Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

Working Paper n° 07/03

Welfare and Output in Third-Degree Price Discrimination: a Note

Francisco Galera

Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

Welfare and Output in Third-Degree Price Discrimination: a Note
Francisco Galera
Working Paper No.07/03
May 2003
JEL Codes: D43, D60, L13

ABSTRACT

In this note I provide examples to show that, contrary to what is widely believed, when changing from an uniform price to a third-degree price discrimination situation, an improvement in welfare is possible with output reduction, if there is more than one firm.

Keywords: price discrimination, imperfect competition, welfare

Francisco Galera
Universidad de Navarra
Departamento de Economía
Campus Universitario
31080 Pamplona
fgalera@unav.es

I Introduction

One of the well-known conclusions about the welfare effects of third-degree price discrimination by a monopolist is that "an increase in total output is a necessary condition for welfare improvement." To avoid repetitions, this affirmation is called "proposition *WO*," or simply, "*WO*." Schmalensee (1981), Varian (1985), and Schwartz (1990) proved *WO* with different levels of generality. The purpose of this note is to show that, although *WO* is valid for a monopolist, it is not extendable to every situation with more than one firm.

To my knowledge, although the welfare effects of third degree price discrimination when competition is present have been studied, *WO* has never been seriously challenged (see Appendix 2). And there is a widely held view that *WO* is true more generally. See, for example, Stole (2001, p.9), Armstrong and Vickers (2001, pp.582-3) and Layson (1994, p. 323).

The logic of *WO* is clear. There is a consumer inefficiency associated with third-degree price discrimination: output is not optimally distributed to consumers because their marginal utilities will be unequal. Proposition *WO* asserts that the *only* way to overcome this consumer inefficiency is a sufficient increase in total output. This is true when there is only one firm or when all the firms share the same costs. But, with heterogeneous firms, costs saving by a better redistribution of output among firms can also overcome the consumer surplus inefficiency. When this is the case, it is no longer true that if output falls when price discrimination is introduced, then welfare must also fall.

I believe that the examples that follow are pertinent because the message of proposition *WO*, even if very probable, is based on one of two unlikely facts in real world: there exists pure monopolies, or all firms in an industry share the same costs. More research is needed to get conditions for proposition *WO* to be valid when there are heterogeneus firms.

II The examples

A dominant firm with zero cost produces a single final product and sells it directly to consumers. Consumers are partitioned into two markets and the dominant firm faces Cournot competition only in one of its two markets from other firm with constant marginal costs, c. The inverse demand functions are $Q_b = a - p_a$ and $Q_b = b - p_b$. We get the following proposition:

Proposition 1: *Neither an increase in the dominant firm's output, nor an increase in total output is a necessary condition for welfare to improve.*

Proof. See the following two examples and Appendix 1.

Example 1. This example shows that *a change from uniform pricing to price discrimination causes a welfare improvement with a reduction of the dominant firm's output, although total output increases*. The dominant firm, Firm 1, sells to two separate markets, A and B. Let us denote its output by q_a and q_b. Demand in market A is $Q_a = 18 - p_a$, and in market B is $Q_b = 10 - p_b$. In market A there is another firm, Firm 2, that competes with Firm 1 in quantities. Let us denote by x_a its output. Costs are zero for both firms. In

Appendix one it is shown that, if Firm 1 is restricted to a uniform price, the equilibrium values, where the superscript U means *uniform pricing*, will be $q_a^U = 6.8$, $x_a^U = 5.6$, $q_b^U = 4.4$; price and welfare (profits plus consumer surplus) will be: $p^U = 5.6$ and $W^U = 180.64$. Firm 1's total output is $Q_1^U = 11.2$. If Firm 1 can price discriminate, then the equilibrium values are (where the superscript D means *price discrimination*): $p_a = 6$, $p_b = 5$, $q_a^D = 6$, $x_a^D = 6$, $q_b^D = 5$ and $W^D = 181.5$. Output of Firm 1 is $Q_1^D = 11$. Welfare is increased under price discrimination with lower output by Firm 1.

Example 2. This example shows that, *with total output reduced, there is an increase in welfare*. In this example, I change demands, and cost of Firm 2. Everything else, notation included, is the same. Demand in market A is $Q_a = 15 - p_a$, and in market B is $Q_b = 18 - p_b$. Firm 2 has marginal cost 6. Again, it is easy to see that, under uniform pricing, the equilibrium values will be $p^U = 7.8$, $q_a^U = 5.4$, $x_a^U = 1.8$, $q_b^U = 10.2$, $W^U = 202.86$. Total output is $Q^U = 17.4$. If Firm 1 can price discriminate, then the equilibrium values are: $p_a = 7$, $p_b = 9$, $q_a^D = 7$, $x_a^D = 1$, $q_b^D = 9$ and $W^D = 203.5$. Total output is $Q^D = 17$. Welfare is increased under price discrimination with lower total output.

Now I provide an example with price competition instead of Cournot competition. A dominant firm with zero cost produces a single final product and sells it directly to consumers. Consumers are partitioned into two markets and the dominant firm faces price competition only in one of its two markets from other firm. We get the following proposition:

Proposition 2: *An increase in total output is not a necessary condition for welfare to improve.*

Proof. See the following example.

Example 3. Demand is formed by 10 consumers; each one of them is willing to buy only one unit of the product. Demand can be divided into two markets. Market A is formed by 6 consumers, each one of them with willingness to pay 2. Market B is formed by 4 consumers, 3 of them have a reservation price of 3, while the remaining consumer is willing to pay 2. Firm 1 sells in both markets and has zero costs. Firm 2 sells only in market A and is a price taker. Marginal cost of Firm 2 is zero for the first unit and 1 for the rest. Firm 2 cannot produce more than 4 units. The price decision by Firm 1 is easy. With uniform price, price will be $p^U = 2$, Firm 2 sells 4 units in market A, and Firm 1 sells 2 units in market A and 4 in market B. Consumer surplus is 3, and profits are 12 for Firm 1 and 5 for Firm 2. Total surplus is $W^U = 20$, and total output is 10. If Firm 1 can price discriminate, prices will be $p_a = 1 - \varepsilon$, with ε being a very small positive number, and $p_b = 3$. Firm 1 sells 5 units in market A and 3 in market B, while Firm 2 sells only one unit. Consumer surplus is $6(1+\varepsilon)$, and profits are $5(1-\varepsilon)+9$ for Firm 1, and $1-\varepsilon$ for Firm 2. Total surplus increase to $W^D = 21$, while total output is reduced in one unit. I provide in Figure 1 a graphical representation of this situation. Shaded areas represent total surplus.

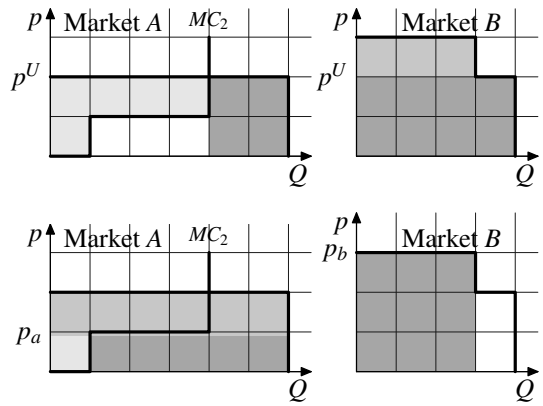

Figure 1: A graphical representation of Example 3

Appendix 1

In examples 1 and 2, there are two markets, A and B, with demands $Q = a - p$ and $Q = b - p$. There are also two firms. Firm 1 has zero costs and sells in both markets. Firm 2 sells only in market A and has a constant marginal cost c. Let q_a, q_b be the quantities of the first firm in both markets and x_a the quantity of the second firm. There is Cournot competition in market A. Superscripts U and D denote uniform pricing and price discrimination respectively.

Whatever price restrictions Firm 1 has, Firm 2 maximizes:

$$\Pi_2 = (a - c - x_a - q_a)x_a \qquad \text{at} \qquad x_a = \frac{1}{2}a - \frac{1}{2}c - \frac{1}{2}q_a.$$

With *uniform pricing*, both markets must have the same price. That is, $p = a - q_a^U - x_a^U$ and $p = b - q_b^U$. Then Firm 1 maximizes:

$$\Pi_1^U = p(a - x_a^U - p + b - p) \qquad \text{at} \qquad p = \frac{1}{4}a - \frac{1}{4}x_a^U + \frac{1}{4}b.$$

If Firm 1 can *price discriminate*, then it maximizes:

$$\Pi_1^D = (a - q_a^D - x_a^D)q_a^D + (b - q_b^D)q_b^D \quad \text{at} \quad q_b^D = \frac{1}{2}b, \quad \text{and} \quad q_a^D = \frac{1}{2}a - \frac{1}{2}x_a^D.$$

With uniform pricing, quantities and prices are obtained by solving the system of equations formed by the reaction curves of both firms:

$$(1) \qquad \left\{ \begin{array}{l} x_a^U = \frac{1}{2}a - \frac{1}{2}c - \frac{1}{2}q_a^U \\ p = \frac{1}{4}a - \frac{1}{4}x_a^U + \frac{1}{4}b \\ p = a - q_a^U - x_a^U \end{array} \right\}, \text{ and the solution is: } \left\{ \begin{array}{l} p = \frac{1}{5}a + \frac{1}{5}b + \frac{1}{5}c, \\ q_a^U = \frac{3}{5}a - \frac{2}{5}b + \frac{3}{5}c, \\ x_a^U = \frac{1}{5}a + \frac{1}{5}b - \frac{4}{5}c. \end{array} \right\}$$

Let us call Q_1 total ouput of Firm 1, and Q total output in the industry. Quantity produced by Firm 1, and total quantity, are respectively:

$$Q_1^U = q_a^U + b - p = \frac{2}{5}a + \frac{2}{5}b + \frac{2}{5}c; \quad \text{and} \quad Q^U = Q_1^U + x_a^U = \frac{3}{5}a + \frac{3}{5}b - \frac{2}{5}c$$

If Firm 1 can price discriminate, quantities and prices are obtained by solving:

$$\left\{ \begin{array}{l} x_a^D = \frac{1}{2}a - \frac{1}{2}c - \frac{1}{2}q_a^D \\ q_b^D = \frac{1}{2}b \\ q_a^D = \frac{1}{2}a - \frac{1}{2}x_a^D \end{array} \right\}, \text{and the solution is:} \left\{ \begin{array}{l} q_a^D = \frac{1}{3}a + \frac{1}{3}c, \\ q_b^D = \frac{1}{2}b, \\ x_a^D = \frac{1}{3}a - \frac{2}{3}c \end{array} \right\} \tag{2}$$

Total quantity produced by Firm 1 when it is allowed to price discriminate, total quantity produced in both markets, and prices are:

$$\begin{array}{ll} Q_1^D = q_a + q_b = \frac{1}{3}a + \frac{1}{3}c + \frac{1}{2}b, & p_a = \frac{1}{3}a + \frac{1}{3}c, \\ Q^D = Q_1^D + x_a^D = \frac{2}{3}a - \frac{1}{3}c + \frac{1}{2}b, & p_b = \frac{1}{2}b. \end{array} \tag{3}$$

With the help of equations 1, 2 and 3, equilibrium values for Example 1 (with $a = 18$ and $b = 10$), and for Example 2 (with $a = 15$, $b = 18$ and $c = 6$) can be found, and they are listed in Tables 1 and 2.

Example 1	p_a	p_b	q_a	q_b	x_a	Q_1	Q
Uniform price	5.6	5.6	6.8	4.4	5.6	11.2	16.8
Price discrimination	6	5	6	5	6	11	17

Table 1: Equilibrium values for Example 1

Example 2	p_a	p_b	q_a	q_b	x_a	Q_1	Q
Uniform price	7.8	7.8	5.4	10.2	1.8	15.6	17.4
Price discrimination	7	9	7	9	1	16	17

Table 2: Equilibrium values for Example 2

Let CS denote consumer surplus. Table 3 shows total welfare, W.

It is easy to see that if Firm 1 increases its output, total output decreases, and vice versa:

$$Q_1^U - Q_1^D = Q^D - Q^U = \frac{2a - 3b + 2c}{30}$$

With some more calculations we can obtain that the difference of welfare between both situations is:

	Example 1		Example 2	
	Uniform price	Price discrimation	Uniform price	Price discrimation
$CS_a = \frac{1}{2}(a-p_a)^2$	$0.5(18-5.6)^2$	$0.5(18-6)^2$	$0.5(15-7.8)^2$	$0.5(15-7)^2$
$CS_b = \frac{1}{2}(b-p_b)^2$	$0.5(10-5.6)^2$	$0.5(10-5)^2$	$0.5(18-7.8)^2$	$0.5(18-9)^2$
$\Pi_1 = p_a q_a + p_b q_b$	$5.6(6.8+4.4)$	$6\cdot 6+5\cdot 5$	$7.8(5.4+10.2)$	$7\cdot 7+9\cdot 9$
$\Pi_2 = (p_a - c)x_a$	$5.6\cdot 5.6$	$6\cdot 6$	$(7.8-6)1.8$	$(7-6)\cdot 1$
$W = CS_a + CS_b + \Pi_1 + \Pi_2$	180.64	181.5	202.86	203.5

Table 3: Welfare

$$W^D - W^U = \frac{51b-14a-134c}{60}\left(Q_1^U - Q_1^D\right) = \frac{51b-14a-134c}{60}\left(Q^D - Q^U\right).$$

Appendix 2

The main point of this paper is that "an increase in total output is *not* a necessary condition for welfare improvement," the value of the paper depending on the novelty of the result. However, Adachi (2002) claims that "By incorporating symmetric interdependency into linear demands, (...), monopolistic third-degree price discrimination can improve social welfare even if total output remains the same.[1]" But in this appendix, I show that this result is misleading. Adachi uses the following model: A monopolist with zero cost face a demand divided into two sub-markets, 1 and 2, with inverse demands: $p_1 = a_1 - q_1 + \eta q_2$ and $p_2 = a_2 - q_2 + \eta q_1$. His main result is enunciated in his **Proposition 3**:

> "Price discrimination improves social welfare if and only if the value of the interdependency exceeds one half $(1/2 < \eta (< 1))$."

Adachi's result is based on calculations to get the change in monopolist's profit, $\Delta\Pi$, and the change in consumer surplus —as he defines it— ΔCS_A, from a situation of uniform price to a regime of price discrimination. Those calculations show that:

$$\Delta\Pi = \frac{1}{8}\frac{(a_1-a_2)^2}{1+\eta},$$

$$\Delta CS_A = -\frac{3}{16}\frac{(a_1-a_2)^2}{(1+\eta)^2}.$$

And in this way welfare change is:

$$\Delta W = \Delta\Pi + \Delta CS_A = (2\eta - 1)\frac{1}{16}\frac{(a_1-a_2)^2}{(\eta+1)^2}.$$

[1] Though the paper is still not published as 2002, October 15, the abstract is published in "The Journal of Industrial Economics," 50, p. 235. I thank Takanori Adachi for kindly making available to me his forthcoming article.

Adachi uses as a measure of consumer surplus the sum of consumer surpluses in both markets, $CS_A = 0.5(q_1^2 + q_2^2)$. But this way of measuring consumer surplus introduces an externality between both markets. To show how welfare results are misleading with this way of measuring consumer surplus we provide the following example.

Suppose that in a competitive market there are two related demands with inverse demands $p_1 = 20 - q_1 + 2q_2/3$ and $p_2 = 10 - q_2 + 2q_1/3$; and that the average cost of production is constant and equal to 2. Then, production is $q_1 = 42$ and $q_2 = 36$. As long as there are no profits, total welfare consists only of consumer surplus: $0.5 \cdot 42^2 + 0.5 \cdot 36^2 = 1530$.

Now, suppose that the good is given free to consumers but the cost of production remains the same. Then quantities demanded are $q_1 = 48$, and $q_2 = 42$, and total welfare is consumer surplus less costs of production: $0.5 \cdot 48^2 + 0.5 \cdot 42^2 - 2(48 + 42) = 1854$. It would be better for society to set a price zero to the good. So, we get this wrong result: *A competitive market does not maximize welfare*. This result is not wrong, of course, when there are externalities.

I have checked the result by Adachi by considering, to measure consumer surplus, that the demand system arises from a representative consumer who maximizes $U(q) - pq$ (see Vives, pp. 144-7) where $U(q_1, q_2) = a_1 q_1 + a_2 q_2 - \frac{1}{2}(q_1^2 + q_2^2 - 2\eta q_1 q_2)$. Maximizing utility with respect to q_1 and q_2, we get the same consumer demand as Adachi:

$$\begin{cases} q_1 = \frac{a_1 + \eta a_2 - p_1 - \eta p_2}{1-\eta^2}, \\ q_2 = \frac{a_2 + \eta a_1 - p_2 - \eta p_1}{1-\eta^2}; \end{cases} \quad \text{or} \begin{cases} p_1 = a_1 - q_1 + \eta q_2, \\ p_2 = a_2 - q_2 + \eta q_1. \end{cases}$$

Substituting the prices into the utility function, we see that consumer surplus, is:

$$\begin{gathered} CS(q_1, q_2) = \\ = a_1 q_1 + a_2 q_2 - \tfrac{1}{2}(q_1^2 + q_2^2 - 2\eta q_1 q_2) - (a_1 - q_1 + \eta q_2) q_1 - (a_2 - q_2 + \eta q_1) q_2 = \\ \tfrac{1}{2}(q_1^2 + q_2^2) - \eta q_1 q_2. \end{gathered}$$

So, we conclude that $CS(q_1, q_2) = CS_A(q_1, q_2) - \eta q_1 q_2$.

Now, we consider the monopolist decisions. With *price discrimination* (superscript D) and *uniform price* (superscript U) regimes, monopolist's profits are:

$$\begin{aligned} \Pi^D(q_1, q_2) &= (a_1 - q_1 + \eta q_2) q_1 + (a_2 - q_2 + \eta q_1) q_2 \\ \Pi^U(p) &= p(a_1 + a_2 - 2p)/(1 - \eta) \end{aligned}$$

In Table 4 are listed the results of maximizing profits in both regimes.

We get the same increment in profits as Adachi:

$$\Delta\Pi = \Pi^{*D} - \Pi^{*U} = \frac{1}{4} \frac{a_1^2 + a_2^2 + 2a_1 a_2 \eta}{1 - \eta^2} - \frac{1}{8} \frac{(a_1 + a_2)^2}{1 - \eta} = \frac{1}{8} \frac{(a_2 - a_1)^2}{1 + \eta}.$$

Price Discrimination	Uniform price
$q_1^D = \frac{1}{2}\frac{a_1+\eta a_2}{1-\eta^2}$	$q_1^U = \frac{1}{4}\frac{(3-\eta)a_1+(3\eta-1)a_2}{1-\eta^2}$
$q_2^D = \frac{1}{2}\frac{\eta a_1+a_2}{1-\eta^2}$	$q_2^U = \frac{1}{4}\frac{(3-\eta)a_2+(3\eta-1)a_1}{1-\eta^2}$
$Q^D = \frac{1}{2}\frac{a_1+a_2}{1-\eta}$	$Q^U = \frac{1}{2}\frac{a_1+a_2}{1-\eta}$
$\Pi^{*D} = \frac{1}{4}\frac{a_1^2+a_2^2+2a_1a_2\eta}{1-\eta^2}$	$\Pi^{*U} = \frac{(a_1+a_2)^2}{8(1-\eta)}$

Table 4: Price Discrimination and Uniform prices

But the increment in consumer surplus is different. We use the fact that total quantity is the same in both regimes:

$$\Delta CS = CS(q_1^D, q_2^D) - CS(q_1^U, q_2^U) =$$
$$\frac{1}{2}\left(q_1^D\right)^2 + \frac{1}{2}\left(q_2^D\right)^2 - \eta q_1^D q_2^D - \left(\frac{1}{2}\left(q_1^U\right)^2 + \frac{1}{2}\left(q_2^U\right)^2 - \eta q_1^U q_2^U\right) =$$
$$= \frac{1}{2}\left(q_1^D + q_2^D\right)^2 - (1+\eta)\, q_1^D q_2^D - \left(\frac{1}{2}\left(q_1^U + q_2^U\right)^2 - (1+\eta)\, q_1^U q_2^U\right) =$$
$$= (q_1^U q_2^U - q_1^D q_2^D)(1+\eta).$$

It is easy to chech that
$\Delta CS = (1+\eta)\left(\frac{(3-\eta)a_1+(3\eta-1)a_2}{4(1-\eta^2)}\frac{(3-\eta)a_2+(3\eta-1)a_1}{4(1-\eta^2)} - \frac{a_1+\eta a_2}{2(1-\eta^2)}\frac{\eta a_1+a_2}{2(1-\eta^2)}\right) = -\frac{3}{16}\frac{(a_2-a_1)^2}{1+\eta}$.
And the total change in welfare is:

$$\Delta W = \Delta\Pi + \Delta CS = \frac{1}{8}\frac{(a_2-a_1)^2}{1+\eta} - \frac{3}{16}\frac{(a_2-a_1)^2}{1+\eta} = -\frac{1}{16}\frac{(a_2-a_1)^2}{1+\eta}.$$

So, total output remains the same with third-degree price discrimination, but *welfare does not increase; moreover, it is reduced*. And **Proposition 3** by Adachi is no longer true. It is easy to verify that if we measure consumer surplus as Adachi, $CS_A(q_1, q_2) = \frac{1}{2}(q_1^2 + q_2^2)$, we get the Adachi's results.

References

[1] Adachi T., 2002, A Note on Third-Degree Price Discrimination with Interdependent Demands, Journal of Industrial Economics, 50, 235 (forthcoming at the Journal of Industrial Economics (JIE) Notes and Comments page: http://www.stern.nyu.edu/ jindec/notes.htm)

[2] Armstrong, M. and J. Vickers, 2001, Competitive price discrimination, RAND Journal of Economics, 32, 579-605.

[3] Layson, Stephen K., 1994, Third-Degree Price Discrimination under Economies of Scale, Southern Economic Journal, 61, 323-327.

[4] Schmalensee, R., 1981, Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination, American Economic Review, 71, 242-247.

[5] Schwartz, M., 1990, Third-Degree Price Discrimination and Output: Generalizing a Welfare Result, American Economic Review, 80, 1259-1262.

[6] Stole, L., 2001, Price Discrimination in Competitive Environments, (preliminary and incomplete), prepared for the Handbook of Industrial Organization, (version November 7, 2001) It can be reached at: "http://gsblas.uchicago.edu/Lars_Stole.html"

[7] Varian, H. R., 1985, Price Discrimination and Social Welfare, American Economic Review, 75, 870-875.

[8] Vives, X., 1999, Oligopoly pricing (MIT press).