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ABSTRACT

We propose in this article a two-step testing procedure of fractional cointegration in macroeconomic time series. It is based on Robinson's (1994) univariate tests and is similar in spirit to the one proposed by Engle and Granger (1987), testing initially the order of integration of the individual series and then, testing the degree of integration of the residuals from the cointegrating relationship. Finite-sample critical values of the new tests are computed and Monte Carlo experiments are conducted to examine the size and the power properties of the tests in finite samples. An empirical application, using the same datasets as in Engle and Granger (1987) is also carried out at the end of the article.

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1. Introduction

Nelson and Plosser (1982) showed, using tests of Dickey and Fuller (1979), that many US macroeconomic time series contained a unit root. These tests, however, were shown to have very low power against certain types of alternatives, and other unit-root tests were proposed in the following years (e.g., Phillips, 1987; Phillips and Perron, 1988; Kwiatkowski et al., 1992, etc.). All these unit-root tests are nested in autoregressive (AR) alternatives. However, the AR model is merely one of the many models that nest a unit root. Robinson (1994) proposes tests for unit roots and other hypotheses, which are embedded in a fractional model of form:

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ... \tag{1}$$

where u_t is I(0), (defined as a covariance stationary process with spectral density function which is positive and finite at the zero frequency), and where the unit root null corresponds to d = 1.

In a multivariate framework, Engle and Granger (1987) noticed that many series may have a common trend and suggested a technique called cointegration, which implies that several series which are I(d) may be related such that there exists at least one linear combination which is I(d-b) with b > 0. If d = b = 1, they proposed a two-step strategy based on Dickey and Fuller (1979). More robust tests in this context of integer d and b were later proposed by Johansen (1988, 1995). The literature on fractional cointegration is relatively new. Kim and Phillips (2000) and Robinson and Hualde (2000) have concentrated on the estimation of the parameters in the cointegrating relationship, while Robinson and Marinucci (1998, 2001) and Robinson and Hualde (2002) have examined the estimation of the orders of integration. In this paper, we take a simpler approach, and propose a two-step procedure, based on Robinson's (1994) tests, for testing the null hypothesis of no cointegration against fractional cointegration. That is, we extend Engle and Granger's (1987) procedure to the case where d and b can be real numbers. The outline of the paper is as follows: Section 2 describes the tests of Robinson (1994). Section 3 presents the procedure for testing fractional cointegration, along with finite-sample critical values obtained by simulation. Section 4 uses Monte Carlo to examine the size and the power properties of the tests. Several examples are carried out in Section 5 and Section 6 contains some concluding comments.

2. The tests of Robinson (1994)

Let's suppose that $\{x_t,\ t = 1,2,\ldots,\ T\}$ is the time series we observe and consider the model given by (1), with u_t with spectral density function given by $f(\lambda;\ \sigma^2;\ \tau) = (\sigma^2/2\pi)g(\lambda;\ \tau)$, where $\sigma^2 = V(\varepsilon_t)$ and τ are unknown but g is known. Robinson (1994) proposes an LM test of:

$$H_o: \quad d = d_o, \tag{2}$$

in (1) for any real value d_o. Specifically, the test statistic is given by:

$$\hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \tag{3}$$

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \quad \psi(\lambda_j) = \log\left|2\,sin\frac{\lambda_j}{2}\right|;$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\,\varepsilon(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\varepsilon(\lambda_j)\,\varepsilon(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\varepsilon(\lambda_j)\,\psi(\lambda_j)'\right); \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}),$$

where $I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1 - L)^{d_o} x_t$, evaluated at $\lambda_j = 2\pi j/T$, and $\hat{\tau}$ is obtained by minimising $\sigma^2(\tau)$. Robinson (1994) showed that under certain regularity conditions:

$$\hat{r} \; \rightarrow_d \; N(0,1) \qquad as \;\; T \rightarrow \infty. \tag{4}$$

Thus, a one-sided 100α%-level test of (2) against H_1: $d > d_o$ ($d < d_o$) is given by the rule: 'Reject H_o if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$)', where the probability that a standard normal variate exceeds z_α is α.

3. Testing of fractional cointegration

The components of a (nx1) vector X_t are fractionally cointegrated of order d,b, ($X_t \sim CI(d,b)$) if a): all components of X_t are integrated of order d ($X_{it} \sim I(d)$), and b): there exists a vector r ($r \neq 0$) such that $N_t = r'X_t$ is integrated of order d-b ($N_t \sim I(d-b)$) with $b > 0$. The vector r is called the cointegrating vector and $r'X_t$ represents an equilibrium constraint operating on the long-run component of X_t. If n is higher than two, then there may be more than one cointegrating vector r, though in what follows we will assume that X_t does have only two components, so that $X_t = (X_{1t}, X_{2t})'$, where X_{1t} and X_{2t} correspond to the variables to be analysed later.

First, we test that both individual series are integrated of the same order, (let's say, e.g., $\bar{d}$). This can be done using Robinson's (1994) univariate tests described in Section 2. Then, we can estimate the cointegrating parameters from the cointegrating regression. Since all linear combinations of X_{1t} and X_{2t} except the one defined by the cointegrating relation will be integrated of order $\bar{d}$, the least squares (LS) estimate from the regression of X_{1t} on X_{2t}, under cointegration, will produce a good estimate of it. In standard cointegration analysis (in which $d = b = 1$), Stock (1987) showed that the LS estimate of the cointegrating parameter was consistent and converged in probability at the rate $T^{1-\delta}$ for any $\delta > 0$. Cheung and Lai (1993), Robinson and Marinucci (1998) and others extended the analysis to the fractional case, and showed that the LS estimate was also consistent though with possible different convergence rates. A problem with this estimator is that suffers from second-order bias which may make it inaccurate in finite samples. In that respect, other estimates like the fully-modified proposed by Kim and Phillips (2000) or the frequency-domain one of Robinson and Hualde (2000) may be preferred. For related results on fractionally cointegrated models, see also Dolado and Marmol (1997) and Jeganathan (1999). However, in order to have exact comparisons in the application below with the results in Engle and Granger (1987), we have decided to use the OLS estimator. Given the consistency of this estimate, we can use Robinson's (1994) tests for testing the order of integration in the equilibrium errors e_t, where $e_t = X_{1t} - \hat{\alpha} X_{2t}$, with $\hat{\alpha}$ as the OLS estimate of the cointegrating parameter, and the test statistic will still remain with the same standard limit distribution. Thus, we could consider the model:

$$(1 - L)^d e_t = v_t, \qquad t = 1, 2, ... \tag{5}$$

with I(0) v_t, and test the null hypothesis: H_o: d = $\bar{d}$ against the alternative: H_a: d < $\bar{d}$. Rejections of H_o against H_a will imply that X_{1t} and X_{2t} are fractionally cointegrated, given that the equilibrium errors display a smaller degree of integration than that of the individuals series. However, since the equilibrium errors are not actually observed but obtained from minimizing the residual variance of the cointegrating regression, the residuals might be biased toward stationarity, and thus, we would expect the null to be rejected more often than suggested by the nominal size of Robinson's (1994) tests. A similar problem arises in Engle and Granger (1987) and Cheung and Lai (1993) when testing cointegration. In order to cope with this problem, the empirical size of the tests in finite samples is obtained using a simulation approach.

(Table 1 about here)

Table 1 reports finite-sample critical values of Robinson's (1994) tests for cointegration, with T = 50, 100, 200 and 300. We use the Monte Carlo method in 50,000 replications, assuming that the true system is of two I(d) processes with Gaussian independent white noise disturbances that are not cointegrated, and take values of d = 0.6, (0.1), 1.5. We assume that v_t is white noise, though we could have extended the analysis to cover the case of autocorrelated disturbances. We see that the critical values are similar across d. They have a negative mean and the values corresponding to the left-hand side distribution, (which is the one required to test cointegration), are smaller than those of the normal distribution, which is consistent with the earlier discussion that, when testing H_o against $d < \bar{d}$, the use of the standard values will result in the cointegration tests rejecting the null hypothesis of no cointegration too often. We also see that the empirical distributions are positively skewed with kurtosis greater than 3, though increasing T, the three statistics (mean, skewness and kurtosis) approximate to the values of the normal distribution.

4. The power of the tests in finite samples

We next examine the power properties of the tests described in Section 3 relative to the ADF and Geweke and Porter-Hudak (GPH, 1983) tests for cointegration. We consider a bivariate system, claimed to be non-cointegrated under the null hypothesis. The ADF unit-root test recommended by Engle and Granger (1987) is given by the usual t-statistic for b_0 in:

$$(1 - L)e_t = b_0 e_{t-1} + b_1(1-L)e_{t-1} + ... + b_p(1 - L)e_{t-p} + \varepsilon_t,$$

where e_t are the equilibrium errors and the lag parameter p can be selected using some model-selection procedures. The GPH test for cointegration proposed by Cheung and Lai (1993) is based on the estimation of the fractional differencing parameter d, in the linear regression:

$$\ln\left(I(\lambda_j)\right) \;=\; \beta_0 \;+\; \beta_1 \ln\left(4\sin^2\left(\frac{\lambda_j}{2}\right)\right) \;+\; u_t,$$

where $\lambda_j = 2\pi j/T$ and $I(\lambda_j)$ is the periodogram of e_t evaluated at the ordinate j. Given that the LS estimate of β_1 provides a consistent estimate of 1-d (see Robinson, 1995), hypothesis testing concerning the value of d is based on the t-statistic of the regression coefficient.

Table 2 reports results of the power function of the three tests (ADF, GPH and Robinson) for cointegration against fractional and AR alternatives. Results for ADF and GPH tests have been taken from Cheung and Lai (1993). The power of a test is measured as the percentage of the time the test can reject a false null hypothesis of no cointegration, and the Monte Carlo experiment is described in Appendix I. We perform Robinson's (1994) statistic, assuming that the differenced series are white noise and AR processes of orders 1, 2 and 3, for 5% and 10% significance levels.

(Table 2 about here)

When testing against fractional alternatives, Robinson's (1994) tests perform better than the ADF and the GPH tests, and this is observed for white noise disturbances but also if they follow AR processes. The highest rejection frequencies are obtained with white noise disturbances if the integration order ranges between 0.05 and 0.75. but when this parameter approximates to 1, better results are obtained for weakly autocorrelated disturbances.

When testing against AR alternatives, again better statistical power properties are observed in Robinson (1994) relative to ADF and GPH tests, with higher rejection frequencies obtained at all values of the AR parameter. If this parameter ranges between 0.05 and 0.55, results are better when the disturbances are white noise, but if it ranges between 0.55 and 0.95, the tests behave better for autocorrelated disturbances. The relative pronounced difference in power between Robinson's (1994) and the ADF and GPH tests for cointegration should not be surprising given that the ADF test assumes a strict I(0) and I(1) distinction and the GPH test requires estimation of the differencing parameter, whereas Robinson (1994) tests do allow fractional differencing and do not require estimation of the fractional differencing parameter.

5. Illustrative examples

We analyse the common behaviour between consumption and income, wages and prices, and nominal GNP and money, using the same dataset as in Engle and Granger (1987), and stock prices and dividends, using the data in Campbell and Shiller (1987). The description of the data is given in Appendix II. All these pairs of variables have been analysed by many authors using classical techniques. However, in the context of fractional models, the literature is scarce. Robinson and Marinucci (1998, 2001) employ a semiparametric method on the consumption and income and the stock prices and dividends relationships and come to the conclusion that both are cointegrated, the order of integration of the residuals being higher than 0.5 but smaller than 1. Robinson and Hualde (2002) also examine these variables along with GNP and money and similarly to the previous works, conclude that the residuals posess long memory with d smaller than 1.

Table 3 reports the results of Robinson's (1994) tests for cointegration. The first two lines of each pair of variables correspond to the analysis of the individual series while the other two correspond to the results based on the OLS regressions in both directions. We look at $\hat{r}$ given by (3), testing H_o (2) for values d_o = 0.6, (0.1), 1.5, with white noise disturbances.

(Table 3 about here)

Starting with consumption and income, we observe that for the individual series, H_o (2) cannot be rejected when d_o = 0.9, 1 and 1.1. However, looking at the residuals, these hypotheses are rejected in favour of alternatives with smaller orders of integration. In fact, the null hypothesis cannot be rejected now when d_o = 0.6, 0.7 and 0.8, the lowest statistic appearing in both cases when d_o = 0.7. Thus, we find evidence of fractional cointegration between consumption and income, with the deviations from an equilibrium following a nonstationary fractional process with the order of integration smaller than one. Engle and Granger (1987) tested the null of nonstationarity I(1) in the estimated residuals from the OLS regressions. Using the Cointegration Regression Durbin-Watson (CRDW) test, the null was rejected at 5% significance level but hardly at 1%, and using the ADF tests, it was rejected for the regression of consumption on income but hardly for the reverse.

The results for prices and wages clearly indicate a lack of cointegration. In fact, H_o (2) cannot be rejected when d_o = 1 and 1.1 for the individual series, and the same result is obtained when testing on the estimated residuals. This result is completely in line with the findings in Engle and Granger (1987). The third example illustrates the relation between nominal GNP and nominal money. This is upon the quantity theory equation: $M \cdot V = P \cdot Y$, and most empirical applications stem from the assumption that velocity is constant or at least stationary. Under this general condition, log M, log P and log Y should be cointegrated with known unit parameters, and similarly, nominal GNP and nominal money should also be cointegrated. Engle and Granger (1987) failed to find cointegration using M1 as the monetary aggregate. The results show that a certain degree of fractional cointegration may appear, with the orders of integration of the individual series ranging between 0.9 and 1.1, but ranging between 0.7 and 0.9 for the residuals.

Finally, we examine the relationship between stock prices (SP) and dividends (D). The idea follows from a present value model, which asserts that an asset price is linear in the present discounted value of future dividends. Campbell and Shiller (1987) applied the ADF tests on both individuals series and their results suggested that both were integrated of order 1. Using DF and ADF tests on the residuals, their results were mixed: the former test rejected the null of no cointegration at the 5% level while the latter narrowly failed to reject it at the 10%. Our results again indicate that this pair of variables may be fractionally cointegrated. Looking at the individual series, the orders of integration range between 0.9 and 1.1 for the stock prices and between 0.9 and 1.3 for dividends, with the lowest statistics appearing in both series at the unit root case (i.e, $d_o = 1$). However, the results for the estimated residuals suggest that the orders of integration are between 0.6 and 0.9, with the lowest statistics appearing in both cases at $d_o = 0.7$, and thus, implying mean reversion in the long run equilibrium relationship. In view of all this, we can conclude by saying that there is some evidence of fractional cointegration between consumption and income, GNP and money and stock prices and dividends, with the equilibrium relationships possessing long memory. Thus, the equilibrium errors are mean reverting, with shocks affecting to them disappearing in the long run.

6. Concluding comments

We have presented a procedure for testing the null hypothesis of no cointegration against fractional cointegration. It is based on Robinson’s (1994) tests and it follows the same methodology as in Engle and Granger (1987). We initially test the order of integration of the individual series and, if all them have the same order, we test the degree of integration on the residuals from the cointegrating regression. There will be a cointegrating relationship if the order of integration of these residuals is smaller than that of the individual series. Finite-sample critical values were computed and experiments conducted via Monte Carlo show that they have better power properties against both fractional and AR alternatives than other existing tests for cointegration.

The tests were employed to analyse the relationship between consumption and income, CPI and wages, nominal GNP and money, and stock prices and dividends. The results indicate that all variables may be individually I(1), and testing the order of integration of the residuals from the OLS regressions, the results show that all pairs of variables (except CPI and wages) may be fractionally cointegrated, with the order of integration of the residuals being greater than 0.5 but smaller than 1. Note that the tests rejected H_o (2) with $d_o = 0.5$ against $d > 0.5$ for all residuals in all series. Thus, the equilibrium errors are non-stationary but display mean reversion, unlike the individual series where shocks seem to persist forever. These results are interesting in that they seem to overcome the mixing conclusions in Engle and Granger (1987) and Campbell and Shiller (1987), the reason being that they only concentrated on I(0) and I(1) specifications and did not consider other possible fractional possibilities. Also, these results are completely in line with those obtained by Robinson and Marinucci (1998, 2001) and Robinson and Hualde (2002). In the first two papers, they use a narrow-band frequency domain least squares estimate to detect the order of integration of the residuals in the relationships between consumption and income and stock prices and dividends. Robinson and Hualde (2002) use a root-n-consistent estimator for the same purpose and extent the analysis to the case of money and GNP. All these papers conclude that there exists some degree of fractional cointegration with d higher than 0.5 but smaller than 1.

This article can be extended in several directions. The finite-sample critical values can be extended to permit more than two variables and also to allow autocorrelated disturbances. Other semiparametric methods of estimating and testing d may also be applied on the residuals from the cointegrating regressions. However, these methods may be too sensitive to the choice of the bandwidth parameter and, in that respect, a fully parametric model like this may be more appropriate. Extensions of the multivariate version of the tests of Robinson (1994) which permit us to test fractional cointegration in a system-based model is also of interest. There exists a reduced-rank procedure suggested by Robinson and Yajima (2000), However, it is not directly applicable here, neither in the simulation study nor in the empirical application since that method assumes I(d) stationarity ($d < 0.5$) for the individual series while we consider I(1) nonstationary processes.

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Appendix I

To illustrate the potential difference in power between the tests of Robinson (1994) and the GPH and the ADF tests of cointegration, a Monte Carlo experiment, similar to that in Engle and Granger (1987) and Cheung and Lai (1993) is conducted. We consider a bivariate system where X_{1t} and X_{2t} are given by

$$X_{1t} + X_{2t} = U_{1t}, \quad t = 1, 2, ... \tag{A1}$$

$$X_{1t} + 2X_{2t} = U_{2t}, \quad t = 1, 2. ..., \tag{A2}$$

where $(1 - L)\, U_{1t} = \varepsilon_{1t}$, and U_{2t} is generated, alternatively, as an autoregressive process

$$(1 - \rho L) U_{2t} = \varepsilon_{2t}, \quad t = 1, 2, \; ... \tag{A3}$$

or as a fractional white noise process

$$(1 - L)^d U_{2t} = \varepsilon_{2t}, \quad t = 1, 2,... , \tag{A4}$$

where the innovations ε_{1t} and ε_{2t} are generated as independent standard normal variates. Thus, if $\rho = 1$ in (A3) or $d = 1$ in (A4), the two series are I(1) and non-cointegrated; if U_{2t} is generated by (A3) and $|\rho| < 1$, X_{1t} and X_{2t} are cointegrated, and (A2) is their cointegrating relationship; alternatively, if U_{2t} is generated by (A4) and $d < 1$, X_{1t} and X_{2t} are fractionally cointegrated. As in Engle and Granger (1987) and Cheung and Lai (1993), we used samples os size $T = 76$, and sample series of X_{1t} and X_{2t} were generated setting the initial values of U_{1t} and U_{2t} equal to zero, creating 126 observations, of which the first 50 were discarded to reduce the effect of the initial conditions.

Appendix II

C_t: US quarterly real per capita consumption on non-durables from 1947.I to 1981.II

Y_t: US quarterly real per capita disposable income from 1947.I to 1981.II.

CPI_t: Log of the US monthly Consumer Price Index from 1950.1 to 1979.12.

W_t: Log of the US monthly production worker wage in manufacturing from 1950.1 to 1979.12.

GNP_t: Log of the US quarterly nominal Gross National Product from 1959.I to 1981.II

$M1_t$: Log of the US quarterly nominal M1 from 1959.I to 1981.II

SP_t: US real annual stock prices from 1871 to 1986.

D_t: US real annual dividends from 1871 to 1986.

The first six series has been taken from Engle and Granger (1987) and the remaining two from Campbell and Shiller (1987).

TABLE 1										
Finite-sample critical values of Robinson (1994) tests for cointegration[*]										
T = 50										
Perc ./ d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-2.94	-2.94	-2.95	-2.93	-2.93	-2.93	-2.93	-2.92	-2.93	-2.92
0.5%	-2.65	-2.66	-2.66	-2.67	-2.66	-2.66	-2.66	-2.66	-2.65	-2.65
1%	-2.51	-2.52	-2.53	-2.52	-2.52	-2.52	-2.52	-2.51	-2.50	-2.50
2.5%	-2.29	-2.30	-2.31	-2.30	-2.30	-2.30	-2.29	-2.29	-2.28	-2.27
5%	-2.09	-2.10	-2.11	-2.11	-2.10	-2.09	-2.08	-2.08	-2.07	-2.07
10%	-1.84	-1.85	-1.85	-1.84	-1.84	-1.84	-1.83	-1.82	-1.82	-1.81
Mean	-0.70	-0.72	-0.72	-0.72	-0.71	-0.70	-0.70	-0.69	-0.68	-0.68
Skewness	0.59	0.59	0.59	0.59	0.58	0.57	0.56	0.56	0.55	0.54
Kurtosis	3.67	3.68	3.69	3.70	3.68	3.64	3.60	3.59	3.53	3.50
T = 100										
Perc ./ d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-2.96	-2.95	-2.95	-2.97	-2.96	-2.94	-2.95	-2.96	-2.96	-2.96
0.5%	-2.64	-2.65	-2.64	-2.63	-2.63	-2.62	-2.62	-2.61	-2.60	-2.60
1%	-2.48	-2.49	-2.48	-2.48	-2.47	-2.47	-2.46	-2.45	-2.45	-2.44
2.5%	-2.23	-2.24	-2.24	-2.23	-2.23	-2.22	-2.21	-2.21	-2.20	-2.20
5%	-2.01	-2.00	-2.00	-2.01	-2.00	-2.00	-1.99	-1.99	-1.99	-1.98
10%	-1.74	-1.75	-1.75	-1.74	-1.74	-1.72	-1.71	-1.71	-1.71	-1.70
Mean	-0.56	-0.57	-0.58	-0.57	-0.56	-0.56	-0.55	-0.54	-0.54	-0.53
Skewness	0.46	0.46	0.46	0.45	0.45	0.45	0.45	0.44	0.44	0.45
Kurtosis	3.41	3.40	3.39	3.39	3.40	3.40	3.38	3.35	3.34	3.34
T = 200										
Perc ./ d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-3.04	-3.08	-3.07	-3.14	-3.19	-3.20	-3.12	-3.12	-3.10	-3.06
0.5%	-2..71	-2.73	-2.70	-2.70	-2.66	-2.64	-2.62	-2.61	-2.63	-2.64
1%	-2.50	-2.48	-2.47	-2.46	-2.45	-2.46	-2.45	-2.44	-2.44	-2.43
2.5%	-2.21	-2.20	-2.20	-2.21	-2.20	-2.20	-2.20	-2.19	-2.18	-2.18
5%	-1.95	-1.97	-1.97	-1.97	-1.97	-1.96	-1.94	-1.94	-1.93	-1.93
10%	-1.64	-1.65	-1.67	-1.66	-1.66	-1.65	-1.63	-1.62	-1.62	-1.61
Mean	-0.44	-0.46	-0.46	-0.46	-0.46	-0.45	-0.44	-0.43	-0.43	-0.42
Skewness	0.31	0.30	0.30	0.31	0.32	0.32	0.33	0.34	0.35	0.36
Kurtosis	3.18	3.17	3.18	3.23	3.26	3.27	3.28	3.28	3.30	3.31
T = 300										
Perc. / d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-2.96	-2.96	-3.04	-3.12	-3.19	-3.22	-3.19	-3.17	-3.17	-3.15
0.5%	-2.52	-2.56	-2.63	-2.61	-2.60	-2.59	-2.60	-2.59	-2.61	-2.61
1%	-2.41	-2.42	-2.44	-2.45	-2.44	-2.44	-2.44	-2.44	-2.44	-2.44
2.5%	-2.17	-2.18	-2.19	-2.20	-2.18	-2.17	-2.16	-2.14	-2.13	-2.13

5%	-1.90	-1.91	-1.92	-1.92	-1.91	-1.90	-1.89	-1.88	-1.87	-1.87
10%	-1.59	-1.60	-1.60	-1.61	-1.60	-1.60	-1.60	-1.59	-1.58	-1.58
Mean	-0.37	-0.39	-0.39	-0.39	-0.39	-0.38	-0.38	-0.37	-0.37	-0.36
Skewness	0.31	0.29	0.28	0.28	0.29	0.29	0.30	0.30	0.30	0.30
Kurtosis	3.07	3.08	3.10	3.13	3.15	3.15	3.15	3.14	3.13	3.13

*: The empirical distribution has been obtained using 50,000 replications in simulation, assuming that the true system is of two non-cointegrated I(d) processes. The test statistic is $\hat{r}$ in (3).

TABLE 2

Power of the ADF, GDH and Robinson tests for cointegration against fractional alternatives*

Size	Test	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05
5%	ADF (ρ = 4)	0.06	0.07	0.10	0.14	0.19	0.26	0.36	0.50	0.61	0.73
	GPH (μ = .55)	0.06	0.09	0.15	0.21	0.30	0.37	0.47	0.56	0.61	0.64
	GPH (μ = .575)	0.06	0.10	0.16	0.24	0.33	0.42	0.53	0.62	0.67	0.71
	GPH (μ = .60)	0.06	0.11	0.18	0.28	0.40	0.52	0.63	0.73	0.78	0.81
	ROB (Wh. N)	**0.07**	**0.22**	**0.50**	**0.78**	**0.94**	**0.99**	**0.99**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.15**	**0.22**	**0.35**	**0.52**	**0.71**	**0.85**	**0.94**	**0.97**	**0.99**	**0.99**
	ROB (AR (2))	**0.22**	**0.26**	**0.31**	**0.41**	**0.54**	**0.67**	**0.78**	**0.86**	**0.92**	**0.95**
	ROB (AR (3))	**0.30**	**0.32**	**0.35**	**0.41**	**0.50**	**0.59**	**0.68**	**0.76**	**0.82**	**0.85**
10%	ADF (ρ = 4)	0.11	0.13	0.18	0.24	0.32	0.41	0.53	0.67	0.78	0.87
	GPH (μ = .55)	0.12	0.17	0.26	0.35	0.46	0.56	0.65	0.72	0.76	0.78
	GPH (μ = .575)	0.12	0.18	0.27	0.38	0.50	0.60	0.71	0.77	0.81	0.83
	GPH (μ = .60)	0.12	0.19	0.30	0.43	0.57	0.68	0.79	0.85	0.88	0.90
	ROB (Wh. N)	**0.16**	**0.37**	**0.66**	**0.88**	**0.97**	**0.99**	**1.00**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.26**	**0.36**	**0.51**	**0.69**	**0.84**	**0.94**	**0.98**	**0.99**	**0.99**	**0.99**
	ROB (AR (2))	**0.32**	**0.37**	**0.45**	**0.57**	**0.69**	**0.81**	**0.89**	**0.94**	**0.97**	**0.98**
	ROB (AR (3))	**0.40**	**0.43**	**0.47**	**0.55**	**0.64**	**0.73**	**0.81**	**0.87**	**0.91**	**0.94**

Power of the ADF, GDH and Robinson tests for cointegration against autoregressive alternatives

Size	Test	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05
5%	ADF (ρ = 4)	0.07	0.16	0.29	0.42	0.53	0.61	0.66	0.73	0.75	0.77
	GPH (μ = .55)	0.07	0.17	0.33	0.49	0.59	0.64	0.67	0.69	0.68	0.66
	GPH (μ = .575)	0.07	0.17	0.35	0.52	0.63	0.69	0.73	0.75	0.74	0.72
	GPH (μ = .60)	0.07	0.18	0.37	0.56	0.70	0.76	0.81	0.84	0.83	0.83
	ROB (Wh. N)	**0.07**	**0.21**	**0.46**	**0.72**	**0.90**	**0.98**	**0.99**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.18**	**0.36**	**0.59**	**0.76**	**0.88**	**0.94**	**0.97**	**0.98**	**0.99**	**0.99**
	ROB (AR (2))	**0.27**	**0.42**	**0.58**	**0.70**	**0.80**	**0.86**	**0.90**	**0.93**	**0.95**	**0.96**
	ROB (AR (3))	**0.37**	**0.49**	**0.60**	**0.69**	**0.75**	**0.80**	**0.83**	**0.86**	**0.87**	**0.88**
10%	ADF (ρ = 4)	0.14	0.28	0.46	0.60	0.71	0.78	0.82	0.86	0.88	0.89
	GPH (μ = .55)	0.14	0.29	0.50	0.66	0.75	0.78	0.81	0.82	0.81	0.79
	GPH (μ = .575)	0.14	0.30	0.52	0.69	0.78	0.82	0.85	0.86	0.85	0.84
	GPH (μ = .60)	0.14	0.30	0.54	0.72	0.82	0.87	0.90	0.91	0.92	0.91
	ROB (Wh. N)	**0.16**	**0.38**	**0.65**	**0.87**	**0.97**	**0.99**	**0.99**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.30**	**0.54**	**0.76**	**0.89**	**0.95**	**0.98**	**0.99**	**0.99**	**0.99**	**0.99**
	ROB (AR (2))	**0.39**	**0.58**	**0.74**	**0.84**	**0.90**	**0.94**	**0.96**	**0.97**	**0.98**	**0.98**
	ROB (AR (3))	**0.47**	**0.63**	**0.74**	**0.82**	**0.87**	**0.90**	**0.92**	**0.93**	**0.94**	**0.95**

*: ADF is augmented Dickey-Fuller test statistic and p is the lag parameter selected using AIC and SIC criteria. GPH is Geweke and Porter-Hudak test statistic and μ is the value used in the sample size function $n=T^{\mu}$. Results for ADF and GPH have been taken from Cheung and Lai (1993), (pages 108 and 109). The critical values of Robinson's (1994) tests with white noise disturbances were taken from Table 1, while those corresponding to AR disturbances were obtained by simulation. The power of each test is based on 10,000 replications.

TABLE 3

Testing fractional cointegration with the tests of Robinson (1994)*

Consumption (C_t) and Income (Y_t)										
Series / d_o	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
C_t	7.13	4.81	2.74	**1.00'**	**-0.40'**	**-1.49'**	-2.33	-2.98	-3.50	-3.91
Y_t	6.74	4.40	2.36	**0.65'**	**-0.73'**	**-1.80'**	-2.63	-3.27	-3.77	-4.16
$C_t - 0.52 - 0.23\ Y_t$	**0.98'**	**-0.24'**	**-1.27'**	-2.12	-2.83	-3.40	-3.87	-4.26	-4.58	-4.85
$Y_t + 0.22 - 4.30\ C_t$	**0.95'**	**-0.26'**	**-1.27'**	-2.12	-2.81	-3.39	-3.86	-4.25	-4.57	-4.84
Consumer Price Index (CPI_t) and Wages (W_t)										
Series / d_o	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
CPI_t	17.44	11.40	6.41	2.60	**-0.17'**	**-1.16'**	-3.60	-4.66	-5.47	-6.10
W_t	27.84	16.41	8.91	3.93	**0.57'**	**-1.73'**	-3.33	-4.47	-5.32	-5.97
$CPI_t - 3.91 - 0.70\ W_t$	35.07	24.95	15.55	8.10	**1.53'**	**-0.68'**	-2.97	-4.49	-5.52	-6.26
$W_t + 5.31 - 13.6\ CPI_t$	32.40	22.63	13.86	7.04	**1.22'**	**-1.02'**	-3.16	-4.60	-5.60	-6.31
Gross National Product (GNP_t) and Money ($M1_t$)										
Series / d_o	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
GNP_t	5.48	3.72	2.10	**0.70'**	**-0.44'**	**-1.35'**	-2.05	-2.60	-3.03	-3.38
$M1_t$	5.65	3.80	2.13	**0.70'**	**-0.46'**	**-1.37'**	-2.07	-2.62	-3.05	-3.39
$GNP_t + 12.1 - 1.54\ M1_t$	3.15	**1.26'**	**-0.22'**	**-1.32'**	-2.13	-2.71	-3.13	-3.44	-3.67	-3.86
$GNP_t + 12.1 - 1.54\ M1_t$	3.24	**1.31'**	**-0.19'**	**-1.31'**	-2.12	-2.70	-3.12	-3.43	-3.67	-3.85
Stock Prices (SP_t) and Dividends (D_t)										
Series / d_o	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
SP_t	6.02	4.03	2.35	**0.95'**	**-0.19'**	**-1.10'**	-2.03	-2.40	-2.86	-3.23
D_t	5.41	4.03	2.84	**1.79'**	**0.86'**	**-0.94'**	**-1.08'**	**-1.30'**	-1.96	-2.30
$SP_t + 0.12 - 30.99\ D_t$	**1.48'**	**0.21'**	**-0.79'**	**-1.58'**	-2.20	-2.68	-3.06	-3.37	-3.62	-3.83
$D_t - 0.005 - 0.027\ SP_t$	**1.10'**	**-0.02'**	**-0.90'**	**-1.60'**	-2.15	-2.60	-2.97	-3.27	-3.52	-3.74

* ' and in bold: Non-rejection values at the 95% significance level. The critical values for the cases corresponding to the OLS regressions are those given in Table 1.