In-Network Algorithm for Passive Sensors in Structural Health Monitoring

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Abstract—Structural health monitoring (SHM) using wireless sensor networks (WSN) has become a popular implementation, due to low maintenance and installation costs. These networks commonly use a centralized approach and battery-powered sensors, leading to energy consumption limitations, in both the central unit and the sensors. Therefore, it is of interest to consider the use of passive sensors and distributed processing in the network. In this letter, we present a distributed algorithm for SHM using wireless passive sensor networks (WPSNs) that allows any passive sensor in the network to obtain the distance to its neighbours via backscattering, and hence to detect and signal changes in the monitored structure.

Index Terms—Structural health monitoring (SHM), wireless passive sensor networks (WPSNs), distributed algorithm, iterative algorithm.

I. INTRODUCTION

T HE process of implementing a real-time damage detection strategy to monitor complex structures such as aerospace, civil and mechanical infrastructures is referred to as Structural Health Monitoring (SHM). A common strategy in SHM is to employ a network of sensors. Traditionally, sensor-based SHM systems were designed using a dense grid of sensors positioned along a structure, connected with wires and with a central processing unit [1], [2]. Wire-based SHM systems have a high installation and maintenance cost. As an alternative, wireless sensor networks (WSN) have emerged as a solution for SHM systems, due to their lower installation and maintenance cost [3], [4].

Sensors in a WSN are usually battery powered. As a result, they have very limited energy resources. Therefore, the problem of network design is usually approached to minimize energy consumption [1], [5], [6], [7]. Using a centralized approach on a wireless battery-powered SHM system has significant limitations, since sensors have to transmit the acquired data to a central entity, leading to high energy consumption. Therefore, for large-size networks it is of great interest to consider distributed processing within the network [8]. Decentralization additionally offers increased robustness and improves the overall system's

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responsiveness [2], [9]. Although distributed algorithms for wireless SHM sensor networks have been proposed in the literature [2], [10], [11], [12], SHM algorithms and distributed processing is still an open research issue [13].

Even though energy consumption can be minimized using distributed processing within the network, batteries eventually run out. Having to change sensor batteries causes maintenance costs to rise and might be particularly critical if the sensors are embedded in the structure or not easily accessible. Nevertheless, since a dense distributed sensor network is being considered, where communications between sensors are at a short distance, a wireless passive sensor network (WPSN) can be used. Power is externally supplied to sensor nodes through an external RF source, and sensor nodes communicate via backscattering [14], [15].

In this letter, we present a distributed algorithm for structural crack detection and monitoring by using a WPSN. Specifically, our algorithm allows any passive sensor in a wireless SHM network to obtain the distance to their neighbours. By each sensor knowing the distance to their neighbours, variances in those distances signal changes (cracks) in the structure being monitored. In [16] a literature review regarding the use of passive sensors to monitor crack growth is presented. None of the works cited in [16] is distributed, that is, in those works the information needed to monitor the structure is captured by the reader instead of the sensors themselves. In addition, the papers referred there have other drawbacks, e.g., crack position should be known a priori, or only surface cracks are detected. To the best of our knowledge, the most closely related work to ours is a tracking algorithm for passive tags that has been proposed in [17], in which sensors determine the distance to one another via backscattering communication. However, that algorithm is only suitable for a two-sensor network, while our algorithm is suitable for any network with an arbitrary number of sensors.

The remainder of this letter is organized as follows: In Section II we introduce the problem of estimating the distances among passive sensors in a wireless SHM network and in Section III we present an iterative fully distributed algorithm that solves the considered problem. In Section IV we present a numerical example to assess the theoretical performance of our algorithm in the presence of multiple reflections and background noise. Finally, we present our conclusions in Section V.

II. PROBLEM STATEMENT

We begin by introducing some notation. If A is a set, |A| denotes its cardinality, and if z is a complex number, Re(z) denotes its real part.

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Fig. 1. Graphical representation of the signals received by node *j*.

Consider a network composed of n passive tags (RF sensors or nodes). These sensors can communicate via backscatter modulation with a certain number of neighbours within their coverage area, which will be the set of sensors located to a threshold distance less than d. Let $\mathcal{N}_j \subset \{1, \ldots, n\}$ denote the local neighbourhood of node j, i.e., the set of nodes within distance d of node j, without including the node j itself. Node j aims to know the distance to its neighbours (see Fig. 1).

Suppose that a continuous wave signal generator (exciter) with angular frequency w is located in the vicinity of the sensor network. Each sensor j will then receive a signal $\hat{s}_j(t)$ resulting from the addition of two terms, i.e., $\hat{s}_j(t) = s_j(t) + \eta_j(t)$. On the one hand, $s_j(t)$ is the signal resulting from the addition of the signal originated at the exciter and the (first) signal reflected at the neighbouring sensors:

$$s_j(t) = a_{E_j} \cos\left(\omega t + \theta_{E_j}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left(\omega t + \theta_{j,k}\right) + \sum_{k \in \mathcal{N}_j} a_{j,k} \cos\left($$

where a_{E_j} and θ_{E_j} are the amplitude and phase of the signal that node j receives directly from the exciter, $a_{j,k}$ and $\theta_{j,k}$ are the amplitude and phase of the signal that node j receives reflected from node k (first reflection), and ψ_k is the reflection phase of node k. On the other hand, $\eta_j(t)$ is the sum of the rest of the signal reflections received by node j and background noise:

$$\eta_j(t) = \sum_{q=1}^{\infty} \eta_{j,q} \cos\left(\omega t + \xi_{j,q}\right) + \mathsf{bg}_j(t)$$

where $\eta_{j,q}$ and $\xi_{j,q}$ are the amplitude and phase of the *q*-th reflection received by node *j*, and $bg_j(t)$ is background noise. It can be shown that $\eta_j(t)$ can be modelled as a Wide Sense Stationary (WSS) process under certain assumptions.¹ This allows to evaluate the algorithm from a theoretical point of view by performing simulations. In the remainder of this letter, for the theoretical design of our algorithm, the noise term $\eta_j(t)$ will be neglected but its effect will be considered in Section IV to assess the performance of the algorithm.

Observe that $s_i(t)$ can be expressed as

$$s_j(t) = \operatorname{Re}\left(a_{E_j}e^{\mathrm{i}(\omega t + \theta_{E_j})}\right) + \sum_{k \in \mathcal{N}_j} \operatorname{Re}\left(a_{j,k}e^{\mathrm{i}(\omega t + \theta_{E_k} + \psi_k + \theta_{j,k})}\right)$$

¹Assumptions: 1) bg_j(t) is a WSS process, 2) $\eta_{j,q}$ and $\xi_{j,p}$ are independent random variables for all q and p, 3) $\xi_{j,q}$ for all q are independent uniform random variables on the interval $[0, 2\pi)$, and 4) $\sum_{q=1}^{\infty} E[\eta_{j,q}^2] < \infty$.

$$= \operatorname{Re}\left(\left(a_{E_j}e^{\mathrm{i}\theta_{E_j}} + \sum_{k \in \mathcal{N}_j} a_{j,k}e^{\mathrm{i}(\theta_{E_k} + \psi_k + \theta_{j,k})}\right)e^{\mathrm{i}\omega t}\right).$$

Observe that since the angular frequency of the signals is known, the distance between nodes j and k can be directly obtained from $\theta_{j,k}$ (obviously, $\theta_{j,k} = \theta_{k,j}$). Therefore, the goal of node j is to know $\theta_{j,k}$ for $k \in \mathcal{N}_j$. Let $\varphi_{j,k} = \theta_{E_j} - \theta_{E_k} - \theta_{j,k}$ and

$$z_j = a_{E_j} e^{\mathrm{i}\theta_{E_j}} + \sum_{k \in \mathcal{N}_j} a_{j,k} e^{\mathrm{i}\left(\theta_{E_j} + \psi_k - \varphi_{j,k}\right)}.$$

Since $s_j(t) = \text{Re}(z_j e^{i\omega t})$, the amplitude of the signal received by node j is the modulus of z_j . Then, if sensor j uses an envelope detector to demodulate the signal s_j , we have that

$$\begin{split} |z_{j}|^{2} &= z_{j}\overline{z_{j}} = a_{E_{j}}^{2} + \sum_{k \in \mathcal{N}_{j}} a_{E_{j}} a_{j,k} \left(e^{\mathrm{i}(\psi_{k} - \varphi_{j,k})} + e^{-\mathrm{i}(\psi_{k} - \varphi_{j,k})} \right) \\ &+ \sum_{k \in \mathcal{N}_{j}} \sum_{\ell \in \mathcal{N}_{j}} a_{j,k} a_{j,\ell} e^{\mathrm{i}(\psi_{k} - \psi_{\ell} + \varphi_{j,\ell} - \varphi_{j,k})} \\ &= a_{E_{j}}^{2} + \sum_{k \in \mathcal{N}_{j}} 2a_{E_{j}} a_{j,k} \cos \left(\psi_{k} - \psi_{\ell} + \varphi_{j,\ell} - \varphi_{j,k}\right) \\ &+ \sum_{k \in \mathcal{N}_{j}} \sum_{\ell \in \mathcal{N}_{j}} a_{j,k}^{2} a_{j,\ell} \cos \left(\psi_{k} - \psi_{\ell} + \varphi_{j,\ell} - \varphi_{j,\ell}\right) \\ &= a_{E_{j}}^{2} + \sum_{k \in \mathcal{N}_{j}} a_{j,k}^{2} a_{j,k} \sin \psi_{k} \sin \varphi_{j,k} \\ &+ \sum_{k \in \mathcal{N}_{j}} 2a_{E_{j}} a_{j,k} \sin \psi_{k} \sin \varphi_{j,k} \\ &+ \sum_{k \in \mathcal{N}_{j}} \sum_{\ell \in \mathcal{N}_{j}} a_{j,k} a_{j,\ell} \cos \left(\psi_{k} - \psi_{\ell}\right) \cos \left(\varphi_{j,k} - \varphi_{j,\ell}\right) \\ &+ \sum_{k \in \mathcal{N}_{j}} \sum_{\ell \in \mathcal{N}_{j}} a_{j,k} a_{j,\ell} \sin \left(\psi_{k} - \psi_{\ell}\right) \sin \left(\varphi_{j,k} - \varphi_{j,\ell}\right) \\ &= \alpha_{j} + \sum_{k \in \mathcal{N}_{j}} \sum_{\ell \neq k} C_{j,k,\ell} \cos \left(\psi_{k} - \psi_{\ell}\right) \\ &+ \sum_{k \in \mathcal{N}_{j}} \sum_{\ell \in \mathcal{N}_{j}} D_{j,k,\ell} \sin \left(\psi_{k} - \psi_{\ell}\right), \tag{1}$$

where $\alpha_j = a_{E_j}^2 + \sum_{k \in \mathcal{N}_j} a_{j,k}^2$ and

$$A_{j,k} = 2a_{E_j}a_{j,k}\cos\varphi_{j,k} \tag{2}$$

$$B_{j,k} = 2a_{E_j}a_{j,k}\sin\varphi_{j,k} \tag{3}$$

$$C_{j,k,\ell} = a_{j,k}a_{j,\ell}\cos\left(\varphi_{j,k} - \varphi_{j,\ell}\right) \tag{4}$$

$$D_{j,k,\ell} = a_{j,k}a_{j,\ell}\sin\left(\varphi_{j,k} - \varphi_{j,\ell}\right).$$
(5)

Since the reflection phase of the neighbouring nodes, ψ_k with $k \in \mathcal{N}_j$, is assumed to be known for node j, (1) is a linear equation with unknowns α_j , $A_{j,k}$, $B_{j,k}$, $C_{j,k,\ell}$, $D_{j,k,\ell}$ for $k, \ell \in \mathcal{N}_j$ and $k \neq \ell$. The number of unknowns is $2|\mathcal{N}_j|^2 + 1$ and therefore, the node j needs at least $2|\mathcal{N}_j|^2 + 1$ linearly independent equations of the form (1),² in order to locally solve for the unknowns. To that end, we consider that all the nodes of the network are able to vary their reflection phase and that this phase is known by their neighbours.³ Once the unknowns α_j , $A_{j,k}$, $B_{j,k}$, $C_{j,k,\ell}$, $D_{j,k,\ell}$ for $k, \ell \in \mathcal{N}_j$ and $k \neq \ell$ are locally solved, from (2), (3), (4), and (5) we have the following overdetermined system of $|\mathcal{N}_j|$ unknowns and $|\mathcal{N}_j|^2$ equations:

$$\varphi_{j,k} = \arctan \frac{B_{j,k}}{A_{j,k}}, \quad k \in \mathcal{N}_j$$
$$\varphi_{j,k} - \varphi_{j,\ell} = \arctan \frac{D_{j,k,\ell}}{C_{j,k,\ell}}, \quad k, \ell \in \mathcal{N}_j, \ k \neq \ell$$

For this system of equations, a least squares problem is also locally solved at node j to obtain $\varphi_{j,k}$ for all $k \in \mathcal{N}_j$.

We recall that $\varphi_{j,k} = \theta_{E_j} - \theta_{E_k} - \theta_{j,k}$ and that $\theta_{j,k}$ provides the sought distance between node j and node k. Therefore, node j knows the following $|\mathcal{N}_j|$ equations:

$$\theta_{E_j} - \theta_{E_k} - \theta_{j,k} = \varphi_{j,k} \qquad k \in \mathcal{N}_j, \tag{6}$$

where θ_{E_j} , θ_{E_k} , and $\theta_{j,k}$ with $k \in \mathcal{N}_j$ are the unknowns. If we consider all the equations known by the nodes of the entire network (equations in (6) and $j \in \{1, \ldots, n\}$), we have a (possibly inconsistent) linear system of $p = \sum_{j=1}^{n} |\mathcal{N}_j|$ equations with $q = n + \frac{1}{2} \sum_{j=1}^{n} |\mathcal{N}_j|$ unknowns, where each node has only a partial information of the whole system of equations. We can express such system of equations in matrix form as Ax = b, where *A* is a $p \times q$ sparse matrix with three non-zero entries per row (see (6)). The goal is to obtain a least squares solution of the whole system of equations in a distributed way.

III. ALGORITHM

In this section, we propose an iterative fully distributed algorithm (Algorithm 2) that allows to locally obtain a least squares solution of the aforementioned whole system of equations (Ax = b). That is, Algorithm 2 allows any passive sensor in a wireless SHM network to know the distances to its neighbours. In the literature there exist various iterative algorithms to approximate a least squares solution of a linear system of equations (see, e.g., [18], [19], [20], [21]). Among them, the Randomized Gauss-Seidel Algorithm is suitable to solve our (possibly inconsistent) linear system of equations [18]. Algorithm 2 is in fact a distributed implementation of the Randomized Gauss-Seidel Algorithm.

For the reader's convenience, we first revisit the Randomized Gauss-Seidel Algorithm (see [19, p. 645] or [18, Sec. 2.3]) for approximating a least squares solution of a linear system of equations of the form Ax = b, being x_0 an arbitrary initial point, e_c a column vector with 1 in the *c*-th position and 0 elsewhere, and $\|\cdot\|_F$ the Frobenius norm (see Algorithm 1).

²In case of having more equations than $2|N_j|^2 + 1$, the least squares solution of the resulting overdetermined system should be obtained.

Algorithm	11:	Randomized	Gauss-Seidel	Algorithm.
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$1: x \leftarrow x_0$				
2: $r \leftarrow b - Ax$				
3: while 1 do				
4: Choose unknown $[x]_{c,1}$ with probability $\frac{ Ae_c _F^2}{ A _F^2}$				
5: $\alpha \leftarrow \frac{(Ae_c)^\top r}{\ Ae_c\ _F^2}$				
6: $x \leftarrow x + \alpha e_c$				
7: $r \leftarrow r - \alpha A e_c$				
8: end while				

Since Algorithm 2 is a distributed implementation of Algorithm 1, the unknowns computed by each node have to be first established. To that end, before running Algorithm 2, we assume that a master-slave relationship has been established between each pair of neighbours, and we denote with $\mathcal{V}_j \subset \mathcal{N}_j$ the set of slave neighbours of node j. Then, the unknowns computed by the node j will be $\theta_{E_j}, \{\theta_{j,k}\}_{k \in \mathcal{V}_j}$. Since the algorithm is distributed, at each step only one node updates the value of one of its unknowns.

Let N be the random variable that signals the node that updates the unknown and let Θ be the random variable that signals the unknown to be updated. From line 4 of Algorithm 1, observe that for $j \in \{1, ..., n\}$

$$P\left(\Theta = \theta_{E_j}\right) = \frac{2|\mathcal{N}_j|}{3\sum_{\ell=1}^n |\mathcal{N}_\ell|},$$
$$P\left(\Theta = \theta_{j,k}\right) = \frac{2}{3\sum_{\ell=1}^n |\mathcal{N}_\ell|} \text{ for all } k \in \mathcal{V}_j.$$

Consequently, for $j \in \{1, ..., n\}$, from the law of total probability we have⁴

$$P(N = j) = \frac{2\left(|\mathcal{N}_j| + |\mathcal{V}_j|\right)}{3\sum_{\ell=1}^n |\mathcal{N}_\ell|},$$

and from Bayes' theorem we have

$$P\left(\Theta = \theta_{E_{j}} | N = j\right) = \frac{\frac{2|\mathcal{N}_{j}|}{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|}}{\frac{2|\mathcal{N}_{j}|}{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|} + |\mathcal{V}_{j}| \frac{2}{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|}}$$
$$= \frac{|\mathcal{N}_{j}|}{|\mathcal{N}_{j}| + |\mathcal{V}_{j}|}$$
$$P\left(\Theta = \theta_{j,k} | N = j\right) = \frac{\frac{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|}{\frac{2|\mathcal{N}_{j}|}{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|}} + |\mathcal{V}_{j}| \frac{2}{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|}}{\frac{2|\mathcal{N}_{j}|}{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|} + |\mathcal{V}_{j}| \frac{2}{3\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|}}$$
$$= \frac{1}{|\mathcal{N}_{j}| + |\mathcal{V}_{j}|}.$$

Furthermore, for $j \in \{1, ..., n\}$, observe that if θ_{E_j} is chosen in line 4 of Algorithm 1, then α in line 5 of Algorithm 1 is given by $\alpha = \frac{1}{2|\mathcal{N}_j|} \sum_{k \in \mathcal{N}_j} (r_{j,k} - r_{k,j})$. On the other hand, if $\theta_{j,k}$ is chosen in line 4 of Algorithm 1 for some $k \in \mathcal{V}_j$, then α in line 5 of Algorithm 1 is given by $\alpha = \frac{1}{2}(-r_{j,k} - r_{k,j})$.

³This can be done either because each node communicates its reflection phase to its neighbours or because the entire network shares a predefined rule for varying the reflection phases.

⁴The value of $\sum_{\ell=1}^{n} |\mathcal{N}_{\ell}|$ can be computed in a distributed way by using, for example, [22].

tances Among Passive Sensors in a Wireless SHM Network. 1: for all nodes $j \in \{1, ..., n\}$ do 2: $P(\Theta = \theta_{E_j} | N = j) \leftarrow \frac{|\mathcal{N}_j|}{|\mathcal{N}_j| + |\mathcal{V}_j|}$ 3: $P(N = j) \leftarrow \frac{2(|\mathcal{N}_j| + |\mathcal{V}_j|)}{3\sum_{\ell=1}^n |\mathcal{N}_\ell|}$ 4: $\theta_{E_i} \leftarrow 0$ for all $k \in \mathcal{V}_j$ do 5: $P(\Theta = \theta_{j,k} | N = j) \leftarrow \frac{1}{|\mathcal{N}_i| + |\mathcal{V}_i|}$ 6: 7: $\theta_{i,k} \leftarrow 0$ end for 8: 9: for all nodes $k \in \mathcal{N}_j$ do 10: $r_{j,k} \leftarrow \varphi_{j,k}$ end for 11: 12: end for 13: while 1 do Choose node j with probability P(N = j)14: Choose among unknowns $\theta_{E_i}, \{\theta_{j,k}\}_{k \in \mathcal{V}_i}$ with 15: probabilities $P(\Theta = \theta_{E_j} | N = j), \{P(\Theta = \theta_{j,k} | N = j)\}_{k \in \mathcal{V}_j}$ 16: **if** θ_{E_i} is chosen **then** $\alpha \leftarrow \frac{1}{2|\mathcal{N}_j|} \left(\sum_{k \in \mathcal{N}_j} r_{j,k} - \sum_{k \in \mathcal{N}_j} r_{k,j}\right)$ 17: $\begin{array}{l} \theta_{E_j} \leftarrow \theta_{E_j} + \alpha \\ \text{for all nodes } k \in \mathcal{N}_j \text{ do} \end{array}$ 18: 19: 20: $\begin{array}{c} r_{j,k} \leftarrow r_{j,k} - \alpha \\ r_{k,j} \leftarrow r_{k,j} + \alpha \end{array}$ 21: 22: end for 23: else if $\theta_{j,k}$ is chosen then $\begin{array}{c} \alpha \leftarrow -\frac{1}{2}(r_{j,k} + r_{k,j}) \\ \theta_{j,k} \leftarrow \theta_{j,k} + \alpha \end{array}$ 24: 25: 26: $r_{j,k} \leftarrow r_{j,k} + \alpha$ 27: $r_{k,j} \leftarrow r_{k,j} + \alpha$ 28: end if 29: If $|\alpha| < \epsilon$ (ϵ denotes the desired accuracy) break 30: end while

Algorithm 2: Distributed Algorithm for Estimating the Dis-

$10^{(}$ +Path with n = 5 nodes \bullet Cycle with n = 5 nodes **-**Grid with n = 16 nodes Average relative error 10 10-4 10^{-6} 10⁻⁸ 10⁻¹⁰ 10 C 20 30 40 50 60 70 Signal-to-noise ratio (dB)

Fig. 2. Average relative error as a function of the SNR.



with n = 5 nodes, (c) Grid with n = 16 nodes.

V. CONCLUSION

In this letter, we have presented a distributed algorithm for Structural health monitoring (SHM) by using a wireless passive sensor network (WPSN). Specifically, our algorithm allows any passive sensor in a wireless SHM network to obtain the distance to their neighbours via backscattering. By knowing the distance to their neighbours, each sensor can detect and signal changes in the structure being monitored. This algorithm is suitable for any network with an arbitrary number of sensors.

We have assessed the theoretical performance of the algorithm in the presence of background noise for three different network topologies, although the algorithm is valid for any network topology. Results show that using the distributed algorithm proposed in this letter, with large WPSN, would be a very useful low cost alternative for SHM. Finally, the algorithm here presented should also be validated by laboratory measurement.

IV. NUMERICAL EXAMPLES

In this section we present a numerical example to assess the theoretical performance of Algorithm 2 in the presence of multiple reflections and background noise. For ease of explanation, we will consider that the noise power received by each sensor is P_{η} , that is, the power of $\eta_j(t)$ is the same for all j. Considering that the exciter generates a continuous wave with power P, the signal-to-noise ratio (SNR) is P/P_{η} . The average relative error is defined as

$$\frac{1}{L}\sum_{j=1}^{n}\sum_{\substack{k\in\mathcal{N}_j\\k>j}}\frac{\left|\widehat{d_{j,k}}-d_{j,k}\right|}{d_{j,k}}$$

where L is the total number of distances to be computed, $d_{j,k}$ is the actual distance between nodes j and k and $d_{j,k}$ is the distance between nodes j and k estimated by using Algorithm 2. Fig. 2 shows the average relative error as a function of the SNR for the three different network topologies shown in Fig. 3.

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