Armstrong meets Rochet-Tirole: On the equivalence of different pricing structures in two-sided markets

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January 23, 2019

Abstract

We formally show that lump-sum participation fees and per-interaction fees charged by a monopoly platform in a two-sided market are not interchangeable in the presence of price distortions, such as ad valorem taxes.

JEL Classification Numbers: D4, L11, L12, H20

Keywords: Two-sided market, Platform pricing, Pricing structure, Taxation, Price distortions.

1 Introduction

In a seminal paper, Armstrong (2006) (A2006 hereafter) states that in two-sided markets with a monopoly platform, "it makes no difference if tariffs are levied on a lump-sum or per-transaction basis" [*RAND Journal of Economics* 37(3), p. 669]. According to him it is equivalent to use a

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monopolistic-platform two-sided market model in which tariffs are levied as a lump-sum fee ("pure subscription tariff") and one in which a platform's end-users pay a per-transaction fee ("pure transaction fee"). Based on this, A2006 develops his model assuming that the platform imposes just lump-sum membership fees on end-users. Contrarily, Rochet and Tirole (2003) (RT2003 hereafter) assume only usage (per-transaction) fees on end-users but not membership charges.[1]

Subsequently, the literature on two-sided markets cited these authors' arguments to choose either a pure membership fee scheme (à la A2006) or a pure usage fee scheme (à la RT2003). For instance, Bolt and Tieman (2008), Weyl (2009), Aloui and Jebsi (2010) and Creti and Verdier (2014) assume a monopoly platform that imposes only usage fees, while Zheng and Kaiser (2013), Hagiu and Hałaburda (2014), Peitz et al. (2017) and Gao (2018) assume a monopoly platform that imposes only membership fees on end-users.

In this paper, we show that the equivalence between a monopolistic-platform two-sided market model in which tariffs on end-users are levied as a lump-sum fee, as a per-transaction fee, and as a combination of both types of fees does not hold in the presence of any kind of price distortions. To show this, we consider the particular case of the distortion created by (ad valorem) taxes levied on the platform or on its end-users. The distortions created by taxes are important because its introduction especially in digital platform markets is a relevant and controversial issue in tax policy. This is reflected in the incipient literature on the taxation of two-sided markets.[2]

In the growing literature on taxation of platform-based two-sided markets frequently the authors exclusively choose one type of tariff or another. For instance, Belleflamme and Toulemonde (2018) consider taxes when the platform charges just access fees, while Kind et al. (2008) and Kind et al. (2009) consider taxes when the platform imposes per usage fees only. However, in the presence of price distortions such as ad valorem taxes, we show that the choice between access fees and usage fees (or a combination of them) is no longer innocuous.

The plan of the paper is as follows. Section 2 introduces a two-sided market environment. Section 3 provides the main results. Section 4 concludes. All the proofs are in the online appendix.

[1] In Section 6, RT2003 derive some results also for lump-sum membership fees.

[2] For a general discussion on the effect of taxes on platform markets, see Tremblay (2018).

2 The theoretical model

We consider a two-sided market with a monopoly platform in the same spirit as in A2006 and RT2003. The platform connects end-users of two types (A and B) who both must be on the platform to interact. End-users obtain an idiosyncratic surplus of joining side $j \in \{A, B\}$, denoted by $\theta_j > 0$. This surplus is frequently called (gross) membership benefit since it is independent of the number of transactions carried out. Next, we assume that there are cross-network effects and introduce a parameter that captures the benefit of an interaction between end-users on different sides. Denote a representative side-j end-user's cross-network effect by $\alpha_j > 0$ for $j \in \{A, B\}$, which is multiplied by the number of participants on the other side of the market, that is, $\alpha_j N_{\text{-}j}$ ($j \neq \text{-}j$). Following the literature (RT2003 and A2006), we do not introduce same-side network effects.[3]

The platform incurs a marginal cost $c \geq 0$ per interaction between two end-users, and a fixed cost $C_j \geq 0$ per side-j member. To compensate these costs, we consider two types of fees that the platform imposes on end-users. First, a lump-sum (fixed) participation fee: when joining side $j \in \{A, B\}$, an end-user pays p_j. Second, a usage (or per-transaction) fee: when interacting with a member on side -j, a side-j end-user pays γ_j. Let us denote a pricing structure or tariff as a quadruple, $\boldsymbol{v} \equiv (\gamma_A, \gamma_B, p_A, p_B)$.

Given all the parameters, a representative end-user's net utility of joining side j is defined as

$$u_j \equiv \theta_j + (\alpha_j - \gamma_j)N_{\text{-}j} - p_j. \tag{1}$$

Note that 1 is similar to equation (1) in Rochet and Tirole (2006) (RT2006 hereafter). Following them, we define the total number of side-j end-users who decide to join the platform, N_j, as

$$N_j = pr(u_j \geq 0), \tag{2}$$

that is, the total number of side-j end-users in the platform is given by the probability that side-j end-users' utility from joining the platform is greater than 0 (outside option).

As RT2006 argue, under usual regularity conditions, the system of two equations embedded in

[3] However, our results easily extend to the case with same-side network effects.

2 has a unique solution characterizing N_j as function of the fees (γ_j, p_j) and $N_{\text{-}j}$, for $j \in \{A, B\}$.[4]

Given the number of agents on each side, the monopoly platform's profit is characterized by

$$\pi(\boldsymbol{v}; \cdot) = (\gamma_A + \gamma_B - c)N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B. \tag{3}$$

RT2003 and A2006 provide the profit-maximizing solution for the monopoly platform absent membership and usage fees, respectively. The characterization of the profit-maximizing solution when both types of fees are present is provided by RT2006—see equation (8) on page 645. However, in our particular case, we shall not focus only on the profit-maximizing solution: the results provided in the next section are shown for any arbitrary fees.

3 Main result and discussion

Next, we consider the following three different tariff schemes: *(i)* $\boldsymbol{v}^1 \equiv (\gamma'_A, \gamma'_B, 0, 0)$, *(ii)* $\boldsymbol{v}^2 \equiv (0, 0, p'_A, p'_B)$, and *(iii)* $\boldsymbol{v}^3 \equiv (\gamma_A, \gamma_B, p_A, p_B)$, where $\gamma'_j > \gamma_j > 0$ and $p'_j > p_j > 0$, for $j \in \{A, B\}$. The latter restrictions ensure that no tariff is (ex-ante) unambiguously more/less profitable for end-users and/or for the platform than another.[5] Case *(i)* corresponds to pure transaction tariffs as in RT2003, case *(ii)* to pure subscription tariffs as in A2006, and case *(iii)* is a combination of both of them.

We define two tariffs as equivalent if they yield *(a)* identical utility for a representative end-user on both sides and *(b)* an identical profit for the platform.

Definition 1. *Given a representative side-j end-user's idiosyncratic surplus θ_j and the cross-network effect α_j, assume that tariff $\boldsymbol{v}$ yields end-users utilities u^*_j, $j \in \{A, B\}$, and platform profit $\pi^*(\boldsymbol{v}; \cdot)$. We say that tariff $\boldsymbol{v}'$ is equivalent to tariff $\boldsymbol{v}$ if and only if $\boldsymbol{v}'$ yields identical end-users' utilities u^*_j and an identical platform profit $\pi^*(\boldsymbol{v}'; \cdot)$.*

We show that, in the presence of price distortions, tariffs $\boldsymbol{v}^1$, $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ cannot be equivalent.

[4] Following RT2006, p. 653, equation (4), N_j is defined as side-j's demand which depends on γ_j, p_j and $N_{\text{-}j}$, which itself is a function of $\gamma_{\text{-}j}$, $p_{\text{-}j}$ and N_j in turn. In the unique solution, we suppress this dependence and for ease of notation, write N_j and $N_{\text{-}j}$, respectively, since both are determined simultaneously and endogenously as functions of the parameters $(\gamma_j, \gamma_{\text{-}j}, p_j, p_{\text{-}j})$.

[5] For instance, if $\gamma'_j < \gamma_j$ for $j \in \{A, B\}$, then $\boldsymbol{v}^1$ offers both a cheaper usage fee and a cheaper membership fee in comparison to $\boldsymbol{v}^3$. If so, $\boldsymbol{v}^1$ is surely more profitable for end-users and less profitable for the platform than $\boldsymbol{v}^3$.

3.1 Price distortions: the case of ad valorem taxes

Taxes in two-sided markets are extremely popular and controversial both in the media and in economics literature. As formally analyzed by Bacache-Beauvallet and Bloch (2018), a large number platforms (especially, the digital ones) rely on peer-to-peer transactions that are often difficult (if not impossible) to tax. These firms explored new pricing schemes to reduce the fiscal burden of taxes on access to the platform and the transactions conducted on the platform. Not surprisingly, some states and countries implemented alternative (often controversial) tax instruments. For instance, some US States introduced taxes on streaming and digital entertainment (the so-called "Netflix tax"). Similarly, France created a "YouTube tax", levied on all streaming videos. At the municipal level, the city of Barcelona recently imposed a tax on "peer-to-peer" rental companies like AirBnb.

We show that, in the presence of price distortions, such as ad valorem taxes, the choice of the pricing structure is no longer innocuous. Therefore, using an alternative fee structure would affect the results of previous authors that considered either one type of fees or the other in the presence of taxes (or some other kind of price distortions) in two-sided markets.[6]

3.1.1 Taxes levied on the platform

First, we assume that an ad valorem tax is levied on the platform and consider two different cases. The government could introduce a sales tax or VAT on a per-transaction basis. That is, the platform pays a tax on the fee that it receives each time that two end-users of different sides interact. Examples are a tax on the real estate agent fee charged when there is a successful transaction between a seller and a buyer, a tax on the recruiting agency fee charged when there is a successful match between an employer and a job seeker, or a tax on the Uber fee taken for a ride booked on this platform.

Let t^p denote the usage tax imposed on the platform. Then, the platform's profit is[7]

$$\pi(\boldsymbol{v}; t^p, \cdot) = [(1 - t^p)(\gamma_A + \gamma_B) - c]\, N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B. \tag{4}$$

[6]See, for instance, Belleflamme and Toulemonde (2018), Kind et al. (2008) and Kind et al. (2009).

[7]An alternative characterization of an ad valorem access tax levied on the platform is

$$\pi(\boldsymbol{v}; t^p, \cdot) = \left(\frac{\gamma_A + \gamma_B}{1 + t^p} - c\right) N_A N_B + (p_A - C_A)N_A + (p_B - C_B),$$

as used by Kind et al. (2008). For this alternative characterization of the tax on the platform, the result is identical.

The government could also introduce a tax on access.[8] Under this alternative tax, the platform pays a tax each time a member joins it. For example, if we regard a dating club as a platform, a tax per subscription sold fits into this category, or a tax per console sold if we regard the video games market as two-sided.

Let τ_j^p denote the access tax imposed on the platform for an additional type-j membership, $j \in \{A, B\}$, and define $\boldsymbol{\tau^p} \equiv (\tau_A^p, \tau_B^p)$. Then, platform's profit is[9]

$$\pi(\boldsymbol{v}; \boldsymbol{\tau^p}, \cdot) = (\gamma_A + \gamma_B - c)N_A N_B + \left[(1 - \tau_A^p)p_A - C_A\right] N_A + \left[(1 - \tau_B^p)p_B - C_B\right] N_B. \tag{5}$$

In Lemma 1 we show that, in the presence of a tax on the platform, if end-users are indifferent between the three aforementioned tariffs, then one tariff yields the platform strictly greater profits. Thus, the platform finds it beneficial to switch from one type of tariff to another.

Lemma 1. *Assume* (i) $\boldsymbol{v}^1 \equiv (\gamma'_A, \gamma'_B, 0, 0)$, (ii) $\boldsymbol{v}^2 \equiv (0, 0, p'_A, p'_B)$, *and* (iii) $\boldsymbol{v}^3 \equiv (\gamma_A, \gamma_B, p_A, p_B)$, *where* $\gamma'_j > \gamma_j > 0$ *and* $p'_j > p_j > 0$ *for* $j \in \{A, B\}$. *Then,*

a) In the presence of an ad valorem per-transaction tax t^p levied on the platform, if end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ then $\pi^*(\boldsymbol{v}^3; t^p, \cdot) > \pi^*(\boldsymbol{v}^1; t^p, \cdot)$, and if end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ then $\pi^*(\boldsymbol{v}^2; t^p, \cdot) > \pi^*(\boldsymbol{v}^3; t^p, \cdot)$.

b) In the presence of ad valorem access taxes τ_j^p levied on the platform, if end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ then $\pi^*(\boldsymbol{v}^1; \boldsymbol{\tau^p}, \cdot) > \pi^*(\boldsymbol{v}^3; \boldsymbol{\tau^p}, \cdot)$, and if end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ then $\pi^*(\boldsymbol{v}^3; \boldsymbol{\tau^p}, \cdot) > \pi^*(\boldsymbol{v}^2; \boldsymbol{\tau^p}, \cdot)$.

For instance, suppose that the government imposes a per-transaction tax on an e-commerce platform such as Amazon. A possible strategy for Amazon is to introduce a subscription fee, such as Amazon prime, and let the user access additional benefits at no additional cost, such as same-day delivery as well as unlimited access to eBooks. In this case, the government does not collect a tax

[8] Notice that for some markets, the government's ability to tax interactions may be hindered due to the anonymity of transactions.

[9] A similar setup is in the model by Belleflamme and Toulemonde (2018). An alternative characterization of an ad valorem access tax levied on the platform is:

$$\pi(\boldsymbol{v}; \boldsymbol{\tau^p}, \cdot) = (\gamma_A + \gamma_B - c)N_A N_B + \left(\frac{p_A}{1 + \tau_A^p} - C_A\right) N_A + \left(\frac{p_B}{1 + \tau_B^p} - C_B\right) N_B,$$

for which our main result also holds.

for the premium delivery service, nor does it if an Amazon prime member downloads an e-book.[10]

Therefore, in the presence of a tax on the platform, tariffs cannot be equivalent.

Proposition 1. *Given $\boldsymbol{v}^1$, $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ as defined in Lemma 1, in the presence of an ad valorem usage tax or ad valorem access tax levied on the platform, $\boldsymbol{v}^1$,$\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ are never equivalent.*

3.1.2 Taxes levied on the end-users

Next, we assume that the tax is levied on end-users. Again, we consider two different cases. In the first one, end-users on side j pay a tax each time they interact with an end-user on side -j. An example is a sales tax paid by consumers who purchase from Amazon.

Let t_j^e denote the per-transaction tax levied on type j end-users, and define $\boldsymbol{t^e} \equiv (t_A^e, t_B^e)$. In this case, type-$j$ end-user's utility is[11]

$$u_j \equiv \theta_j + [\alpha_j - \gamma_j(1 + t_j^e)]N_i - p_j. \tag{6}$$

In the second case, we assume that there is a tax on access levied on end-users. An example is a tax paid on the subscription fee when joining a dating club.

Let τ_j^e denote the access tax imposed on type-j end-users, $j \in \{A, B\}$, and define $\boldsymbol{\tau^e} \equiv (\tau_A^e, \tau_B^e)$. In this case, type-$j$ end-user's utility is[12]

$$u_j \equiv \theta_j + (\alpha_j - \gamma_j)N_i - p_j(1 + \tau_j^e). \tag{7}$$

Lemma 2. *Assume $\boldsymbol{v}^1$, $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ as in Lemma 1.*

a) In the presence of ad valorem per-transaction taxes t_j^e levied on end-users, if end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ then $\pi^*(\boldsymbol{v}^3; \boldsymbol{t^e}, \cdot) > \pi^*(\boldsymbol{v}^1; \boldsymbol{t^e}, \cdot)$, and if end-users are indifferent

[10] This result is also consistent with the example provided by Snider (2017) on entertainment (movie) platforms.

[11] An alternative characterization of an ad valorem access tax is

$$u_j \equiv \theta_j + \left(\alpha_j - \frac{\gamma_j}{1 + t_j^e}\right) N_i - p_j,$$

for which our main result also holds.

[12] An alternative characterization of an ad valorem access tax is

$$u_j \equiv \theta_j + (\alpha_j - \gamma_j)N_i - \frac{p_j}{1 - \tau_j^e},$$

for which the main results are similar.

between $\boldsymbol{v}^2$ *and* $\boldsymbol{v}^3$ *then* $\pi^*(\boldsymbol{v}^2;\boldsymbol{t}^{\boldsymbol{e}},\cdot) > \pi^*(\boldsymbol{v}^3;\boldsymbol{t}^{\boldsymbol{e}},\cdot)$.

b) In the presence of an ad valorem access tax τ_j^e *levied on end-users, if end-users are indifferent between* $\boldsymbol{v}^1$ *and* $\boldsymbol{v}^3$ *then* $\pi^*(\boldsymbol{v}^1;\boldsymbol{\tau}^{\boldsymbol{e}},\cdot) > \pi^*(\boldsymbol{v}^3;\boldsymbol{\tau}^{\boldsymbol{e}},\cdot)$, *and if end-users are indifferent between* $\boldsymbol{v}^2$ *and* $\boldsymbol{v}^3$ *then* $\pi^*(\boldsymbol{v}^3;\boldsymbol{\tau}^{\boldsymbol{e}},\cdot) > \pi^*(\boldsymbol{v}^2;\boldsymbol{\tau}^{\boldsymbol{e}},\cdot)$.

In Lemma 2, we show that if the three tariffs considered yield the same utility to consumers, then it is strictly profitable for the platform to choose one of them. As a consequence, the three tariffs cannot be equivalent.

Proposition 2. *Given* $\boldsymbol{v}^1$, $\boldsymbol{v}^2$ *and* $\boldsymbol{v}^3$ *as defined in Lemma 1, in the presence of ad valorem usage taxes or ad valorem access taxes levied on end-users,* $\boldsymbol{v}^1$,$\boldsymbol{v}^2$ *and* $\boldsymbol{v}^3$ *are never equivalent.*

4 Conclusion

The previous literature on two-sided markets considered that the platform imposes either a pure membership tariff or a pure usage tariff. Previous authors explicitly relied on the equivalence between both—as stated by A2006—to restrict their analysis to only one type of tariff. However, we show that this equivalence does not hold in the presence of market distortions, such as taxes.

Acknowledgements

We wish to acknowledge helpful comments from Florencio López-de-Silanes, an anonymous referee, an editor of the journal and seminar participants in Pamplona. Raúl Bajo-Buenestado gratefully acknowledges financial support from Fundación Ramón Areces (grant number CISP16A4779). Markus Kinateder gratefully acknowledges financial support from the Spanish Ministry of Economics (grant number ECO2017-85503-R).

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Online Appendix: Proofs

For the sake of simplicity we suppress the dependence of the profit π on the access or usage tax.

First, in the following Lemma we identifiy conditions on the fees under which the platform's profit π and an agent's utility on side j for $j \in \{A, B\}$, are identical given the tariffs $\boldsymbol{v}^1$, $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$. This result is used to prove Lemma 1 later on.

Lemma A1. *Assume $\boldsymbol{v}^1$, $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ as in Lemma 1. Then,*

a) $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ are equivalent $\Leftrightarrow p_j = (\gamma'_j - \gamma_j)N_i$, and

b) $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ are equivalent $\Leftrightarrow \gamma_j = \frac{p'_j - p_j}{N_i}$.[*]

Proof. First we show a): $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ are equivalent $\Leftrightarrow p_j = (\gamma'_j - \gamma_j)N_i$.

$\Rightarrow$: if $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ are equivalent, then, given θ_j and α_j, in equilibrium end-users utilities must be the same, that is,

$$\theta_j + (\alpha_j - \gamma'_j)N_i = \theta_j + (\alpha_j - \gamma_j)N_i - p_j, \tag{A.1}$$

and platform's profit must be the same, that is, $\pi(\boldsymbol{v}^1; \cdot) = \pi(\boldsymbol{v}^3; \cdot)$. Solving A.1 for p_j we arrive at

$$p_j = (\gamma'_j - \gamma_j)N_i, \tag{A.2}$$

which is the unique solution for which A.1 is satisfied. Then, we just need to verify that $\pi(\boldsymbol{v}^1; \cdot) = \pi(\boldsymbol{v}^3; \cdot)$ if A.2 is satisfied:

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma_A + \gamma_B - c)N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B. \tag{A.3}$$

Substituting A.2 in A.3, we arrive at

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma_A + \gamma_B - c)N_A N_B + [(\gamma'_A - \gamma_A)N_B - C_A]N_A + [(\gamma'_B - \gamma_B)N_A - C_B]N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = (-c)N_A N_B + [\gamma'_A N_B - C_A]N_A + [\gamma'_B N_A - C_B]N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma'_A + \gamma'_B - c)N_A N_B - C_A N_A - C_B N_B.$$

We know that:

$$\pi(\boldsymbol{v}^1; \cdot) = (\gamma'_A + \gamma'_B - c)N_A N_B - C_A N_A - C_B N_B.$$

That is, $\pi(\boldsymbol{v}^1; \cdot) = \pi(\boldsymbol{v}^3; \cdot)$, as required.

$\Leftarrow$: assume that $p_j = (\gamma'_j - \gamma_j)N_i$ holds. Then we need to show that, if so, both tariffs are equivalent. Under $\boldsymbol{v}^3$, agent j's utility is:

$$u_j = \theta_j + (\alpha_j - \gamma_j)N_i - p_j. \tag{A.4}$$

[*]The third possible relationship, that is, the equivalence between $\boldsymbol{v}^1$ and $\boldsymbol{v}^2$ can be trivially obtained from the two others.

Substituting $p_j = (\gamma_j' - \gamma_j)N_i$ into A.4:

$$u_j = \theta_j + (\alpha_j - \gamma_j)N_i - (\gamma_j' - \gamma_j)N_i,$$

$$u_j = \theta_j + (\alpha_j - \gamma_j')N_i, \tag{A.5}$$

which is agent j's utility under $\boldsymbol{v}^1$.

We have already verified that if $p_j = (\gamma_j' - \gamma_j)N_i$, then $\pi(\boldsymbol{v}^1;\cdot) = \pi(\boldsymbol{v}^3;\cdot)$ (see above).

Second we show b): $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ are equivalent $\Leftrightarrow \gamma_j = \frac{p_j' - p_j}{N_i}$.

$\Rightarrow$: if $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ are equivalent, then, given θ_j and α_j, in equilibrium end-users utilities must be the same

$$\theta_j + \alpha_j N_i - p_j' = \theta_j + (\alpha_j - \gamma_j)N_i - p_j, \tag{A.6}$$

and platform's profit must be the same, that is, $\pi(\boldsymbol{v}^2;\cdot) = \pi(\boldsymbol{v}^3;\cdot)$. Solving A.6 for γ_j we arrive at

$$\gamma_j = \frac{p_j' - p_j}{N_i}, \tag{A.7}$$

which is the unique solution for which A.6 is satisfied. Then, we just need to verify that $\pi(\boldsymbol{v}^2;\cdot) = \pi(\boldsymbol{v}^3;\cdot)$ if A.7 is satisfied.

Platform's profit under $\boldsymbol{v}^3$ is given by equation A.3. Substituting A.7 in A.3, we arrive at

$$\pi(\boldsymbol{v}^3;\cdot) = \left(\frac{p_A' - p_A}{N_B} + \frac{p_B' - p_B}{N_A} - c\right) N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B,$$

$$\pi(\boldsymbol{v}^3;\cdot) = -cN_A N_B + p_A' N_A + p_B' N_B - C_A N_A - C_B N_B,$$
$$\pi(\boldsymbol{v}^3;\cdot) = (p_A' - C_A)N_A + (p_B' - C_B)N_B - cN_A N_B.$$

We know that:

$$\pi(\boldsymbol{v}^2;\cdot) = (p_A' - C_A)N_A + (p_B' - C_B)N_B - cN_A N_B.$$

That is, $\pi(\boldsymbol{v}^2;\cdot) = \pi(\boldsymbol{v}^3;\cdot)$, as required.

$\Leftarrow$: assume that $\gamma_j = \frac{p_j' - p_j}{N_i}$ holds. Then need to show that, if so, both tariffs are equivalent. Under $\boldsymbol{v}^3$, agent j's utility is given by A.4. Substituting $\gamma_j = \frac{p_j' - p_j}{N_i}$ into A.4:

$$u_j = \theta_j + \left(\alpha_j - \frac{p_j' - p_j}{N_i}\right) N_i - p_j,$$

$$u_j = \theta_j + \alpha_j N_i - p_j',$$

which is agent j's utility under $\boldsymbol{v}^2$.

We have already verified that if $\gamma_j = \frac{p_j' - p_j}{N_i}$, then $\pi(\boldsymbol{v}^2;\cdot) = \pi(\boldsymbol{v}^3;\cdot)$ (see above). □

Proof of Lemma 1. a) By Lemma A1, end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ if and only if $p_j = (\gamma'_j - \gamma_j)N_i$ for $j \in \{A, B\}$.

In the presence of an ad valorem per-transaction tax levied on the platform, its profit under $\boldsymbol{v}^3$ is given by:

$$\pi(\boldsymbol{v}^3; \cdot) = [(1 - t^p)(\gamma_A + \gamma_B) - c]\, N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B. \tag{A.8}$$

Substituting $p_j = (\gamma'_j - \gamma_j)N_i$ in A.8, we arrive at

$$\pi(\boldsymbol{v}^3; \cdot) = [(1 - t^p)(\gamma_A + \gamma_B) - c]\, N_A N_B + [(\gamma'_A - \gamma_A)N_B - C_A]N_A + [(\gamma'_B - \gamma_B)N_A - C_B]N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = \gamma'_A N_A N_B + \gamma'_B N_A N_B - C_A N_A - C_B N_B - cN_A N_B - t^p(\gamma_A + \gamma_B)N_A N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma'_A + \gamma'_B - c)N_A N_B - C_A N_A - C_B N_B - t^p(\gamma_A + \gamma_B)N_A N_B. \tag{A.9}$$

Platform's profit under $\boldsymbol{v}^1$ is given by:

$$\pi(\boldsymbol{v}^1; \cdot) = \left[(1 - t^p)(\gamma'_A + \gamma'_B) - c\right] N_A N_B - C_A N_A - C_B N_B,$$

$$\pi(\boldsymbol{v}^1; \cdot) = (\gamma'_A + \gamma'_B - c)N_A N_B - C_A N_A - C_B N_B - t^p(\gamma'_A + \gamma'_B)N_A N_B. \tag{A.10}$$

Recall that $\gamma'_j > \gamma_j$ for $j \in \{A, B\}$. Therefore, combining A.9 and A.10, we conclude that $\pi(\boldsymbol{v}^3; \cdot) > \pi(\boldsymbol{v}^1; \cdot)$ for $N_j > 0$, for $j \in \{A, B\}$.

By Lemma A1, end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ if and only if $\gamma_j = \frac{p'_j - p_j}{N_i}$ for $j \in \{A, B\}$. Substituting this condition into A.8, we arrive at

$$\pi(\boldsymbol{v}^3; \cdot) = \left[(1 - t^p)\left(\frac{p'_A - p_A}{N_B} + \frac{p'_B - p_B}{N_A}\right) - c\right] N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = (p'_A - p_A)N_A + (p'_B - p_B)N_B - cN_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B - \left[t^p\left(\frac{p'_A - p_A}{N_B} + \frac{p'_B - p_B}{N_A}\right)\right] N_A N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = (p'_A - C_A)N_A + (p'_B - C_B)N_B - cN_A N_B - \left[t^p\left(\frac{p'_A - p_A}{N_B} + \frac{p'_B - p_B}{N_A}\right)\right] N_A N_B. \tag{A.11}$$

Platform's profit under $\boldsymbol{v}^2$ is given by:

$$\pi(\boldsymbol{v}^2; \cdot) = (p'_A - C_A)N_A + (p'_B - C_B)N_B - cN_A N_B. \tag{A.12}$$

Thus, combining A.11 and A.12, we arrive at:

$$\pi(\boldsymbol{v}^3; \cdot) = \pi(\boldsymbol{v}^2; \cdot) - \left[t^p\left(\frac{p'_A - p_A}{N_B} + \frac{p'_B - p_B}{N_A}\right)\right] N_A N_B.$$

Since $p'_j > p_j$ for $j \in \{A, B\}$, then $\pi(\boldsymbol{v}^2; \cdot) > \pi(\boldsymbol{v}^3; \cdot)$ for $N_j > 0$, for $j \in \{A, B\}$.

b) By Lemma A1, end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ if and only if $p_j = (\gamma'_j - \gamma_j)N_i$ for $i \in \{A, B\}$.

In the presence access taxes levied on the platform, platform's profit under $\boldsymbol{v}^3$ is given by:

$$\pi(\boldsymbol{v}^3;\cdot) = (\gamma_A + \gamma_B - c)N_A N_B + \left[(1-\tau_A^p)p_A - C_A\right] N_A + \left[(1-\tau_B^p)p_B - C_B\right] N_B. \tag{A.13}$$

Substituting $p_j = (\gamma_j' - \gamma_j)N_i$ in A.13, we arrive at

$$\pi(\boldsymbol{v}^3;\cdot) = (\gamma_A+\gamma_B-c)N_A N_B+\left[(1-\tau_A^p)(\gamma_A' - \gamma_A)N_B - C_A\right] N_A+\left[(1-\tau_B^p)(\gamma_B' - \gamma_B)N_A - C_B\right] N_B,$$

$$\pi(\boldsymbol{v}^3;\cdot) = -cN_A N_B+\gamma_A' N_A N_B-\tau_A^p(\gamma_A'-\gamma_A)N_A N_B-C_A N_A+\gamma_B' N_A N_B-\tau_B^p(\gamma_B'-\gamma_B)N_A N_B-C_B N_B,$$

$$\pi(\boldsymbol{v}^3;\cdot) = (\gamma_A'+\gamma_B'-c)N_A N_B-C_A N_A-C_B N_B-\tau_A^p(\gamma_A'-\gamma_A)N_A N_B-\tau_B^p(\gamma_B'-\gamma_B)N_A N_B. \tag{A.14}$$

Platform's profit under $\boldsymbol{v}^1$ is given by:

$$\pi(\boldsymbol{v}^1;\cdot) = (\gamma_A' + \gamma_B' - c)N_A N_B - C_A N_A - C_B N_B. \tag{A.15}$$

Thus, combining A.14 and A.15, we arrive at:

$$\pi(\boldsymbol{v}^3;\cdot) = \pi(\boldsymbol{v}^1;\cdot) - \tau_A^p(\gamma_A' - \gamma_A)N_A N_B - \tau_B^p(\gamma_B' - \gamma_B)N_A N_B.$$

Since $\gamma_A' > \gamma_A$, then $\pi(\boldsymbol{v}^1;\cdot) > \pi(\boldsymbol{v}^3;\cdot)$ for $N_j > 0$, for $j \in \{A, B\}$.

By Lemma A1, end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ if and only if $\gamma_j = \frac{p_j'-p_j}{N_i}$ for $i \in \{A, B\}$. Substituting this condition in A.13, we arrive at

$$\pi(\boldsymbol{v}^3;\cdot) = \left(\frac{p_A' - p_A}{N_B} + \frac{p_B' - p_B}{N_A} - c\right) N_A N_B + \left[(1-\tau_A^p)p_A - C_A\right] N_A + \left[(1-\tau_B^p)p_B - C_B\right] N_B,$$

$$\begin{aligned}\pi(\boldsymbol{v}^3;\cdot) = p_A' N_A - p_A N_A + p_B' N_B - p_B N_B + p_A N_A - \tau_A^p p_A N_A + p_B N_B - \tau_B^p p_B N_B \\ - C_A N_A - C_B N_B - cN_A N_B,\end{aligned}$$

$$\pi(\boldsymbol{v}^3;\cdot) = p_A' N_A + p_B' N_B - \tau_A^p p_A N_A - \tau_B^p p_B N_B - C_A N_A - C_B N_B - cN_A N_B. \tag{A.16}$$

Platform's profit under $\boldsymbol{v}^2$ is given by:

$$\pi(\boldsymbol{v}^2;\cdot) = \left[(1-\tau_A^p)(p_A' - C_A)\right] N_A + \left[(1-\tau_B^p)(p_B' - C_B)\right] N_B - cN_A N_B,$$

$$\pi(\boldsymbol{v}^2;\cdot) = p_A' N_A + p_B' N_B - \tau_A^p p_A' N_A - \tau_B^p p_B' N_B - C_A N_A - C_B N_B - cN_A N_B. \tag{A.17}$$

Recall that $p_j' > p_j$ for $j \in \{A, B\}$. Therefore, combining A.16 and A.17, we conclude that $\pi(\boldsymbol{v}^3;\cdot) > \pi(\boldsymbol{v}^2;\cdot)$ for $N_j > 0$, for $j \in \{A, B\}$. □

The following Lemma is useful to prove Lemma 2.

Lemma A2. *Assume $\boldsymbol{v}^1$, $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ as in Lemma 1.*

a) In the presence of ad valorem per-transaction taxes levied on end-users:

i) end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ $\Rightarrow p_j = (\gamma_j' - \gamma_j)(1 + t_j^e)N_i$,

ii) end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ $\Rightarrow \gamma_j = \frac{p_j'-p_j}{(1+t_j^e)N_i}$.

b) In the presence of ad valorem access taxes levied on end-users:

i) end-users are indifferent between $\boldsymbol{v}^1$ *and* $\boldsymbol{v}^3 \Rightarrow p_j = \frac{(\gamma_j' - \gamma_j)N_i}{1+\tau_j^e}$,

ii) end-users are indifferent between $\boldsymbol{v}^2$ *and* $\boldsymbol{v}^3 \Rightarrow \gamma_j = \frac{(p_j' - p_j)(1+\tau_j^e)}{N_i}$.

Proof. a) *i)* Type-j end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ implies that

$$\theta_j + \left[\alpha_j - \gamma_j'(1+\tau_j^e)\right] N_i = \theta_j + \left[\alpha_j - \gamma_j(1+\tau_j^e)\right] N_i - p_j,$$

$$\gamma_j'(1+\tau_j^e)N_i = \gamma_j(1+\tau_j^e)N_i + p_j.$$

$$\text{Therefore, } p_j = (\gamma_j' - \gamma_j)(1+t_j^e)N_i.$$

ii) Type-j end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ implies that

$$\theta_j + \alpha_j N_i - p_j' = \theta_j + \left[\alpha_j - \gamma_j(1+\tau_j^e)\right] N_i - p_j,$$

$$p_j' = \gamma_j(1+\tau_j^e)N_i + p_j.$$

$$\text{Therefore, } \gamma_j = \frac{p_j' - p_j}{(1+t_j^e)N_i}.$$

b) *i)* Type-j end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ implies that

$$\theta_j + (\alpha_j - \gamma_j')N_i = \theta_j + (\alpha_j - \gamma_j)N_i - p_j(1+\tau_j^e),$$

$$\gamma_j' N_i = \gamma_j N_i + p_j(1+\tau_j^e).$$

$$\text{Therefore, } p_j = \frac{(\gamma_j' - \gamma_j)N_i}{1+\tau_j^e}.$$

ii) Type-j end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ implies that

$$\theta_j + \alpha_j N_i - p_j'(1+\tau_j^e) = \theta_j + (\alpha_j - \gamma_j)N_i - p_j(1+\tau_j^e),$$

$$p_j'(1+\tau_j^e) = \gamma_j N_i + p_j(1+\tau_j^e).$$

$$\text{Therefore, } \gamma_j = \frac{(p_j' - p_j)(1+\tau_j^e)}{N_i}.$$

□

Proof of Lemma 2. a) By Lemma A2, if end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ then $p_j = (\gamma_j' - \gamma_j)(1+t_j^e)N_i$ for $j \in \{A, B\}$.

Platform's profit under $\boldsymbol{v}^3$ is given by:

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma_A + \gamma_B - c)\, N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B. \tag{A.18}$$

Substituting $p_j = (\gamma_j' - \gamma_j)(1+t_j^e)N_i$ in A.18, we arrive at

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma_A + \gamma_B - c)\, N_A N_B + (\gamma_A' - \gamma_A)(1+t_A^e)N_A N_B + (\gamma_B' - \gamma_B)(1+t_B^e)N_A N_B - C_A N_A - C_B N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = \left(\gamma_A' + \gamma_B' - c\right) N_A N_B - C_A N_A - C_B N_B + t_A^e(\gamma_A' - \gamma_A)N_A N_B + t_B^e(\gamma_B' - \gamma_B)N_A N_B. \tag{A.19}$$

Platform's profit under $\boldsymbol{v}^1$ is given by:

$$\pi(\boldsymbol{v}^1;\cdot) = (\gamma'_A + \gamma'_B - c)N_A N_B - C_A N_A - C_B N_B. \tag{A.20}$$

Thus, combining A.19 and A.20, we arrive at:

$$\pi(\boldsymbol{v}^3;\cdot) = \pi(\boldsymbol{v}^1;\cdot) + t^e_A(\gamma'_A - \gamma_A)N_A N_B + t^e_B(\gamma'_B - \gamma_B)N_A N_B.$$

Since $\gamma'_j > \gamma_j$, then $\pi(\boldsymbol{v}^3;\cdot) > \pi(\boldsymbol{v}^1;\cdot)$ for $N_j > 0$, for $j \in \{A, B\}$.

By Lemma A2, if end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ then $\gamma_j = \frac{p'_j - p_j}{(1+t^e_j)N_i}$ for $j \in \{A, B\}$. Substituting $\gamma_j = \frac{p'_j - p_j}{(1+t^e_j)N_i}$ in A.18, we arrive at

$$\pi(\boldsymbol{v}^3;\cdot) = \left[\frac{p'_A - p_A}{(1+t^e_A)N_B} + \frac{p'_B - p_B}{(1+t^e_B)N_A} - c\right] N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B,$$

$$\pi(\boldsymbol{v}^3;\cdot) = \frac{p'_A N_A - p_A N_A + p_A N_A + t^e_A p_A N_A}{(1+t^e_A)} + \frac{p'_B N_B - p_B N_B + p_B N_B + t^e_B p_B N_B}{(1+t^e_B)} - C_A N_A - C_B N_B - c N_A N_B,$$

$$\pi(\boldsymbol{v}^3;\cdot) = \frac{p'_A N_A + t^e_A p_A N_A}{(1+t^e_A)} + \frac{p'_B N_B + t^e_B p_B N_B}{(1+t^e_B)} - C_A N_A - C_B N_B - c N_A N_B.$$

Since $p'_j > p_j$ for $j \in \{A, B\}$, we can define $p'_j \equiv p_j + k_j$, where $k_j > 0$. Then,

$$\pi(\boldsymbol{v}^3;\cdot) = \frac{p_A N_A + k_A N_A + t^e_A p_A N_A}{(1+t^e_A)} + \frac{p_B N_B + k_B N_B + t^e_B p_B N_B}{(1+t^e_B)} - C_A N_A - C_B N_B - c N_A N_B,$$

$$\pi(\boldsymbol{v}^3;\cdot) = \frac{k_A N_A}{(1+t^e_A)} + \frac{k_B N_B}{(1+t^e_B)} + p_A N_A + p_B N_B - C_A N_A - C_B N_B - c N_A N_B.$$

Platform's profit under $\boldsymbol{v}^2$ is given by:

$$\pi(\boldsymbol{v}^2;\cdot) = (p'_A - C_A)N_A + (p'_B - C_B)N_B - c N_A N_B. \tag{A.21}$$

Substituting $p'_j \equiv p_j + k_j$ into A.21, we arrive at

$$\pi(\boldsymbol{v}^2;\cdot) = k_A N_A + k_B N_B + p_A N_A + p_B N_B - C_A N_A - C_B N_B - c N_A N_B.$$

Since $k_j N_j > \frac{k_j N_j}{(1+t^e_j)}$ for $j \in \{A, B\}$, then we can conclude that $\pi(\boldsymbol{v}^2;\cdot) > \pi(\boldsymbol{v}^3;\cdot)$.

b) By Lemma A2, if end-users are indifferent between $\boldsymbol{v}^1$ and $\boldsymbol{v}^3$ then $p_j = \frac{(\gamma'_j - \gamma_j)N_i}{1+\tau^e_j}$ for $j \in \{A, B\}$.

Platform's profit under $\boldsymbol{v}^3$ is given by:

$$\pi(\boldsymbol{v}^3;\cdot) = (\gamma_A + \gamma_B - c)\, N_A N_B + (p_A - C_A)N_A + (p_B - C_B)N_B. \tag{A.22}$$

Substituting $p_j = \frac{(\gamma_j' - \gamma_j)N_i}{1+\tau_j^e}$ in A.22, we arrive at

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma_A + \gamma_B - c)\, N_A N_B + \frac{(\gamma_A' - \gamma_A)N_B}{1+\tau_A^e} N_A + \frac{(\gamma_B' - \gamma_B)N_A}{1+\tau_B^e} N_B - C_A N_A - C_B N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = \left(\gamma_A + \gamma_B + \frac{\gamma_A'}{1+\tau_A^e} - \frac{\gamma_A}{1+\tau_A^e} + \frac{\gamma_B'}{1+\tau_B^e} - \frac{\gamma_B}{1+\tau_B^e}\right) N_A N_B - C_A N_A - C_B N_B - c N_A N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = \left(\frac{\tau_A^e \gamma_A}{1+\tau_A^e} + \frac{\tau_B^e \gamma_B}{1+\tau_B^e} + \frac{\gamma_A'}{1+\tau_A^e} + \frac{\gamma_B'}{1+\tau_B^e}\right) N_A N_B - C_A N_A - C_B N_B - c N_A N_B.$$

Since $\gamma_j' > \gamma_j$ for $j \in \{A, B\}$, we can define $\gamma_j' \equiv \gamma_j + h_j$, where $h_j > 0$. Then,

$$\pi(\boldsymbol{v}^3; \cdot) = \left[\frac{\tau_A^e(\gamma_A' - h_A)}{1+\tau_A^e} + \frac{\tau_B^e(\gamma_B' - h_B)}{1+\tau_B^e} + \frac{\gamma_A'}{1+\tau_A^e} + \frac{\gamma_B'}{1+\tau_B^e}\right] N_A N_B - C_A N_A - C_B N_B - c N_A N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = (\gamma_A' + \gamma_B' - c) N_A N_B - C_A N_A - C_B N_B - \left(\frac{\tau_A^e h_A}{1+\tau_A^e} + \frac{\tau_B^e h_B}{1+\tau_B^e}\right) N_A N_B.$$

Platform's profit under $\boldsymbol{v}^1$ is given by:

$$\pi(\boldsymbol{v}^1; \cdot) = (\gamma_A' + \gamma_B' - c) N_A N_B - C_A N_A - C_B N_B.$$

Since $h_j > 0$ for $j \in \{A, B\}$, we can conclude that $\pi(\boldsymbol{v}^1; \cdot) > \pi(\boldsymbol{v}^3; \cdot)$.

By Lemma A2, if end-users are indifferent between $\boldsymbol{v}^2$ and $\boldsymbol{v}^3$ then $\gamma_j = \frac{(p_j' - p_j)(1+\tau_j^e)}{N_i}$ for $j \in \{A, B\}$.

Substituting $\gamma_j = \frac{(p_j' - p_j)(1+\tau_j^e)}{N_i}$ in A.22, we arrive at

$$\pi(\boldsymbol{v}^3; \cdot) = \left[\frac{(p_A' - p_A)(1+\tau_A^e)}{N_B} + \frac{(p_B' - p_B)(1+\tau_B^e)}{N_A}\right] N_A N_B + (p_A - C_A) N_A + (p_B - C_B) N_B - c N_A N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = p_A'(1+\tau_A^e) N_A - p_A \tau_A^e N_A + p_B'(1+\tau_B^e) N_B - p_B \tau_B^e N_B - C_A N_A - C_B N_B - c N_A N_B,$$

$$\pi(\boldsymbol{v}^3; \cdot) = (p_A' - C_A) N_A + (p_B' - C_B) N_B - c N_A N_B + (p_A' - p_A)\tau_A^e N_A + (p_B' - p_B)\tau_B^e N_B. \quad (A.23)$$

Platform's profit under $\boldsymbol{v}^2$ is given by:

$$\pi(\boldsymbol{v}^2; \cdot) = (p_A' - C_A) N_A + (p_B' - C_B) N_B - c N_A N_B. \quad (A.24)$$

Thus, combining A.23 and A.24, we arrive at:

$$\pi(\boldsymbol{v}^3; \cdot) = \pi(\boldsymbol{v}^2; \cdot) + (p_A' - p_A)\tau_A^e N_A + (p_B' - p_B)\tau_B^e N_B.$$

Since $p_j' > p_j$, then $\pi(\boldsymbol{v}^3; \cdot) > \pi(\boldsymbol{v}^2; \cdot)$ for $N_j > 0$, for $j \in \{A, B\}$. $\square$