)	**1.06 (0.99, 1.15)**	1.06 (0.99, 1.15)
Series: LOW	No terms	An intercept	A linear trend
Spain	0.99 (0.93, 1.06)	**1.03 (0.96, 1.12)**	1.03 (0.96, 1.12)
France	0.99 (0.93, 1.05)	**0.99 (0.92, 1.08)**	0.99 (0.92, 1.08)
Germany	0.99 (0.93, 1.06)	**1.05 (0.97, 1.14)**	1.05 (0.97, 1.14)
Italy	0.99 (0.93, 1.06)	**1.06 (0.99, 1.15)**	1.06 (0.99, 1.15)
U.K.	0.99 (0.93, 1.06)	**1.02 (0.94, 1.11)**	1.01 (0.94, 1.11)
Series: RANGE	No terms	An intercept	A linear trend
Spain	0.43 (0.38, 0.49)	0.35 (0.30, 0.42)	**0.34 (0.28, 0.41)**
France	0.47 (0.42, 0.54)	0.40 (0.34, 0.46)	**0.40 (0.34, 0.48)**
Germany	0.47 (0.42, 0.54)	0.40 (0.35, 0.46)	**0.41 (0.36, 0.47)**
Italy	0.49 (0.44, 0.55)	0.41 (0.36, 0.48)	**0.42 (0.35, 0.49)**
U.K.	0.48 (0.42, 0.55)	0.39 (0.34, 0.46)	**0.40 (0.34, 0.47)**

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values. The coefficients in bold are those of the models selected on the basis of the statistical significance of the regressors.

Table 4
Results with WEEKLY data and AUTOCORRELATED disturbances (Model given by Eq. (4) with autocorrelated u_t).

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.98 (0.89, 1.08)	**0.98 (0.89, 1.09)**	0.98 (0.89, 1.09)
France	0.98 (0.89, 1.08)	**0.94 (0.84, 1.07)**	0.94 (0.84, 1.06)
Germany	0.98 (0.90, 1.08)	1.00 (0.88, 1.14)	**1.00 (0.88, 1.14)**
Italy	0.98 (0.89, 1.08)	**0.98 (0.89, 1.11)**	0.98 (0.89, 1.11)
U.K.	0.99 (0.89, 1.08)	0.91 (0.81, 1.02)	**0.91 (0.81, 1.02)**
Series: LOW	No terms	An intercept	A linear trend
Spain	0.97 (0.88, 1.09)	**0.84 (0.75, 0.94)**	0.84 (0.75, 0.94)
France	0.97 (0.90, 1.09)	0.78 (0.69, 0.89)	**0.78 (0.69, 0.89)**
Germany	0.97 (0.89, 1.09)	0.79 (0.70, 0.91)	**0.80 (0.71, 0.91)**
Italy	0.99 (0.89, 1.10)	**0.84 (0.75, 0.96)**	0.84 (0.75, 0.96)
U.K.	0.97 (0.89, 1.09)	0.74 (0.66, 0.85)	**0.76 (0.68, 0.87)**
Series: RANGE	No terms	An intercept	A linear trend
Spain	0.44 (0.38, 0.53)	0.32 (0.26, 0.42)	**0.30 (0.20, 0.41)**
France	0.47 (0.38, 0.55)	0.36 (0.28, 0.44)	**0.36 (0.27, 0.48)**
Germany	0.50 (0.43, 0.58)	0.40 (0.32, 0.51)	**0.43 (0.34, 0.54)**
Italy	0.44 (0.36, 0.53)	0.33 (0.26, 0.41)	**0.31 (0.22, 0.42)**
U.K.	0.47 (0.39, 0.58)	0.35 (0.28, 0.43)	**0.35 (0.27, 0.47)**

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values. The coefficients in bold are those of the models selected on the basis of the statistical significance of the regressors.

of Bloomfield (1973), the latter being a non-parametric approach approximating ARMA processes with very few parameters.

Tables 1–6 show the estimates of d along with the 95 % confidence intervals of the non-rejection values based on Robinson's (1994) method; values of d outside the corresponding interval imply a rejection of the null of d being equal to its estimate at the 5 % level.[3] The results are reported for the three cases of i) no deterministic terms (i.e., $\beta_0 = \beta_1 = 0$ in.(4)), ii) an intercept ($\beta_1 = 0$) and iii) an intercept and a linear time trend (i.e., with β_0 and β_1 being estimated from the data). In each table, panel i) displays the results for the high prices, panel ii) for the low prices, and panel iii) for the range. Table 1 and 2 provide the results for the monthly series, Table 3 and 4 for the weekly ones, and Table 5 and 6 for the daily ones; in all cases the estimates are reported for the two cases of white noise and autocorrelated (Bloomfield) disturbances.[4]

In the case of the monthly series, under the assumption of white noise disturbances for both high and low prices the estimates of d are around 1 (sometimes below 1) and the unit root null hypothesis cannot be rejected in any case (see Table 1). However, those estimates are much smaller for the range, ranging between 0.27 (UK) and 0.43 (France), and the unit root null hypothesis is decisively rejected in all countries in favour of mean reversion and cointegration (d < 1). Interestingly, the null hypothesis d = 0 (consistent

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[3] Thus, for example, if the interval is (0.65, 0.93) the null hypothesis H_o: d = 0.50 is rejected at the 5% level in favour of the alternative H_a: d > 0.50. Similarly, the unit root null, i.e. H_o: d = 1 is also rejected in this case, though in this case in favour of the alternative H_a: d < 1.

[4] The coefficients of the preferred specifications are in bold.

Table 5
Results with DAILY data and UNCORRELATED disturbances (Model given by Eq. (4) with autocorrelated u_t).

Series: HIGH	No terms	An intercept	A linear trend
Spain	1.00 (0.97, 1.03)	**1.06 (1.02, 1.10)**	1.06 (1.02, 1.10)
France	1.00 (0.96, 1.03)	**1.04 (1.00, 1.08)**	1.04 (1.00, 1.08)
Germany	1.00 (0.97, 1.03)	**1.05 (1.02, 1.09)**	1.05 (1.02, 1.09)
Italy	1.00 (0.97, 1.03)	**1.07 (1.03, 1.11)**	1.07 (1.03, 1.11)
U.K.	1.00 (0.97, 1.03)	**1.09 (1.05, 1.14)**	1.09 (1.05, 1.14)
Series: LOW	No terms	An intercept	A linear trend
Spain	1.00 (0.97, 1.03)	**1.06 (1.02, 1.10)**	1.06 (1.02, 1.10)
France	1.00 (0.97, 1.03)	**1.05 (1.01, 1.09)**	1.05 (1.01, 1.09)
Germany	1.00 (0.97, 1.03)	**1.06 (1.02, 1.10)**	1.06 (1.02, 1.10)
Italy	1.00 (0.97, 1.03)	**1.08 (1.04, 1.13)**	1.08 (1.04, 1.13)
U.K.	1.00 (0.97, 1.03)	**1.06 (1.02, 1.11)**	1.06 (1.02, 1.11)
Series: RANGE	No terms	An intercept	A linear trend
Spain	0.39 (0.36, 0.41)	0.35 (0.32, 0.38)	**0.34 (0.31, 0.37)**
France	0.40 (0.37, 0.42)	0.37 (0.34, 0.40)	**0.37 (0.34, 0.40)**
Germany	0.38 (0.35, 0.40)	**0.36 (0.33, 0.38)**	0.35 (0.33, 0.38)
Italy	0.39 (0.36, 0.42)	0.36 (0.35, 0.39)	**0.36 (0.34, 0.39)**
U.K.	0.37 (0.34, 0.39)	**0.34 (0.32, 0.37)**	0.34 (0.32, 0.37)

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values. The coefficients in bold are those of the models selected on the basis of the statistical significance of the regressors.

Table 6
Results with DAILY data and AUTOCORRELATED disturbances (Model given by Eq. (4) with autocorrelated u_t).

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.99 (0.95, 1.04)	**0.94 (0.89, 1.00)**	0.94 (0.89, 1.00)
France	1.00 (0.95, 1.05)	**0.93 (0.88, 0.99)**	0.93 (0.88, 0.99)
Germany	0.99 (0.95, 1.05)	**0.97 (0.92, 1.03)**	0.97 (0.92, 1.03)
Italy	1.00 (0.95, 1.06)	**0.96 (0.91, 1.01)**	0.96 (0.91, 1.01)
U.K.	1.00 (0.95, 1.06)	**0.91 (0.86, 0.99)**	0.91 (0.86, 0.99)
Series: LOW	No terms	An intercept	A linear trend
Spain	1.00 (0.95, 1.05)	**0.89 (0.85, 0.94)**	0.89 (0.85, 0.94)
France	1.00 (0.95, 1.06)	**0.87 (0.83, 0.92)**	0.87 (0.83, 0.92)
Germany	1.00 (0.95, 1.06)	0.91 (0.87, 0.96)	**0.91 (0.87, 0.96)**
Italy	1.00 (0.95, 1.05)	**0.90 (0.86, 0.95)**	0.90 (0.86, 0.95)
U.K.	1.00 (0.95, 1.05)	**0.90 (0.86, 0.94)**	0.90 (0.86, 0.94)
Series: RANGE	No terms	An intercept	A linear trend
Spain	0.44 (0.40, 0.48)	0.39 (0.34, 0.43)	**0.37 (0.33, 0.42)**
France	0.45 (0.41, 0.49)	**0.41 (0.37, 0.46)**	0.40 (0.36, 0.45)
Germany	0.44 (0.41, 0.48)	**0.41 (0.37, 0.45)**	0.41 (0.37, 0.45)
Italy	0.46 (0.41, 0.50)	**0.42 (0.37, 0.45)**	0.41 (0.36, 0.46)
U.K.	0.49 (0.45, 0.53)	**0.45 (0.41, 0.50)**	0.45 (0.41, 0.50)

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values. The coefficients in bold are those of the models selected on the basis of the statistical significance of the regressors.

with the classical definition of cointegration) is also rejected this time in favour of d > 0. As for the results under the assumption of autocorrelated errors (Table 2), the estimates of d for high and low prices are slightly smaller than the previous ones and the unit root null is almost never rejected.[5] The values of d for the range are much smaller, the null hypothesis d = 1 being rejected in all cases, which implies cointegration. For Spain and Italy, the null d = 0 cannot be rejected, which suggests that classical cointegration holds in these two countries.

Concerning the weekly series, with white noise residuals (see Table 3) the estimates of d are again around 1, although in some cases (Spain, France, Germany and Italy with high prices) the unit root null is rejected in favour of d > 1, while the corresponding estimates for the range are between 0.34 (Spain) and 0.41 (Italy). With autocorrelated disturbances (see Table 4) the I(1) hypothesis cannot be rejected in any case for the high price series, whilst it is rejected in favour of mean reversion for the low price series; the estimates of d for the range are similar to the previous ones, lying in the interval between 0.30 (Spain) and 0.43 (Germany).

Finally, for the daily series, with white noise errors (see Table 5) the estimates of d for high and low prices are slightly above 1 and the I(1) hypothesis is rejected in favour of d > 1 in all cases; the estimate of d for the range is between 0.34 (UK) and 0.37 (France). However, with autocorrelated disturbances (see Table 6) mean reversion occurs in most cases for both high and low prices and the estimated value of d for the range is now between 0.37 (Spain) and 0.45 (UK).

Table 7–9 summarise the results for the monthly, weekly and daily series respectively. In brief, in all cases, the order of

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[5] In this case we found evidence of mean reversion (i.e., d < 1) for low prices in France and Germany.

Table 7
Summary of the results for the monthly series.

Country	Series	No autocorrelation	Autocorrelation
Spain	High	1.09 (0.95, 1.29)	0.88 (0.65, 1.18)
	Low	1.05 (0.90, 1.26)	0.74 (0.49, 1.07)
	Range	0.34 (0.18, 0.55)	0.09 (-0.21, 0.61)
France	High	1.09 (0.93, 1.32)	0.77 (0.53, 1.13)
	Low	0.93 (0.78, 1.14)	0.61 (0.33, 0.95)
	Range	0.43 (0.28, 0.63)	0.32 (0.05, 0.83)
Germany	High	0.96 (0.82, 1.15)	0.81 (0.54, 1.15)
	Low	0.96 (0.81, 1.17)	0.66 (0.43, 0.97)
	Range	0.38 (0.26, 0.53)	0.49 (0.22, 1.06)
Italy	High	1.05 (0.92, 1.23)	0.83 (0.59, 1.11)
	Low	0.97 (0.83, 1.17)	0.70 (0.49, 1.03)
	Range	0.39 (0.24, 0.59)	0.18 (-0.11, 0.63)
UK	High	1.01 (0.87, 1.20)	0.88 (0.65, 1.27)
	Low	0.86 (0.74, 1.03)	0.74 (0.53, 1.05)
	Range	0.27 (0.14, 0.43)	0.37 (0.08, 0.84)

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values.

Table 8
Summary of the results for the weekly series.

Country	Series	No autocorrelation	Autocorrelation
Spain	High	1.11 (1.04, 1.19)	0.98 (0.89, 1.09)
	Low	1.03 (0.96, 1.12)	0.84 (0.75, 0.94)
	Range	0.34 (0.28, 0.41)	0.30 (0.20, 0.41)
France	High	1.07 (1.00, 1.15)	0.94 (0.84, 1.07)
	Low	0.99 (0.92, 1.08)	0.78 (0.69, 0.89)
	Range	0.40 (0.34, 0.48)	0.36 (0.27, 0.48)
Germany	High	1.07 (1.00, 1.14)	1.00 (0.88, 1.14)
	Low	1.05 (0.97, 1.14)	0.80 (0.71, 0.91)
	Range	0.41 (0.36, 0.47)	0.43 (0.34, 0.54)
Italy	High	1.16 (1.09, 1.25)	0.98 (0.89, 1.11)
	Low	1.06 (0.99, 1.15)	0.84 (0.75, 0.96)
	Range	0.42 (0.35, 0.49)	0.31 (0.22, 0.42)
UK	High	1.06 (0.99, 1.15)	0.91 (0.81, 1.02)
	Low	1.02 (0.94, 1.11)	0.76 (0.68, 0.87)
	Range	0.40 (0.34, 0.47)	0.35 (0.27, 0.47)

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values.

Table 9
Summary of the results for the daily series.

Country	Series	No autocorrelation	Autocorrelation
Spain	High	1.06 (1.02, 1.10)	0.94 (0.89, 1.00)
	Low	1.06 (1.02, 1.10)	0.89 (0.85, 0.94)
	Range	0.34 (0.31, 0.37)	0.37 (0.33, 0.42)
France	High	1.04 (1.00, 1.08)	0.93 (0.88, 0.99)
	Low	1.05 (1.01, 1.09)	0.87 (0.83, 0.92)
	Range	0.37 (0.34, 0.40)	0.41 (0.37, 0.46)
Germany	High	1.05 (1.02, 1.09)	0.97 (0.92, 1.03)
	Low	1.06 (1.02, 1.10)	0.91 (0.87, 0.96)
	Range	0.36 (0.33, 0.38)	0.41 (0.37, 0.45)
Italy	High	1.07 (1.03, 1.11)	0.96 (0.91, 1.01)
	Low	1.08 (1.04, 1.13)	0.90 (0.86, 0.95)
	Range	0.36 (0.34, 0.39)	0.42 (0.37, 0.45)
UK	High	1.09 (1.05, 1.14)	0.91 (0.86, 0.99)
	Low	1.06 (1.02, 1.11)	0.90 (0.86, 0.94)
	Range	0.34 (0.32, 0.37)	0.45 (0.41, 0.50)

Notes. The reported coefficients are the estimated values of d, and in parentheses the 95 % confidence bands of its non-rejection values.

Table 10

Number of breaks in each series (High and Low prices series) using Bai and Perron (2003) and Gil-Alana (2008) methods.

i) Monthly data		
Series	N. breaks	Break dates
FRANCE	2	September 2013 and March 2017
GERMANY	2	(July 2010), (March 2012), September 2013 and December 2016
ITALY	2	August 2011 and January 2014
SPAIN	2	August 2011, (September 2013) and December 2015
U.K.	2	(July 2010), January 2013 and December 2016
ii) Weekly data		
Series	N. breaks	Break dates
FRANCE	2	(August 2011), April 2013, (March 2015) and March 2017
GERMANY	2	(July 2010), July 2013, (January 2015) and December 2016
ITALY	4	August 2011, October 2013, Novemebr 2015 and May 2017
SPAIN	4	August 2011, September 2013, December 2015 and June 2017
U.K.	2	(July 2010), January 2013, (June 2015) and December 2016
iii) Daily data		
Series	N. breaks	Break dates
FRANCE	4	June 2012, September 2013, January 2015, March 2017
GERMANY	4	August 2012, November 2013, February 2015, December 2016
ITALY	4	March 2012, October 2013, January 2016 nd March 2017
SPAIN	4	March 2012, September 2013, December 2016 and February 2017
U.K.	4	February 2013, May 2014, August 2015 nd December 2016

Table 11

Estimates of d for each country and each subsample: High Prices Series.

Country	Subsamples	Monthly	Weekly	Daily
FRANCE	1st subsample	0.937	0.834	1.021
	2nd subsmple	0.957	0.851	0.859
	3rd subsample	0.891	1.040	0.763
	4rd subsample	—	—	0.935
	5th subsample	—	—	0.882
GERMANY	1st subsample	0.699	0.731	1.240
	2nd subsmple	0.890	0.898	0.764
	3rd subsample	1.247	1.199	0.810
	4rd subsample	—	—	1.133
	5th subsample	—	—	0.934
ITALY	1st subsample	0.969	0.737	1.096
	2nd subsmple	0.569*	0.637	1.037
	3rd subsample	1.261	0.751	0.891
	4rd subsample	—	0.991	0.839
	5th subsample	—	0.936	0.877
SPAIN	1st subsample	1.222	0.862	0.926
	2nd subsmple	1.019	0.981	1.018
	3rd subsample	1.369	0.534*	0.937
	4rd subsample	—	0.932	0.715*
	5th subsample	—	0.510*	0.932
UK	1st subsample	0.961	0.804	0.922
	2nd subsmple	0.687*	0.977	0.552*
	3rd subsample	1.253	0.792	0.767
	4rd subsample	—	—	0.770
	5th subsample	—	—	0.912

* Evidence of mean reversion ($d < 1$) at the 5 % level.

integration of the range is lower than that of the original series, which implies the existence of a long-run equilibrium relationship between high and low prices. Further, the estimated fractional differencing parameter is positive in all cases, which represents evidence of long memory. Therefore, in comparison to the standard cointegration case the dynamic adjustment can take much longer.[6]

The possibility of structural breaks has also been taken into account. Specifically, we carry out both the Bai and Perron's (2003) tests for multiple breaks and those suggested by Gil-Alana (2008), which are an extension of the former to the fractional case. The results are reported in Table 10; the detected breaks are the same for both high and low prices, whilst there is no evidence of any breaks in the range, which is consistent with breaks in the two former series occurring at the same time.

More specifically, two breaks are identified in the monthly series, whilst in the weekly case two breaks are found for France,

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[6] Almost identical results were obtained when using a semi-parametric (Whittle) approach. These are available from the authors upon request.

Germany and the UK, and four breaks for Italy and Spain; finally, four breaks are detected in all series at the daily frequency, the most common break dates being around September 2013, December 2016 and March 2017. The first corresponds to the anticipation of the US federal government shutdown of October 2013 and the resulting concerns about growth in the US, which had an adverse effect on stock markets throughout the world. The second one coincides with the increase in interest rates decided by the Fed (only the second since the global financial crisis of 2008), which again had a negative impact on financial markets worldwide. Finally, the third one is likely to be related to the release of strong US GDP data and the resulting surge in the Dow Jones and other stock market indices.

Next, we estimate the differencing parameter for each subsample, using here a semi-parametric method where no functional form is imposed on the error term. We use the local Whittle method proposed by Robinson (1995) and later extended by Phillips and Shimotsu (2005); Abadir et al. (2007) and others. We choose this approach because of its simplicity and the fact that, apart from the bandwidth, it does not require additional user-chosen parameters, to which the results could be sensitive. Table 11 displays the estimates of d for the high prices series and for bandwidth $m = (T)^{0.5}$ which has been widely used in the empirical literature. It can be seen that there are a few cases when mean reversion occurs, specifically in Italy (2nd subsample) and the UK (2nd subsample) with monthly data, in Spain (3rd and 5th subsamples) and the UK (2nd subsample) with weekly data, and again in Spain (4th subsample) and in the UK (2nd subsample) with daily data. In all the other cases, the null of I(1) or explosive behaviour (I(d, d > 1) cannot be rejected. Almost identical results were obtained in case of the low prices series. Note that the results presented in Table 11 should be taken with caution, especially in the case of the monthly data, because of the relatively short subsamples being analysed. Further, they are not directly comparable to those reported in Tables 1–6 firstly because of the different sample sizes and methods used (parametric in Tables 1–6 and a semiparametric in Table 11). However, they are very similar, the estimates of d being relatively high and close to 1 in most cases.

5. Conclusions

This paper has used fractional integration techniques to examine the stochastic behaviour of high and low stock prices in Europe and then to test for the possible existence of long-run linkages by looking at the range, i.e., the difference between the two logged series. Specifically, monthly, weekly and daily data on the following five European stock market indices have been analysed: DAX30 (Germany), FTSE100 (UK), CAC40 (France), FTSE MIB40 (Italy) and IBEX35 (Spain). The methods used are more general and flexible than the standard ones applied in previous studies such as Cheng et al. (2009) since they allow for the differencing parameter to take fractional values and therefore are able to capture a much greater variety of dynamic and long-run behaviours.

The empirical findings suggest that the range is mean-reverting in all cases, which implies the existence of a long-run cointegrating relationship between these two series. This confirms the well-known finding in the literature that high and low prices move together in the long run also in the case of the European stock markets and when adopting a much more general empirical framework. Further, our results indicate the presence of long-memory behaviour in both high and low prices, since the estimated value of d is always positive. This evidence of persistence goes contrary to the EMH (see Fama, 1970). As for break tests, these detect various break dates in both high and low prices but not in the range, which is consistent with the breaks occurring at the same time in the former two series.

Future research could investigate whether or not the range exhibits long memory in the US case as well. Further, alternative fractional cointegration methods such as the FCVAR model proposed by Johansen and Nielsen (2010, 2012) could also be used as a robustness check. In addition, other issues such as determining which variable is driving the cointegrating relationships could be examined. Finally, it would be interesting to investigate whether similar results can be obtained in the case of absolute and/or squared returns as a proxy for volatility, and to examine the forecasting properties of the range.

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