Consumer surplus bias and the welfare effects of price discrimination*

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Abstract

A well-known result with important policy implications is that an output increase is a necessary condition for social welfare to increase with third-degree price discrimination. In this paper, we explore the robustness of this result to the introduction of an assumption that is different than the conventional approach, namely preferences not being quasilinear. We show that in the presence of income differences among consumers, the aggregate utility of consumers may increase with price discrimination while total output remains constant. **This result questions** the general policy recommendation that third-degree price discrimination should be disapproved **because it reduces welfare unless output increases. Our** result highlights the crucial role of the assumption of quasilinear preferences in standard welfare calculations. In the presence of income differences, consumer surplus may be a biased welfare measure, thus potentially leading to incorrect conclusions when assessing the impact of specific policies.

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1 Introduction

A central question in Economics is the welfare consequences of government intervention. In fact, competition authorities and regulators are typically concerned about the welfare of consumers and of society as a whole. This calls for the need of a measure of welfare that may be used to evaluate whether or not society is better off after a given policy is adopted. According to Economic Theory, consumer surplus is an accurate measure of welfare, as it reflects consumers' willingness-to-pay. However, the validity of this measurement, especially when dealing with aggregate consumer surplus, relies on the assumption of a constant utility of income, typically associated with the assumption of quasilinear preferences. This consideration may have important effects on the evaluation of government policies. Since standard welfare calculations that rely on the concept of consumer surplus may be giving a greater weight to individuals or markets where income levels are higher, these calculations may be distorted, potentially leading to incorrect conclusions. This is precisely the issue that we address in this paper, for the specific case of third-degree price discrimination.

Specifically, a well-known result in the literature (Schmalensee, 1981; Varian, 1985) is that – assuming quasilinear preferences, and therefore a constant marginal utility of income– an output increase is a necessary condition for welfare to increase with price discrimination. Theorists and policy-makers have taken the implications of this result as generally valid for market regulation, which has led to the widespread view that banning price discrimination is desirable unless total output increases with discrimination. For instance, in a recent background note by the OECD's Directorate for Financial and Enterprise Affairs (OECD, 2016), it is argued that "one clear test for identifying the impact of discrimination on consumers is to ask whether the discrimination significantly increases output or not."

In contrast, we **propose a theoretical model to** show that this result may fail to hold if the marginal utility of income is not constant: consumers may be strictly better off with price discrimination even if output remains constant when the monopolist is allowed to price discriminate. **In order for this result to arise, we need to use a utility function where the marginal utility of income is not constant. Our choice of the particular utility function that we use in our model was driven by its simplicity.** Given that discrimination never reduces the profits of the producer, we focus on consumer surplus to evaluate whether social welfare increases. We argue that since a greater willingness-to-pay may stem from having higher income rather than a greater utility of consumption, if the marginal utility of income is not constant across consumers then consumer surplus may be a biased measure of how well off consumers in a particular market are.

The welfare effects of price discrimination by a monopolist have long been the object of study in the Industrial Organization literature. An early formalization of the argument was included in Schmalensee (1981), who concluded that output increase is a necessary condition for third-degree price discrimination to enhance social welfare, defined as the addition of consumer and producer surplus. Even if the condition was initially established only for the case of independent demands and constant marginal costs, its validity was soon generalized. Building upon quasilinear utility functions, Varian (1985) extended this conclusion also to the case of increasing marginal costs.

However, recent contributions have explored the limitations of this result, showing for instance that this proposition cannot be generalized to an oligopoly with asymmetric costs (Galera and Zaratiegui, 2006), with convex costs in the presence of demand uncertainty (Galera et al., 2014), externalities (Adachi, 2005), quality differences (Galera et al., 2017a), or seasonal demands (Galera et al., 2017b). In all these cases, price discrimination can be welfare-increasing even if output does not increase. In this paper we study the robustness of the fundamental result in Schmalensee (1981) and Varian (1985) if we abandon the usual assumption of quasilinear utility functions.

In fact, the use of quasilinear utility functions is the standard procedure for aggregating satisfaction levels of individuals. It is argued in Varian (1985) that "for this class of preferences (...) not only does consumer's surplus serve as a legitimate measure of individual welfare, but also that the individual consumers' utility functions can be added up to form a social utility function, so that aggregate consumers' surplus is also meaningful." However, this approach, even if unintentionally, biases the welfare calculations in favor of the individuals with higher income levels. Our main point is therefore that the use of quasilinear utility functions may be analytically convenient, but this modeling assumption necessarily constrains the scope of the analysis and its conclusions to the case of constant marginal utility of income. In other words, the policy maker should be concerned about taking at face value the recommendations emanating from an analysis that relies on the use of quasilinear utilities, which could only be taken as accurate for individuals, or groups of individuals, who have identical purchasing power.

The concern regarding this issue is not precisely new. Over a century ago, Wright (1917) argued that "in the case of a community whose members differ widely with respect to money income, the demand curve and the price no longer even approximately represent conditions of utility. The utility to the first purchaser, a millionaire, may be low, while the utility to the marginal purchaser, a worker in a sweat-shop, may be very high. The millionaire takes the first unit not because its utility to him is higher than to the worker but because the sacrifice involved in purchasing it is very much less". This argument, as Wright explicitly mentions, is inspired in Marshall (1890): "A pound's worth of satisfaction to an ordinary poor man is a much greater thing than a pound's worth of satisfaction for an ordinary rich man".

Two different problems must be distinguished when using consumer surplus as a measure of consumer satisfaction. The first one refers to the existence of income effects, which makes it difficult to accurately define the consumer surplus. Willig (1976) manages to (partially) solve the problem by approximation: given that income effects are usually small, he claims that the consumer surplus is a good proxy measure of welfare anyway. But we disclose here a different aspect, which arises when comparing utility levels across individuals. For this comparison to be properly done, the marginal utility of money ought to be similar for all the consumers; but this will hardly be the case, as income levels differs substantially across individuals.[1] Yet, the limitations associated with the consumer surplus to evaluate welfare in real markets, which used to be widely acknowledged a few decades ago, have progressively been forgotten, due presumably to the technical advantages of this tool. Moreover, its validity for economic analysis and policy decision making has generally been taken for granted, as in, for instance, Willig (1976), Pindyck and Rubinfeld (2012), Mankiw (2014).

Therefore, we contribute to the literature in two ways. First, we highlight the risk of misunder-

[1] Notice further that various theoretical analyses have been based upon the condition that the incremental consumer's surplus is independent of the consumer's income. See, for instance, Willig (1978). Also Foster and Neuburger (1974) address the problems with the behavior of the marginal utility of money. They actually refer to the fact that: "Marshall originally developed his consumer surplus as a measure of cardinal utility. In order to do this he required the condition of constant marginal utility of money. Thus, for him the measure was never anything but a good approximation".

standing the scope of Varian (1985), whose findings we claim to depend critically on the assumption of quasilinear utility functions. We then examine the extent to which the current consensus –concerning the requirement that increasing social welfare is not possible unless output increases– can be legitimate in practice. Furthermore, we show that welfare enhancing policies can result from adopting third-degree price discrimination, even if total output does not increase. In line with this conclusion, further research is needed to establish the conditions under which preventing price discrimination may be harmful to society. Second, and more importantly, our paper challenges the validity of the consumer surplus as a definitive way to evaluate welfare.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model that will be used to derive the main result of the paper. Section 3 discusses this result and proposes an alternative interpretation of the model. Finally, Section 4 offers some concluding comments.

2 The model

We consider the existence of two markets, 1 and 2. **There is a representative consumer in each market. Both** consumers have the same preferences on goods x and y. Specifically, we assume that the utility function of any individual is given by

$$U(x,y) = u(x) + v(y),$$

where $u(\cdot)$ and $v(\cdot)$ are concave functions. In particular, so as to simplify the analysis, let us assume that

$$u(z) = v(z) = z - \frac{z^2}{2},$$

with $0 \leq z \leq 1$. This is to ensure that marginal utilities are bounded away from zero. **Notice that this function attains a maximum at $z = 1$, implying that the individual's utility function exhibits satiation at $x = 1$ or $y = 1$. We introduce below a restriction on the parameter values, namely Assumption 1, which guarantees that satiation does not occur.**

The consumer's income level in market i is m_i. Consumers in both markets are identical, except for their income levels, which is assumed to be higher in market 2. It is easy to anticipate that such disparity in income levels will allow for the possibility of price discrimination, and that the monopolist will seek to raise the price in market 2 –the strong market–, if allowed to price discriminate. **Additionally, we assume that both income levels are sufficiently high. These two facts are reflected in the following assumption:**

Assumption 1 $m_2 > m_1 > 1$.

As it will become clear below, the introduction of this assumption will greatly simplify the analysis, since it will be a sufficient condition for both individuals to consume strictly positive amounts of goods x and y, while not reaching satiation, both under a uniform price and under price discrimination. After proving the main result of our paper under Assumption 1, we will discuss the welfare implications for the cases when this assumption does not hold.

Finally, regarding the welfare of consumers, we assume that there is a utilitarian social welfare function (Mas-Colell et al., 1995), where the **the welfare of consumers is simply the sum of the utilities of the two representative consumers, that is,**

$$W(x_1,y_1,x_2,y_2) = \sum_{i=1}^{2}\left(x_i - \frac{x_i^2}{2} + y_i - \frac{y_i^2}{2}\right). \tag{1}$$

Since the marginal utility of income is not constant, we can not simply add firm profits to the level of utility of the consumers. For this reason, we will focus on consumer welfare as defined in (1) and we will study the effect of price discrimination on (1), conditional on price discrimination not reducing firm profits.

2.1 Optimal prices and quantities

On the supply side, we assume that good y is produced by a competitive industry and that it is sold at a price equal to its marginal cost, which is assumed to be one. In contrast, good x is produced at zero cost by a monopolist, who can sell it in both markets. We will analyze first the case of the monopolist being constrained to choosing a uniform price with that in which the monopolist is able to price discriminate. Call $p_{i,j}$ the price of good x in market $i = \{1,2\}$, under price regime $j = \{U,D\}$, where U is uniform price and D is price discrimination.

The representative consumer in market i maximizes the (identical) utility function given above, subject to the budget constraint $p_{i,j}x + y \leq m_i$. Therefore, the optimal consumption levels of x and y in market i and under price regime j are given by

$$x^*_{i,j}\left(p_{i,j}, 1, m_i\right) = \frac{p_{i,j}\, m_i + 1 - p_{i,j}}{1 + p^2_{i,j}}, \qquad y^*_{i,j}\left(p_{i,j}, 1, m_i\right) = \frac{m_i + p^2_{i,j} - p_{i,j}}{1 + p^2_{i,j}}. \tag{2}$$

In the case of price discrimination, the monopolist chooses the optimal price $p_{i,D}$ in each market so as to maximize profits, which, since the marginal cost is zero, are simply given by $x_{i,j} \cdot p_{i,j}$. From the first-order condition of the profit maximization problem, the optimal prices are given by the following expressions:

$$p_{1,D} = m_1 - 1 + \sqrt{(m_1 - 1)^2 + 1}, \qquad p_{2,D} = m_2 - 1 + \sqrt{(m_2 - 1)^2 + 1}. \tag{3}$$

In contrast, the demand function of a monopolist adopting a single uniform price is given by the sum of the demand in both markets. Accordingly, the profits of the monopolist are in this case

$$\Pi(p_U) = p_U \left(\frac{p_U\, m_1 - p_U + 1}{p^2_U + 1} + \frac{p_U\, m_2 - p_U + 1}{p^2_U + 1} \right) = p_U \frac{p_U (m_1 + m_2 - 2) + 2}{p^2_U + 1}. \tag{4}$$

To obtain the price of a non-discriminating monopolist, we equate to zero the derivative of (4) with respect to p_U. Solving for p_U yields

$$p_U = \frac{m_1 + m_2 - 2 + \sqrt{(m_1 + m_2 - 2)^2 + 4}}{2}. \tag{5}$$

From equations (3) and (5), we can see that the formula of the pricing policy that the monopolist uses is

$$p(m) = m - 1 + \sqrt{(m - 1)^2 + 1},$$

where m is the income level at each market, under price discrimination. Under a uniform price regime, m is average income.

In order to simplify the manipulation of equations, in expressions (3) and (5) we call $r = m_1 - 1$, and $s = m_2 - 1$. Rewriting the previous expressions, we obtain:

$$p_{1,D} = r + \sqrt{1+r^2}, \;\; p_{2,D} = s + \sqrt{1+s^2}, \;\text{ and } p_U = \frac{s+r+\sqrt{4+(r+s)^2}}{2}. \tag{6}$$

Given optimal uniform and discrimination prices, we now proceed to calculate the quantities exchanged in the market **under** both regimens. Given demands, optimal consumption levels in market 1 are:

$$x_{1,D} = \frac{1}{2}, \quad y_{1,D} = 1 - \frac{\sqrt{1+r^2}-r}{2}. \tag{7}$$

Similarly, optimal consumption levels in market 2 are:

$$x_{2,D} = \frac{1}{2}, \quad y_{2,D} = 1 - \frac{\sqrt{1+s^2}-s}{2}. \tag{8}$$

Since $y_{1,D}$ and $y_{2,D}$ are positive as long as m_1 and m_2 are greater than $\frac{1}{4}$, therefore they will be positive under Assumption 1. Furthermore, both $y_{1,D}$ and $y_{2,D}$ are always strictly less than one, hence satiation does not occur under price discrimination. Regarding quantities consumed with a uniform price, on the one hand, in market 1, where **consumer's income** is m_1, we have

$$x_{1,U} = \frac{1}{2} - \frac{(s-r)}{2\sqrt{4+(r+s)^2}}, \quad y_{1,U} = \frac{r+2}{2} - \frac{(2+rs+r^2)}{2\sqrt{4+(r+s)^2}}; \tag{9}$$

Finally, in the case of market 2, where consumer income is $m_2 > m_1$, quantities with a single uniform price are

$$x_{2,U} = \frac{1}{2} + \frac{(s-r)}{2\sqrt{4+(r+s)^2}}, \quad y_{2,U} = \frac{s+2}{2} - \frac{(2+s^2+rs)}{2\sqrt{4+(r+s)^2}}. \tag{10}$$

If Assumption 1 holds, all four quantities in (9) and (10) are both positive and less than one. Therefore, under Assumption 1, both consumers consume a strictly positive amount of both goods, x and y, and in no case the level of satiation is reached, that is, $0 < x_{i,j} < 1 \; \forall i, j$. In contrast, if Assumption 1 does not hold, we may find combinations of m_1 and m_2 such that $x_{1,U} = 0$, $y_{1,U} = 0$, $y_{2,U} = 0$, $y_{1,D} = 0$, and/or $y_{2,D} = 0$. While these are perfectly valid solutions,

the expressions for the welfare functions computed below would not be correct, and the proof of the main result of the paper would have to be carried out considering a large number of different cases.

Recall that our objective is to show that there are parameter values such that consumers are better off with price discrimination, even in the absence of an output increase. We proceed to show that, **insofar as Assumption 1 is fulfilled,** total consumption of good x remains constant when the producer of good x is allowed to price discriminate. In fact, from (7) and (8), it can be easily seen that **total consumption of good x under price discrimination equals one.** Similarly, from (9) and (10), we see that $x_{1,U}+x_{2,U}=1$. Therefore, under either price regime, the per capita consumption of good x is one **if Assumption 1 holds**.

It is then clear that, in our model, total output remains the same regardless of whether the producer of good x may engage in price discrimination. That is, either if a single uniform price is applied, or when adopting price discrimination, the total amount of x sold by the monopolist is $x=1$. Of course, the monopolist will not be worse off under price discrimination than under a uniform price, since under price discrimination the monopolist can always choose the same price in both markets and producers of the other good do not react strategically (Galera et al., 2017a) to the choice of price by the producer of good x. Then, provided that the monopolist is not worse off with price discrimination, we need to examine whether consumers are made worse off or, on the contrary, there are some configuration of parameter values such that they are better off if the monopolist price discriminates.

2.2 Welfare

We now proceed to compute the welfare of consumers. With price discrimination, **and making use of the expressions for quantities (7) and (8), which are valid under Assumption 1,** we find that

$$W_D=\frac{s\sqrt{1+s^2}+r\sqrt{1+r^2}+6-s^2-r^2}{4}. \tag{11}$$

Whereas the in the case of a uniform price, **under Assumption 1,**

$$W_U = \left[\frac{6 - s^2 - r^2}{4} + \frac{(s+r)\left(2+s^2+r^2\right)\sqrt{4+(r+s)^2}}{16+4(r+s)^2} \right]. \tag{12}$$

By computing the difference between the two welfare levels, and after some calculations, we find that

$$\Delta W = W_D - W_U = \frac{\sqrt{4+(s+r)^2}(s\sqrt{1+s^2}+r\sqrt{1+r^2}) - (s+r)(2+s^2+r^2)}{4\sqrt{4+(s+r)^2}}. \tag{13}$$

We now proceed to present the main proposition of this paper:

Proposition 1 *Under Assumption 1, the welfare of consumers is greater with price discrimination than with a single uniform price, while total consumption of good x is constant.*

Proof. Recall that if Assumption 1 holds, total consumption of good x equals one both with a uniform price and with price discrimination. On the other hand, remember that $r = m_1 - 1$ and $s = m_2 - 1$. Let us study the numerator of equation (13) as a function:

$$F(r,s) = \sqrt{4+(s+r)^2}\left(s\sqrt{1+s^2}+r\sqrt{1+r^2}\right) - (s+r)(2+s^2+r^2). \tag{14}$$

We then use the Cauchy inequality, which establishes that **for vectors** (a_1,a_2) **and** (b_1,b_2)

$$\left(\sum_{k=1}^{2} a_k b_k\right)^2 \leq \left(\sum_{k=1}^{2} a_k^2\right)\left(\sum_{k=1}^{2} b_k^2\right),$$

both sides being equal if and only if vectors (a_1,a_2) **and** (b_1,b_2) **are linearly dependent. Taking** the square root, and defining $a_1 = 2$, $a_2 = s+r$, $b_1 = s$ and $b_2 = s^2$, or also $b_1' = r$ and $b_2' = r^2$. For $s > r > 0$, we conclude that

$$2s + s^2(s+r) \leq \sqrt{4+(r+s)^2}\sqrt{s^2+s^4} \tag{15}$$

$$2r + r^2(s+r) \leq \sqrt{4+(r+s)^2}\sqrt{r^2+r^4}. \tag{16}$$

The sum of the two inequalities leads to

$$\sqrt{4+(s+r)^2}\sqrt{s^2+s^4}+\sqrt{4+(s+r)^2}\sqrt{r^2+r^4}\geq (r+s)(2+r^2+s^2).$$

This result means that $F(r,s)>0$, for any $s>r>0$. And, in this case, expression (13) is positive.

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If Assumption 1 does not hold, we need to consider a number of different cases. For instance, in the case $0>s>r$, if quantities consumed of x and y are strictly positive for both consumers under both regimes, then expression (13) is still a valid measure of welfare. For instance, this is the case if $m_1=0.3$ and $m_2=0.5$, that is, if $r=-0.7$ and $s=-0.5$. Then, since $F(r,s)=-F(-r,-s)$, the welfare of consumers decreases with price discrimination. However, while the welfare of consumers decrease, we should keep in mind the fact that the firm's profits would increase at the same time.

The case $r<0<s$ is more complicated, even if it seems to follow the same patterns as those we have just proved. In fact, it seems to be always the case that for meaningful solutions $r+s>0\Leftrightarrow F(r,s)>0$, and $r+s<0\Leftrightarrow F(r,s)<0$; but finding a general proof of such a result is complicated and **beyond the scope of the present analysis**.

3 Discussion

In our model, we found that there is scope for price discrimination to be welfare-increasing even in the absence of an output increase. According to the analysis in Varian (1985), the use of price discrimination implies transferring a certain amount of the good from the strong to the weak market. Of course, according to the willingness to pay, the consumer surplus associated to the transferred good is greater in the country with high income than in the country where income is low. For this reason, if the overall amount sold by a monopoly is the same regardless of the chosen policy –uniform price and price discrimination–, welfare decreases with price discrimination. However, the greater willingness to pay may be due to having a higher income, rather than to the larger utility of consumption.

Indeed, it is well-known that price discrimination makes output to increase or decrease depending on the shape of the demand functions (Aguirre et al., 2010; Cowan, 2018). For the sake of simplicity, and to make our point more clear, we have chosen **a utility function** such that the total consumption of good x does not vary with the pricing regime. **In fact, this result arises if we consider that the utility function is $U(x,y) = u(x) + \alpha u(y)$ for $\alpha > 0$ and $\alpha \neq 1$, that is, if both goods enter the consumer's utility function with different weights.**[2] Under price discrimination, the consumption levels of the agents (in the two markets) will surely be more balanced than under a uniform price. This is due to the fact that the monopolist reduces the price of x in market 1 while increasing it in market 2. Actually, when the monopoly chooses to price discriminate, there is a two-sided effect. On the one hand, the consumer's satisfaction increases, given the more balanced consumption levels we observe now (as compared to those of the uniform price). On the other hand, the payment made for consuming the initial amount of x is now greater, and the individual will necessarily reduce the amount of the other good, y, that the consumer can afford. Of course, the final net effect on welfare depends on the relative impact of these two factors.

What is the rationality behind our results? At a first glance, the intuition points to the difference in income as the crucial feature to determine when price discrimination should be preferred. Instead, our analysis suggests that the issue depends on whether or not the sum of income levels is greater than a certain threshold. To understand this finding, remember the two basic elements involved here: (a) price discrimination brings about greater balance in the consumption levels in both markets (which is positive in terms of welfare); (b) but at the same time, price discrimination makes the consumer to allocate a higher income share to the consumption of good x, which naturally reduces the income available for buying y.

Therefore, one of the implications of our model is that banning third-degree price discrimination may indeed harm consumers. Actually, our analysis indicates that price discrimination may bring about welfare gains, thereby allowing for the implementation of economically efficient programs. For instance, some initiatives have been carried out to implement policies to give access, at a

[2] If $\alpha \neq 1$, the result in Proposition 1 would still hold under Assumption 1. A proof of this result may be obtained from the authors upon request.

reduced cost, to new technologies or to education, such as the as granting low-cost access to mobile phone technology in African countries or as the "One Laptop Per Child (OLPC) program". The latter relied to the provision by Microsoft of Windows XP for $3 per computer, a clear case of third-degree price discrimination. Of course, the public sector can always grant subsidies to enable low-income individuals getting access to cultural or medical goods, but this may no longer be necessary if price discrimination is allowed.

At the very least, we claim that –by imposing restraints in price discrimination– policymakers may sometimes operate against their intended aim of enhancing social welfare. Moreover, we advocate here that price discrimination may make consumers better off in the aggregate. If this is the case, and given that discrimination always yields larger profits to the monopolist, banning price discrimination could be mistaken in order to enhance social welfare. Moreover, we question the accuracy of consumer surplus as a measure os consumer welfare. The conventional way for aggregating satisfaction levels of the individuals consists of using quasilinear utility functions. While this procedure has clear analytical advantages, arising from the assumption of a constant utility of income, consumer surplus, even if unintentionally, bias the calculations in favor of individuals who are endowed with the largest income levels. Dropping the assumption of quasilinear preferences, our analysis has revealed the critical influence of income on demand. A least, one must be aware of the misleading conclusions that are obtained from using quasilinear utility functions. Thus, there is a risk of reaching the wrong policy conclusions if taking the result in Varian (1985) at face value.

We finish this section by discussing an alternative interpretation of our model, one in which there is no aggregation of utility across consumers. In particular, in our theoretical model, we have considered two consumers with identical preferences, but different income levels. Given our definition of social welfare function, which follows a utilitarian approach and is thus simply the addition of utility levels, the same results arise if we assume instead that there are two identical consumers, with the same preferences on goods x and y. Each consumer lives half a year in market 1, and the other half in market 2, where income levels are m_1 and m_2, respectively. Assuming that income cannot be transferred across markets, the individual is constrained to spending his income

m_i in each half of the year. Assuming no time discounting, the utility of each individual is

$$U(x_1, y_1, x_2, y_2) = u(x_1) + v(y_1) + u(x_2) + v(y_2),$$

which is identical to the utilitarian social welfare function (1). Furthermore, we assume that whenever one of the consumers is in market 1, the other consumer is in market 2. This way, the monopolist always faces the same demand in each market. Notice that this assumption is consistent with a market that operates over the whole year, which permits the monopoly to sell its product simultaneously in the two markets and, hence, to price discriminate (otherwise, the monopolist will simply seek to maximize its profits in the one market that is working at each six-month period).

Within this alternative framework, we can then study whether, relative to a uniform price being set, the implementation of third-degree price discrimination makes each individual consumer better off or worse off. The advantage of this alternative approach is that it does not require the aggregation of utilities across consumers, since both consumers are present in both markets, 1 and 2, although at different times. It is easy to see that the problem that the monopolist faces both under a uniform price and under price discrimination is identical to that discussed in Section 2. Therefore, optimal prices in these two regimes are also given by (3) and (5). Hence, total output is not affected by price discrimination and, from Proposition 1, if $1 \leq m_1 \leq m_2$, both consumers are better off, even if consumption of x does not increase with price discrimination.

4 Conclusions

The main contribution of this paper is to highlight the crucial role of assuming that consumers have quasilinear preferences when evaluating the impact of a given policy on the welfare of society. In particular, we focus on third-degree price discrimination. We assume that consumers in two different markets, with income levels being different across markets, have preferences on two goods, x and y, that are not quasilinear, hence potentially giving rise to differences in the marginal utility of money, arising from differences in income levels across consumers. We show that there are values of the two income levels such that total consumption of good x remains constant and yet, consumers are better off with price discrimination if we consider a utilitarian welfare function.

Therefore, we find cases where price discrimination brings about greater social welfare, **–in contrast with standard analyses of the welfare effects of price discrimination, which assume quasilinear utility functions and implicitly rule out this effect–** thus questioning the generality of the result in Varian (1985). Then the recommendation that price discrimination must be constrained, or even banned, may be not appropriate, at least whenever consumption depends on income. Even if assuming quasilinear utilities may be convenient for developing sound theoretical arguments, it also makes the analysis less general and realistic. In other words, the conclusions reached if using quasilinear utilities should be carefully taken as a nothing but guidance for the regulation of price discrimination.

Despite its simplicity, our analysis yields interesting results. First, it reveals that third-degree price discrimination may cause gains for the monopolist as well as for consumers. Second, our study is all the more relevant regarding the provision of primary goods, such as health or education services, to low-income populations. Consider, for instance, how relevant it might be for people to afford the medicine or vaccination they need to overcome a particular illness. Third and more importantly, the scope of our analysis reaches beyond the specific utility functions we have used here. In fact, the specification of our utility function was chosen for being convenient to stress our major point, but the implications of our analysis are not constrained to this particular specification. Finally, even though our main conclusion needs no additional analysis, we consider that the study of utility functions other than the quasilinear specification suggests an interesting research line. Further research could also be made to explore the cases where the level of output decreases with price discrimination, unlike in our analysis, where it remains constant.

Notice also that this paper ultimately resumes the debate on the validity of the consumer surplus, as the conventional way to evaluate social welfare. In line with this concern, the conclusion emerges other complementary approaches, to the consumer surplus, could be needed for evaluating social welfare. We do not claim that the consumer surplus is a wrong welfare measure, but that it may sometimes be a biased measure of consumer satisfaction. Then, according to our analysis, policymakers must try to use improved criteria for judging policies in terms of welfare. Further research must anyway be encouraged, in order to generalize our findings or to learn how far-reaching are the implications of adopting our approach.

References

Adachi, T. (2005). Third-degree price discrimination, consumption externalities and social welfare. *Economica 72*(285), 171 – 178.

Aguirre, I., S. Cowan, and J. Vickers (2010). Monopoly price discrimination and demand curvature. *American Economic Review 100*(4), 1601 – 1615.

Cowan, S. (2018). Regulating monopoly price discrimination. *Journal of Regulatory Economics 54*(1), 1–13.

Foster, C. D. and H. L. I. Neuburger (1974). The ambiguity of the consumer's surplus measure of welfare change. *Oxford Economic Papers 26*(1), 66 – 77.

Galera, F., M. Kinateder, and P. Mendi (2014). The cost effect in third-degree price discrimination. *Journal of Economics 113*(2), 175–186.

Galera, F., P. Mendi, and J. C. Molero (2017a). Quality Differences, Third-Degree Price Discrimination, and Welfare. *Economic Inquiry 55*(1), 339–351.

Galera, F., P. Mendi, and J. C. Molero (2017b). Seasonality, price discrimination, and welfare. *Journal of Economics 120*(3), 269–277.

Galera, F. and J. M. Zaratiegui (2006). Welfare and output in third-degree price discrimination: A note. *International Journal of Industrial Organization 24*(3), 605 – 611.

Mankiw, N. (2014). *Principles of Microeconomics*. Cengage Learning.

Marshall, A. (1890). *Principles of Economics*. Number v. 1 in Principles of Economics. Macmillan and Company.

Mas-Colell, A., M. Whinston, and J. Green (1995). *Microeconomic Theory*. Oxford University Press.

OECD (2016). Price Discrimination. Background note by the Secretariat. Technical report, Competition Committee, Directorate for Financial and Enterprise Affairs. https://one.oecd.org/document/DAF/COMP(2016)15/en/pdf.

Pindyck, R. and D. Rubinfeld (2012). *Microeconomics*. Pearson Education.

Schmalensee, R. (1981). Output and welfare implications of monopolistic third-degree price discrimination. *American Economic Review 71*(1), 242 – 247.

Varian, H. R. (1985). Price discrimination and social welfare. *American Economic Review 75*(4), 870 – 875.

Willig, R. D. (1976). Consumer's surplus without apology. *American Economic Review 66*(4), 589 – 597.

Willig, R. D. (1978). Incremental consumer's surplus and hedonic price adjustment. *Journal of Economic Theory 17*(2), 227 – 253.

Wright, P. G. (1917). Total utility and consumers' surplus under varying conditions of the distribution of income. *Quarterly Journal of Economics 31*, 307 – 318.