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ABSTRACT

This article complements the structural New-Keynesian macro framework with a no-arbitrage affine term structure model. Whereas our methodology is general, we focus on an extended macro-model with an unobservable time varying inflation target and the natural rate of output which are filtered from macro and term structure data. We obtain large and significant estimates of the Phillips curve and real interest rate response parameters. Our model also delivers strong contemporaneous responses of the entire term structure to various macroeconomic shocks. The inflation target dominates the variation in the "level factor" whereas the monetary policy shocks dominate the variation in the "slope and curvature factors".

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# 1 Introduction

Structural New-Keynesian models, featuring dynamic aggregate supply (AS), aggregate demand (IS) and monetary policy equations are becoming pervasive in macroeconomic analysis. In this article we complement this structural macroeconomic framework with a no-arbitrage term structure model.

Our analysis overcomes three deficiencies in previous work on New-Keynesian macro models. First, the parsimony of such models implies very limited information sets for both the monetary authority and the private sector. It is well known, however, that monetary policy is conducted in a data-rich environment. Recent research by Bernanke and Boivin (2003) and Bernanke, Boivin and Elias (2005) collapses multiple observable time series into a small number of factors and embeds them in standard vector autoregressive (VAR) analyses. In this article we use perhaps the most efficient information of all, term structure data. The critical variables in most macro models are the output gap, expected inflation and a short-term interest rate. It is unlikely that lags of inflation, the output gap and the short-term interest rate suffice to adequately forecast their future behavior. However, under the null of the Expectations Hypothesis, term spreads embed all relevant information about future interest rates. Additionally, a host of studies have shown that term spreads are very good predictors of future economic activity (see, for instance, Harvey (1988), Estrella and Mishkin (1998), Ang, Piazzesi and Wei ((forthcoming))) and of future inflation (Mishkin (1990) or Stock and Watson (2003)). In our proposed models, the conditional expectations of inflation and the de-trended output are a function of the past realizations of macro variables and of unobserved components which are extracted from term structure data through a no-arbitrage pricing model.

Second, the additional information from the term structure model transforms a version of a New-Keynesian model with a number of unobservable variables into a

very tractable linear model which can be efficiently estimated by maximum likelihood or the general method of moments (GMM). Hence, the term structure information helps recover important structural parameters, such as those describing the monetary transmission mechanism, in an econometrically efficient manner.

Third, incorporating term structure information leads to a simple VAR on macro variables and term spread information but the reduced-form model for the macro variables is a complex VARMA model. This is important because one disadvantage of most structural New-Keynesian models is the absence of sufficient endogenous persistence. We generate additional channels of endogenous persistence by introducing unobservable variables in the macro model which must be identified from the term structure.

The approach set forth in this paper also contributes to the term structure literature. In this literature it is common to have latent factors drive most of the dynamics of the term structure of interest rates. These factors are often interpreted ex-post as level, slope and curvature factors. A classic example of this approach is Dai and Singleton (2000), who construct an arbitrage-free three factor model of the term structure.<sup>1</sup> While the Dai and Singleton (2000) model provides a satisfactory fit of the data, it remains silent about the economic forces behind the latent factors. In contrast, we construct a no-arbitrage term structure model where all the factors have a clear economic meaning. Apart from inflation, the de-trended output and the short term interest rate, we introduce two unobservable variables in the underlying macro model. While there are many possible implementations, our main application here introduces a time-varying inflation target and natural rate of output. Consequently, we construct a 5 factor affine term structure model that obeys New-Keynesian structural relations.

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<sup>1</sup>Other examples include Knez, Litterman and Scheinkman (1994) and Pearson and Sun (1994).

Our main empirical findings are as follows. First, the model matches the persistence displayed by the three macro variables despite being nested in a parsimonious VAR(1) for macro variables and term spreads. Second, in contrast to previous maximum likelihood (MLE) or GMM estimations of the standard New-Keynesian model, we obtain large and significant estimates of the Phillips curve and real interest rate response parameters. Third, our model exhibits strong contemporaneous responses of the entire term structure to the various structural shocks in the model.

Our article is part of a rapidly growing literature exploring the relation between the term structure and macro economic dynamics. Kozicki and Tinsley (2001) and Ang and Piazzesi (2003) were among the first to incorporate macroeconomic factors in a term structure model to improve its fit. Evans and Marshall (2004) use a VAR framework to trace the effect of macroeconomic shocks on the yield curve whereas Dewachter and Lyrío ((forthcoming)) assign macroeconomic interpretations to standard term structure factors. Our paper differs from these articles in that all the macro variables obey a set of structural macro relations. This facilitates a meaningful economic interpretation of the term structure dynamics. For instance, we can trace not only the impact of macroeconomic shocks but also of changes in the behavior of the private sector and the monetary authority on the term structure. Moreover, the implied interactions between macro and term structure factors are more general in our framework than in the articles we mentioned. Diebold, Rudebusch and Aruoba ((forthcoming)) empirically characterize the dynamic interactions between the macro economy and the term structure. They find that macro factors have strong effects on future movements in interest rates and that the reverse effect is much weaker, which seems to contradict some of the earlier work on term structure based forecasts of output and inflation. In our framework, we can explore the structural origin of these dynamic interactions. Our effort is contemporaneous to articles by Rudebusch and Wu (2004) and Hordahl, Tristani and Vestin (2004) who also append a term structure

model to a New-Keynesian macro model. We discuss how our approach differs from theirs below.

The remainder of the paper is organized as follows. Section 2 describes the structural macroeconomic model, whereas Section 3 outlines how to combine the macro model with an affine term structure model. Section 4 provides some preliminary data analysis. Section 5 discusses the estimation methodology and considers the fit of different model variants with the data. Section 6 analyzes the macroeconomic implications while section 7 studies the term structure implications of our model. Section 8 concludes.

## **2 New-Keynesian Macro Models with Unobservable State Variables**

We present a standard New-Keynesian model featuring AS, IS and monetary policy equations with two additions. First, we assume the existence of a natural rate of output which follows a potentially persistent stochastic process. Second, the inflation target is assumed to vary through time according to a persistent linear process. The monetary authorities react to the output gap which is the deviation of output from the natural rate of output. We allow for endogenous persistence in the AS, IS and monetary policy equations. The resulting model requires numerical techniques to solve for the linear Rational Expectations (RE) equilibrium and the equilibrium may not be unique. Our solution approach closely follows Cho and Moreno (2004). This method essentially selects the solution that generates a stationary equilibrium. In what follows, we describe each equation in turn and describe the model solution. In the appendix we describe the microfoundations of the AS and IS equations. Related theoretical derivations can be found in Clarida, Galí and Gertler (1999) or Woodford

(2003).

## 2.1 The IS Equation

A standard intertemporal IS equation is usually derived from the first-order conditions for a representative agent with power utility as in the original Lucas (1978) economy. Standard estimation approaches have experienced difficulty pinning down the risk aversion parameter, which is at the same time an important parameter underlying the monetary transmission mechanism. Another discomfoting feature implied by a standard IS equation is that it typically fails to match the well-documented persistence of output. We derive an alternative IS equation from a utility maximizing framework with external habit formation similar to Fuhrer (2000). We also explicitly model labor supply in the utility function but assume additive separability. In particular, we assume that the representative agent maximizes:

$$E_t \sum_{s=t}^{\infty} \psi^{s-t} U(C_s, N_s; F_s) = E_t \sum_{s=t}^{\infty} \psi^{s-t} \left[ \frac{F_s C_s^{1-\sigma} - 1}{1-\sigma} - \frac{N_s^{1+\chi}}{1+\chi} \right] \quad (1)$$

where  $C_t$  is the composite index of consumption,  $N_t$  is the labor supply,  $F_t$  represents an aggregate demand shifting factor;  $\psi$  denotes the time discount factor,  $\sigma$  is the inverse of intertemporal elasticity of consumption and  $\chi$  represents the inverse of elasticity of labor supply. We specify  $F_t$  as follows:

$$F_t = H_t G_t \quad (2)$$

where  $H_t$  is an external habit level, that is, the agent takes  $H_t$  as exogenously given, even though it may depend on past consumption.  $G_t$  is an exogenous aggregate demand shock that can also be interpreted as a preference shock. Following Fuhrer (2000), we assume that  $H_t = C_{t-1}^\eta$  where  $\eta$  measures the degree of habit dependence

on the past consumption level. It is this assumption that delivers endogenous output persistence.

Since labor supply does not affect the marginal utility of consumption, we show in the appendix that the Fuhrer-type IS equation is preserved:

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1 - \mu)y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \quad (3)$$

where  $i_t$  is the short term interest rate;  $\phi = \frac{1}{\sigma + \eta}$  and  $\mu = \sigma\phi$ . The IS shock,  $\epsilon_{IS,t} = \phi \ln G_t$ , is assumed to be independently and identically distributed with homoskedastic variance  $\sigma_{IS}^2$ .

## 2.2 The AS Equation (Phillips Curve)

The standard New-Keynesian aggregate supply (AS) curve relates inflation to the output gap. The Calvo (1983) pricing model implies a positive relation between real marginal cost and inflation. Even though under certain conditions the output gap is proportional to marginal cost, in practice, structural estimates of the Phillips curve based on output gap measures seem less successful than those based on marginal cost (see Galí and Gertler (1999)). Moreover, the standard AS curve fails to account for the observed inflation persistence. By assuming that the fraction of price-setters which does not adjust prices optimally, indexes their prices to past inflation, we obtain endogenous persistence in the AS equation:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta)\pi_{t-1} + \varpi \hat{s}_t \quad (4)$$

where  $\pi_t$  is inflation and  $\hat{s}_t$  is the deviation of log of real marginal cost from its steady state level.  $\varpi$  captures the short run tradeoff between inflation and the real marginal cost and  $(1 - \delta)$  characterizes the endogenous persistence of inflation.



We consider two specifications for the real marginal cost. First, we assume that real marginal cost is proportional to the current output gap, which is standard in the literature. Second, the real marginal cost is assumed proportional to the past output gap as well as the current output gap. This specification follows from considering an explicit labor market equilibrium condition coupled with external habit. We refer to Appendix B for the derivation. Consequently, we analyze the following two AS curves:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \kappa(y_t - y_t^n) + \epsilon_{AS,t} \quad (5)$$

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \kappa((y_t - y_t^n) - \lambda(y_{t-1} - y_{t-1}^n)) - \zeta \epsilon_{IS,t} + \epsilon_{AS,t} \quad (6)$$

where  $y_t$  is de-trended output,  $y_t^n$  is the natural rate of output that would prevail in the case of perfectly flexible prices and  $\epsilon_{AS,t}$  is a negative technology shock with standard deviation  $\sigma_{AS}$ . The IS shock,  $\epsilon_{IS,t}$ , enters in the AS equation because real marginal cost depends on the exogenous aggregate demand shock,  $G_t$ . The parameter  $\lambda$  captures the endogenous persistence of output and equals  $\frac{\eta}{\sigma + \chi}$  in terms of the structural parameters. It is the one-period lag structure in the external habit specification that drives this result. Appendix C shows how the model produces endogenously the dynamics for  $y_t^n$ :

$$y_t^n = \alpha_{y^n} + \lambda y_{t-1}^n + \epsilon_{y^n,t} \quad (7)$$

where  $\alpha_{y^n}$  is a constant and  $\epsilon_{y^n,t}$  can be interpreted as a negative markup shock with standard deviation  $\sigma_{y^n}$ .

## 2.3 The Monetary Policy Rule

We assume that the monetary authority specifies the nominal interest rate target,  $i_t^*$ , as in the forward-looking Taylor rule proposed by Clarida et al. (1999):

$$i_t^* = [\bar{i}_t + \beta(E_t\pi_{t+1} - \pi_t^*) + \gamma(y_t - y_t^n)] \quad (8)$$

where  $\pi_t^*$  is a time-varying inflation target and  $\bar{i}_t$  is the desired level of the nominal interest rate that would prevail when  $E_t\pi_{t+1} = \pi_t^*$  and  $y_t = y_t^n$ . We present three alternative specifications for  $\bar{i}_t$ :

$$\bar{i}_t = \bar{r} + E_t\pi_{t+1} \quad (9)$$

$$\bar{i}_t = \bar{r}_t + E_t\pi_{t+1} \quad (10)$$

$$\bar{i}_t = \bar{i} \quad (11)$$

Specification (9) makes the desired nominal rate consistent with the Fisher hypothesis, so that the real rate in the IS equation is a constant whenever inflation hits the target and output equals potential. Because expected inflation enters the desired nominal rate in specifications (9) and (10), the long-run response of the interest rate to expected inflation - a typical measure of the Fed's stance against inflation - is  $1 + \beta$ .<sup>2</sup> Equation (10) is analogous to (9), except that the desired rate of interest,  $\bar{r}_t$ , is allowed to vary over time. We specify  $\bar{r}_t$  as in Woodford (2003), so that:

$$\bar{r}_t = \frac{1}{\phi} [\mu E_t y_{t+1}^n + (1 - \mu)y_{t-1}^n - y_t^n]. \quad (12)$$

That is, the desired rate of interest follows from the IS equation evaluated at the natural rate of output. The desired real rate thus coincides with the Wicksellian real

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<sup>2</sup>Replacing  $E_t\pi_{t+1}$  by  $\pi_t^*$  in equations (9) and (10) leads to observationally equivalent rules where this long-run response is captured by  $\beta$ .

rate, the real rate at which there are no monetary pressures on either the output gap or inflation. Finally, equation (11) specifies a constant desired nominal interest rate.

Our motivation for considering alternative monetary policy specifications is twofold. First, different researchers have estimated alternative monetary policy rules. For instance, Rudebusch and Wu (2004) use our specification (9), whereas Hordahl et al. (2004) employ equation (11). We can assess whether alternative specifications imply different estimates for the structural parameters or different dynamics for the macro and term structure variables and which specification fits the data better. Second, our study lets both the desired real rate and the inflation target vary through time, so that the desired nominal rate variation can stem from alternative sources.

We further assume that the monetary authority sets the short term interest rate as a weighted average of the interest rate target and a lag of the short term interest rate to capture the tendency by central banks to smooth interest rate changes (see Clarida et al. (1999)):

$$i_t = \rho i_{t-1} + (1 - \rho) i_t^* + \epsilon_{MP,t} \quad (13)$$

where  $\rho$  is the smoothing parameter and  $\epsilon_{MP,t}$  is an exogenous monetary policy shock, assumed to be *i.i.d.* with standard deviation,  $\sigma_{MP}$ . The resulting monetary policy rule for the interest rate is given by:

$$i_t = \rho i_{t-1} + (1 - \rho) [\bar{i}_t + \beta (E_t \pi_{t+1} - \pi_t^*) + \gamma (y_t - y_t^n)] + \epsilon_{MP,t} \quad (14)$$

## 2.4 Inflation Target $\pi_t^*$

We close our model by specifying a stochastic process for the inflation target,  $\pi_t^*$ . Little is known about how the monetary authority sets the inflation target. Presumably, the inflation target is anchored in the expectations of long-run inflation by the

private sector, in addition to some exogenous information. Therefore, we define  $\pi_t^{LR}$  as the conditional expected value of a weighted average of all future inflation rates.

$$\pi_t^{LR} = (1 - d) \sum_{j=0}^{\infty} d^j E_t \pi_{t+j} \quad (15)$$

with  $0 \leq d \leq 1$ . This equation can be succinctly written as:

$$\pi_t^{LR} = dE_t \pi_{t+1}^{LR} + (1 - d)\pi_t \quad (16)$$

When  $d$  equals 0,  $\pi_t^{LR}$  collapses to current inflation, when  $d$  approaches 1, long-run inflation approaches unconditional expected inflation. We assume that the monetary authority anchors its inflation target around  $\pi_t^{LR}$ , but smooths target changes, so that:

$$\pi_t^* = \omega \pi_{t-1}^* + (1 - \omega) \pi_t^{LR} + \epsilon_{\pi^*, t} \quad (17)$$

We view  $\epsilon_{\pi^*, t}$  as an exogenous shift in the policy stance regarding the long term rate of inflation or the target, and assume it to be *i.i.d.* with standard deviation  $\sigma_{\pi^*}$ . Substituting out  $\pi_t^{LR}$  in Equation (17) using equation 16, we obtain:

$$\pi_t^* = \varphi_1 E_t \pi_{t+1}^* + \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_t + \epsilon_{\pi^*, t} \quad (18)$$

where  $\varphi_1 = \frac{d}{1+d\omega}$ ,  $\varphi_2 = \frac{\omega}{1+d\omega}$  and  $\varphi_3 = 1 - \varphi_1 - \varphi_2$ .

We call equation (18) the new inflation target specification. We also consider an inflation target specification which is quite popular in the literature:

$$\pi_t^* = \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_{t-1} + \epsilon_{\pi^*, t} \quad (19)$$

This equation can be theoretically justified postulating that long-run inflation is

formed in a backward-looking manner.<sup>3</sup> We call equation (19) the standard inflation target specification.

## 2.5 The Full Model

Bringing together all the equations, we have a 5 variable system with three observed and two unobserved macro factors:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \kappa [(y_t - y_t^n) - \lambda DU_t (y_{t-1} - y_{t-1}^n)] - DU_t \zeta \epsilon_{IS,t} + \epsilon_{AS,t} \quad (20)$$

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi (i_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \quad (21)$$

$$i_t = \alpha_{MP} + \rho i_{t-1} + (1 - \rho) [\bar{i}_t + \beta (E_t \pi_{t+1} - \pi_t^*) + \gamma (y_t - y_t^n)] + \epsilon_{MP,t} \quad (22)$$

$$y_t^n = \alpha_{y^n} + \lambda y_{t-1}^n + \epsilon_{y^n,t} \quad (23)$$

$$\pi_t^* = \varphi_1 E_t \pi_{t+1}^* + \varphi_2 \pi_{t-1}^* + \varphi_3 \Upsilon_t + \epsilon_{\pi^*,t} \quad (24)$$

For the  $\pi_t^*$  process, we either have  $\Upsilon_t = \pi_t$  or we set  $\varphi_1 = 0$  and  $\Upsilon_t = \pi_{t-1}$ . There are two specifications for  $\hat{s}_t$  depending on whether  $DU_t = 1$  or  $DU_t = 0$  and three specifications for  $\bar{i}_t$ . Consequently, we have a total of 12 possible models. They can be expressed in matrix form as:

$$Bx_t = \alpha + AE_t x_{t+1} + Jx_{t-1} + C\epsilon_t \quad (25)$$

where  $x_t = [\pi_t \ y_t \ i_t \ y_t^n \ \pi_t^*]'$  and  $\epsilon_t = [\epsilon_{AS,t} \ \epsilon_{IS,t} \ \epsilon_{MP,t} \ \epsilon_{y^n,t} \ \epsilon_{\pi^*,t}]'$ .  $\alpha$  is a  $5 \times 1$  vector of constants and  $B, A, J, C$  are appropriately defined  $5 \times 5$  matrices. The Rational

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<sup>3</sup>In particular, when  $\pi_t^{LR} = (1 - d) \sum_{j=0}^{\infty} d^j \pi_{t-j-1}$  and  $\pi_t^* = \pi_t^{LR} + \epsilon_{\pi^*,t}$ , the inflation target follows this process:  $\pi_t^* = d\pi_{t-1}^* + (1 - d)\pi_{t-1} + \epsilon_{\pi^*,t}$

Expectations (RE) equilibrium can be written as a first-order VAR:

$$x_t = c + \Omega x_{t-1} + \Gamma \epsilon_t \tag{26}$$

Hence, the implied model dynamics are a simple VAR subject to a set of non-linear restrictions.<sup>4</sup> Note that  $\Omega$  cannot be solved analytically in general. We solve for  $\Omega$  numerically using the QZ method (see Klein (2000) and Cho and Moreno (2004)). Once  $\Omega$  is solved,  $\Gamma$  and  $c$  follow straightforwardly.

It can be shown that the reduced-form representation of the vector of observable macro variables follows a VARMA(3,2) process. By adding unobservables, we potentially deliver more realistic joint dynamics for inflation, the output gap and the interest rate, and overcome the lack of persistence implied by previous studies.

### 3 Incorporating Term Structure Information

We derive the term structure model implicit in the IS curve that we presented in section 2. In contrast, Rudebusch and Wu (2004) and Hordahl et al. (2004) formulate exogenous kernels, not linked to a utility function. This effort results in an easily estimable linear system in observable macro variables and term structure spreads.

#### 3.1 Affine Term Structure Models with New-Keynesian Factor Dynamics

Affine term structure models require linear state variables dynamics and a linear pricing kernel process with conditionally normal shocks (see Duffie and Kan (1996)).

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<sup>4</sup>A closed-form solution is available when the model is fully forward-looking. Because the observed macro factors do not Granger-cause the term structure in this case the model is empirically uninteresting.

For the state variable dynamics implied by the New-Keynesian model in equation (26) to fall in the affine class, we assume that the shocks are conditionally normally distributed,  $\epsilon_t \sim N(0, D_{t-1})$ . The pricing kernel process  $M_{t+1}$  prices all securities such that:

$$E_t[M_{t+1}R_{t+1}] = 1 \quad (27)$$

In particular, for an  $n$ -period bond,  $R_{t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}$  with  $P_{n,t}$  the time  $t$  price of an  $n$ -period zero-coupon bond. If  $M_{t+1} > 0$  for all  $t$ , the resulting returns satisfy the no-arbitrage condition (Harrison and Kreps (1979)). In affine models, the log of the pricing kernel is modelled as a conditionally linear process. Consider, for instance:

$$m_{t+1} = \ln(M_{t+1}) = -i_t - \frac{1}{2}\Lambda_t' D_t \Lambda_t - \Lambda_t' \epsilon_{t+1} \quad (28)$$

Here  $\Lambda_t = \Lambda_0 + \Lambda_1 x_t$ , where  $\Lambda_0$  is a  $5 \times 1$  vector and  $\Lambda_1$  is a  $5 \times 5$  matrix. First, setting  $D_t = D$ , we obtain a Gaussian price of risk model. Dai and Singleton (2002) study such a model and claim that it accounts for the deviations of the Expectations Hypothesis (EH) observed in U.S. term structure data. An alternative model sets  $\Lambda_t = \Lambda$  and  $\epsilon_t \sim N(0, D_{t-1})$  with  $D_t = D_0 + D_1 \text{diag}(x_t)$ , where  $\text{diag}(x_t)$  is the diagonal matrix with the vector  $x_t$  on its diagonal. This model introduces heteroskedasticity of the square-root form and has a long tradition in finance (see Cox, Ingersoll and Ross (1985)). Finally, setting  $\Lambda_t = \Lambda_0$  and  $D_t = D$  results in a homoskedastic model.

All three of these models imply an affine term structure. That is, log bond prices,  $p_{n,t}$  are an affine function of the state variables. The maturity-dependent coefficients follow Ricatti difference equations. The three models have different implications for the behavior of term spreads and holding period returns. First, the homoskedastic model implies that the EH holds: there may be a term premium but it does not vary through time. Both the Gaussian prices of risk model and the square root model imply time-varying term premiums. Second, our model includes inflation as a state

variable and the real pricing kernel (the kernel that prices bonds perfectly indexed against inflation) and inflation are correlated. It is this correlation that determines the inflation risk premium. If the covariance term is constant, the risk premium is constant over time and this will be true in a homoskedastic model.

The kernel model implied by the IS curve derived above fits in the homoskedastic class. It is possible to modify the pricing framework into one of the two other models, but we defer this to future work. Bekaert, Hodrick and Marshall (2001) show that a model with minimal variation in the term premium suffices to match the evidence regarding the Expectations Hypothesis for the US.

### 3.2 The term structure model implied by the macro model

Because our derivation of the IS curve assumed a particular preference structure, the pricing kernel is given by the intertemporal consumption marginal rate of substitution of the model. That is:

$$m_{t+1} = \ln \psi - \sigma y_{t+1} + (\sigma + \eta)y_t - \eta y_{t-1} + (g_{t+1} - g_t) - \pi_{t+1} \quad (29)$$

The no-arbitrage condition holds by construction. In a log-normal model, pricing a one period bond implies

$$E_t[m_{t+1}] + 0.5V_t[m_{t+1}] = -i_t \quad (30)$$

Hence, we can express the pricing kernel as:

$$m_{t+1} = -i_t - \frac{1}{2}\Lambda'D\Lambda - \Lambda'\epsilon_{t+1} \quad (31)$$



where  $\Lambda$  is a vector of prices of risk entirely restricted by the structural parameters,

$$\Lambda' = [1 \ \sigma \ 0 \ 0 \ 0]\Gamma - [0 \ (\sigma + \eta) \ 0 \ 0 \ 0] \quad (32)$$

The bond pricing equation is affine:

$$p_{n,t} = a_n + b_n' x_t \quad (33)$$

where, by log-normality,

$$p_{n,t} = E_t(m_{t+1} + p_{n-1,t+1}) + \frac{1}{2} V_t(m_{t+1} + p_{n-1,t+1}) \quad (34)$$

Using an induction argument and equations (26) and (31), we find:

$$\begin{aligned} a_n &= a_{n-1} + b_{n-1}' c + 0.5 b_{n-1}' \Gamma D \Gamma' b_{n-1} - \Lambda' D \Gamma' b_{n-1} \\ b_n' &= -e_3' + b_{n-1}' \Omega \end{aligned}$$

with  $e_3$  a  $5 \times 1$  vector of zeros with a 1 in the third row. Therefore, the bond yields are an affine function of the state variables:

$$y_{n,t} = -\frac{a_n}{n} - \frac{b_n'}{n} x_t \quad (35)$$

Term spreads are also an affine function of the state variables:

$$sp_{n,t} = -\frac{a_n}{n} - \left(\frac{b_n}{n} + e_3\right)' x_t \quad (36)$$

where  $sp_{n,t} \equiv -\frac{p_{n,t}}{n} - i_t$  is the spread between the  $n$  period yield and the short rate.

This model provides a particular convenient form for the joint dynamics of the macro variables and the term spreads. Let  $z_t = [\pi_t \ y_t \ i_t \ sp_{n_1,t} \ sp_{n_2,t}]'$ , where  $n_1$  and  $n_2$

refer to two different yield maturities for the long-term bond in the spread. Then

$$x_t = c + \Omega x_{t-1} + \Gamma \epsilon_t \quad (37)$$

$$z_t = A_z + B_z x_t \quad (38)$$

where

$$A_z = \begin{bmatrix} 0_{3 \times 1} \\ -\frac{a_{n_1}}{n_1} \\ -\frac{a_{n_2}}{n_2} \end{bmatrix}, B_z = \begin{bmatrix} I_3 & 0_{3 \times 2} \\ -(\frac{b_{n_1}}{n_1} + e_3)' \\ -(\frac{b_{n_2}}{n_2} + e_3)' \end{bmatrix}$$

Using  $x_t = B_z^{-1}(z_t - A_z)$ , we find:

$$z_t = a_z + \Omega_z z_{t-1} + \Gamma_z \epsilon_t \quad (39)$$

where

$$\begin{aligned} \Omega_z &= B_z \Omega B_z^{-1} \\ \Gamma_z &= B_z \Gamma \\ a_z &= B_z c + (I - B_z \Omega B_z^{-1}) A_z \end{aligned}$$

In other words, the macro variables and the term spreads follow a first-order VAR with complex cross-equation restrictions.

## 4 A First Look at the Data

### 4.1 Data Description

The sample period is from the first quarter of 1961 to the fourth quarter of 2003. We measure inflation with the CPI (collected from the Bureau of Labor Statistics)

but check robustness using the GDP deflator, from the National Income and Product Accounts (NIPA). We measure de-trended output as linearly de-trended output. Output is real GDP from NIPA. We use the 3-month T-bill rate, taken from the Federal Reserve of St. Louis database, as the short-term interest rate. Finally, our analysis uses term-structure data at the one, three and five year maturities from the CRSP database.

## 4.2 Macro and Term Structure Interactions

If term spreads indeed predict macro variables, expanding the agents' information set adding term structure information may lead to a more accurate estimation of the structural parameters. Diebold et al. ((forthcoming)) stress the strength of the predictive power of macro variables for term structure variables. Consequently, we perform Granger-causality (GC) tests in both directions. To identify macroeconomic shocks and structural parameters, it is at least as important to correctly identify contemporaneous correlations. Therefore, we also assess the significance of contemporaneous projection coefficients.

We investigate a first-order VAR containing inflation, de-trended output, the interest rate and two term spreads for the 3 and 5 year maturities. Table 1 reports the results for two inflation measures, the CPI and GDP deflator. The Granger causality results are rather mixed. Term spreads Granger-cause inflation when it is measured using the CPI index, but not when it is measured using the GDP deflator. There is no evidence of term spreads predicting output.<sup>5</sup> In contrast, the evidence of macro variables Granger-causing the long term spreads is more significant and uniform. This conclusion remains valid if the interest rate is omitted from the macro variables. The evidence for significant contemporaneous correlations is much stronger and most of

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<sup>5</sup>Studies finding that spreads predict future output typically use output growth rather than de-trended output (Estrella and Mishkin (1998)).

our tests reject the null of no correlation at the 5% level. This suggests that there are indeed important interactions between macro and term structure variables, but they may not necessarily be the ones most stressed in the literature to date, which primarily focused on feedback parameters.

### 4.3 Persistence

One motivation for introducing additional unobserved variables into a standard macro-model is to generate more persistent dynamics. Estimates of empirical VAR models use many lags, sometimes as many as 12, which leads to over-parameterized systems. However, structural macro-models have difficulty generating endogenous persistence sufficient to match the persistence in the data.

In our model the unobserved variables also inject more persistence into the endogenous dynamics for inflation and the output gap. We can provide some preliminary data-based motivation for why our approach may be successful. First, consider a standard lag selection criterion, in particular the Schwarz criterion (BIC). We contrast the number of lags the BIC criterion would select in an empirical VAR system with only macro variables (inflation, the output gap and the interest rate) with how many lags would be necessary for a VAR that also embeds term spreads. We find that the optimal lag length for the macro system is 2 (the Akaike's criterion selects 3), requiring the estimation of 18 feedback parameters. If we look at individual equations, the BIC criterion selects three lags for de-trended output and the interest rate. Hence, we take as our empirical benchmark, a VAR of the three macro variables with three lags, which we call the macro VAR(3). Our model can potentially replicate its dynamics, because the reduced form for the macro variables implied by our model is a VARMA(3,2). When we add the term spreads, the optimal lag length selected by BIC is 1 (the Akaike criterion selects 2). In individual equations, the BIC criterion

selects two lags only for the inflation and output equations. Consequently, the empirical evidence is generally consistent with our model, which has a first-order VAR reduced form when the term spreads are added. Of course, our model tries to fit the feedback dynamics of the system (which unconstrained has 25 parameters) in a structural fashion, using only 10 parameters.

Second, we investigate the autocorrelograms of the data directly. Panel A of Table 2 produces the empirical autocorrelograms of the 5 variables and Panel B shows the autocorrelogram implied by an unconstrained first order VAR. Note that the inflation and interest rate autocorrelograms decay slower than what is implied for a first-order autoregressive model. However, for both output and the term spreads the opposite is true. The unconstrained first-order VAR still fails to fully match these patterns, but it is possible that our structural model will perform better.

## 5 Estimation and Model Fit

In this section, we first present the general estimation methodology and then analyze the goodness of fit of the various models we estimate.

### 5.1 Estimation Methodology

All the models we consider imply a first-order VAR on  $z_t$ , with complex cross-equation, non-linear restrictions. Because we are not interested in the drifts, we perform the estimation on de-meaned data,  $\bar{z}_t = z_t - \hat{E}z_t$  with  $\hat{E}z_t$  the sample mean of  $z_t$ . The structural parameters to be estimated are therefore  $\theta = (\delta \ \kappa \ \sigma \ \eta \ \rho \ \beta \ \gamma \ \lambda \ \omega \ d \ \sigma_{AS} \ \sigma_{IS} \ \sigma_{MP} \ \sigma_{y^n} \ \sigma_{\pi^*})$ . Assuming normal errors, it is straightforward to write down the likelihood function for this problem and produce Full Information Likelihood Estimates (FIML) estimates. To accommodate possible deviations from the strong

normality and homoskedasticity assumptions underlying maximum likelihood, we use a version of GMM instead. To do so, re-write the model in the following form:

$$\bar{z}_t = \Omega_z \bar{z}_{t-1} + \Gamma_z \epsilon_t = \Omega_z \bar{z}_{t-1} + \Gamma_z \Sigma u_t \quad (40)$$

where  $u_t = \Sigma^{-1} \epsilon_t \sim (0, I_5)$  and  $\Sigma = \text{diag}([\sigma_{AS} \ \sigma_{IS} \ \sigma_{MP} \ \sigma_{y^n} \ \sigma_{\pi^*}]')$ , that is  $\Sigma^2 = D$ .

To construct the moment conditions, consider the following vector valued processes:

$$h_{1,t} = u_t \otimes \bar{z}_{t-1} \quad (41)$$

$$h_{2,t} = \text{vech}(u_t u_t' - I_5) \quad (42)$$

$$h_t = [h'_{1,t} \ h'_{2,t}]' \quad (43)$$

where  $\text{vech}$  represents an operator stacking the elements on or below the principle diagonal of a matrix. The model imposes  $E[h_t] = 0$ . The 25  $h_{1,t}$  moment conditions capture the feedback parameters; the 15  $h_{2,t}$  moment conditions capture the structure imposed by the model on the variance-covariance matrix of the innovations. Rather than using an initial identity matrix as the weighting matrix, which may give rise to poor first-stage estimates, we use a weighting matrix implied by the model under normality. That is under the null of the model, the weighting matrix must be:

$$W_t = (E[h_t h_t'])^{-1} \quad (44)$$

Using normality and the error structure implied by the model, it is then straightforward to show that the optimal weighting matrix is given by:

$$\hat{W}_0 = \begin{bmatrix} I \otimes \frac{1}{T} \sum_{t=1}^T \bar{z}_{t-1} \bar{z}'_{t-1} & 0_{25 \times 15} \\ 0_{15 \times 25} & I_{15} + \text{vech}(I_5) \text{vech}(I_5)' \end{bmatrix}^{-1} \quad (45)$$

This weighting matrix does not depend on the parameters. Then we minimize the standard GMM objective function:

$$Q = \left( \hat{E}[h_t] \right)' \hat{W}_0 \left( \hat{E}[h_t] \right) \quad (46)$$

where  $\hat{E}[h_t] = \frac{1}{T} \sum_{t=1}^T h_t$ . This gives rise to estimates that are quite close to what would be obtained with maximum likelihood. Given these estimates, we produce a second-stage weighting matrix allowing for heteroskedasticity and 5 Newey-West (Newey and West (1987)) lags in constructing the variance covariance matrix of the orthogonality conditions. We iterate this system until convergence. This estimation proved overall rather robust with parameter estimates varying little after the first round, except for the micro-founded model, where we failed to obtain convergence for all of the model variants but one. Hence, we consider a total of 7 specifications.

## 5.2 Model Fit

Before we examine the goodness of fit, Table 3 proposes a taxonomy for the models we estimated. There are 3 different Taylor rule specifications and two inflation target specifications. All the acronyms in the table refer to the standard AS curve model. Because there is only one micro-founded model that converged, we simply add the MF qualifier for this model. The micro-founded model that converged has a constant desired nominal rate and the new inflation target specification.

The standard GMM test of the over-identifying restrictions follows a  $\chi^2$  distribution with 25 degrees of freedom because there are 40 moment conditions but only 15 parameters. We find that the test fails to reject all models at the 5% level when 5 Newey-West lags are used in the construction of the weighting matrix (the p-value is around 17%), but rejects strongly when only three Newey-West lags are used. This in

itself suggests that the orthogonality conditions still display substantial persistence. The test values are similar across models and hence do not yield a useful criterion to differentiate models.

We consider several points statistics that compare the fit of the model with different features of the dynamics of the data. There is not a clear “winning” model, with certain models fitting particular aspects of the data better than others. For example, the micro-founded model does best in matching the feedback coefficients of a first-order VAR but does less well in matching individual autocorrelograms. Overall, it is one of the top two models, together with the [CR,EI,N] model. Let’s illustrate the performance of the various models with respect to important features of the data. We start with persistence.

Table 4 focusses on how well the model fits the autocorrelogram of the 5 variables in  $z_t$ . For each variable we set a loss function in terms of the autocorrelogram fit as follows:  $0.5 (\rho_1 - \rho_{1,mod}) + 0.3 (\rho_4 - \rho_{4,mod}) + 0.2 (\rho_8 - \rho_{8,mod})$ , where  $\rho_j$  represent data correlations at the j-th lag and *mod* refers to the autocorrelations implied by the model. Our point statistic is the square of this value divided by an estimate of its standard deviation. The table produces the statistic values plus a p-value based on the  $\chi^2(1)$  distribution. For inflation, none of the models does particularly well. The output gap persistence is hard to fit too, but models [MF], [CR,EI,N] and [TVR,EI,N] do a reasonable job. The lowest function value is for model [TVR,EI,N]. Most models have no problem fitting interest rate dynamics. The lowest function value occurs for model [CR,CI,N]. The spread dynamics are hard to fit, but model [CR,CI,N] again does very well for both spreads. Nevertheless, model [CR,EI,N] does even better for the 5-year spread. Model [CR,EI,N] is the only model that fails to reject at the 5% level in 4 out of 5 cases. This is the model we will focus on in what follows.

To view the fit of persistence differently, Figure 1 compares the autocorrelograms



implied by the model [CR,EI,N] and the macro VAR(3). The macro VAR(3) fits the data very well, but our model also manages to generate very slow decay. For the inflation process, this comes at the cost of a too high first-order autocorrelation coefficient. Nevertheless, the autocorrelogram for each of the macro variables implied by the model is within the 95 % confidence interval around the data correlogram.

We also explore whether the various models replicate the dynamic behavior of  $z_t$ . In Table 5, we compare the reduced form feedback coefficients for model [CR,EI,N] with their counterparts for an unconstrained VAR(1) model. The model captures the relative magnitude of the diagonal elements quite well, generating strong autocorrelation feedback for the output gap, the interest rate and the five-year spread and less so for inflation and the three-year spread. Most of the off-diagonal elements are insignificantly different from zero in the data. Nevertheless, de-trended output has predictive power for all of the term structure variables, including the interest rate, and the model reproduces this feature near perfectly. The data also show strong cross-feedback between the two term spreads. The model gets the sign right, but makes the effects even larger. Whereas some other feedback coefficients do not appear to be as well matched, they are mostly not significant. Moreover, comparing the magnitude of individual coefficients may be misleading. For instance, the two term spread predict inflation and interest rates with the wrong sign, but the spreads are highly correlated so the joint effect is likely more important.

To see this more clearly, Table 6 shows correlation coefficients between forecasts using a VAR(1) and model-based forecasts at different horizons. For ease of interpretation, we average the forecast correlations over different horizons, producing short-term, medium-term and long-term correlations. The results are similar across models. Short-run projections by the model are highly correlated with those by a VAR for all five variables. That correlation decreases monotonically for all variables at medium and long horizons. Whereas the correlations of the spread projections be-

come negative at long-horizons (8-10 quarters) the correlations remain very high for interest rates and moderately high for inflation. For the output gap, the correlation is very much model dependent, with the MF model and the standard inflation target specification performing the best. When we compute a quadratic statistic based on the deviations of the model feedback parameters from the VAR(1) parameters using the inverse of the empirical variance-covariance matrix as the weighting matrix, the MF model performs best, followed by the [CR,CI,N] and the [CR,EI,N] models.

In Table 7, we compare the correlation structure of the innovations implied by the model with those implied by an unconstrained VAR(1). The correlations of the macro variables with the term structure variables display a poor fit with the VAR implied ones, with the signs being mostly reversed. However, the correlations are not significantly different from zero in the unrestricted VAR. The correlations between the term structure variables are fit well by the model.

Note that in Tables 5 and 7 we use bootstrapped standard errors. We were concerned that the GMM estimation may under-predict the sampling error of the parameter estimates (see also below) and therefore conducted the following bootstrap experiment. We bootstrap from the 172 observations on the vector of structural standard errors ( $\epsilon_t$ ) with replacement and re-create a sample of artificial data using the estimated parameter matrices ( $\Omega, \Gamma$ ) and historical initial values. For each replication, we create a sample of 672 observations, discard the first 500 and retain the last 172 observations to create a sample of length equal to the data sample. We then re-estimate the model, obtain parameters, impulse responses, and other statistics for these artificial data. We use 1,000 replications to create small sample distributions and use the standard deviation of the empirical distribution as an estimate of the true sampling error of the parameter, or derived statistic (for instance, VAR feedback parameter, impulse response, or regression slope).<sup>6</sup>

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<sup>6</sup>Our bootstrap also reveals a number of biases in estimated parameter coefficients. We defer a

## 6 Macroeconomic Implications

### 6.1 Structural Parameter Estimates

The second to fourth columns in Table 8 show the parameter estimates of model [CR,EI,N] and their GMM and bootstrap standard errors. All the parameter estimates have the expected sign and are significantly different from zero with the exception of  $\gamma$ , the response of the monetary authority to the output gap. Columns 5 through 8 show the mean, standard error, maximum and minimum parameter estimates within the set of seven models which were estimated. All the estimations yielded a stationary and unique solution. Interestingly, most of the parameter estimates are similar across specifications, that is, the cross-model standard deviation is small. Consequently, unless otherwise noted, our discussion below focusing on model [CR,EI,N] is robust to the specification of the Taylor rule and the inflation target.

A first important finding is the size and significance of  $\kappa$ , the Phillips curve parameter. As Galí and Gertler (1999) point out, previous studies fail to obtain reasonable and significant estimates of  $\kappa$  with quarterly data. Galí and Gertler (1999) do obtain larger and significant estimates using a measure for marginal cost replacing the output gap. Our estimates of  $\kappa$ , using the output gap and term spreads are even larger than those obtained by Galí and Gertler (1999). The estimate is also highly significant. While the standard error may be under-estimated, the cross-model standard deviation of the  $\kappa$  estimate still suggests a large and significant  $\kappa$ . Using the (larger) standard error from the bootstrap,  $\kappa$  remains statistically significantly different from zero. The forward-looking parameter in the AS equation is estimated close to 0.61 consistent with previous studies.

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discussion of small sample inference in this context to future work but refer the reader to Cho and Moreno (2004) and Fuhrer and Rudebusch (2004) for related analyzers of small sample biases in the estimation of New-Keynesian models.

When structural models are estimated with efficient techniques such as GMM or MLE, they often give rise to large estimates of  $\sigma$  rendering the IS equation a rather ineffective channel of monetary policy transmission. Two examples are Ireland (2001) and Cho and Moreno (2004)). As Lucas (2003) points out, a curvature parameter in the representative agent's utility function consistent with most macro and public finance models should be between 1 and 4. While the Lucas' statement does not strictly apply to models with habit persistence, in our multiplicative habit model  $\sigma$  still represents local risk aversion and our estimation yields a small and significant estimate of  $\sigma$ . Note that  $\sigma$ 's bootstrapped standard error is substantially higher than the asymptotic one, making its significance more marginal. Model [CR,EI,N] yields a significant estimate of  $\sigma$  slightly larger than 3, but the  $\sigma$  estimate is never larger than 5 in any model. Smets and Wouters (2003) and Lubik and Schorfheide (2004) find small estimates of  $\sigma$  using Bayesian estimation techniques. Rotemberg and Woodford (1998) and Boivin and Giannoni (2003) also find small estimates of  $\sigma$  but they modify the estimation procedure towards fitting particular impulse responses. Our model exhibits large habit persistence effects, as the habit persistence parameter,  $\eta$ , is close to 4. Other studies have also found an important role for habit persistence (Fuhrer (2000), Boldrin, Christiano and Fisher (2001)).<sup>7</sup> In summary, the parameter estimates for the AS and IS equations imply that our model delivers large economic effects of monetary policy on inflation and output.

Why do we obtain large and significant estimates of  $\kappa$  and  $\sigma$ ? Two channels seem to be at work: First, expectations are based on both observable and unobservable macro variables. Therefore, an important variable in the AS equation, such as expected inflation is directly affected by the inflation target. As a result, changes in the inflation target shift the AS curve. As we show below in the variance decomposi-

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<sup>7</sup>Our habit persistence parameter is not directly comparable to that derived by Fuhrer (2000). There is however a linear relationship between them:  $\eta = (\sigma - 1)h$ , where  $h$  is the Fuhrer (2000) habit persistence parameter. Our implied  $h$  is close to 2, larger than in previous studies.

tions, the inflation target shock contributes significantly to variation in the inflation rate. Similarly, the natural rate shock significantly contributes to the dynamics of de-trended output. Second, our measure of the output gap is different from the usual de-trended output and contains additional valuable information extracted from the term structure. For instance, the first order autocorrelation of the implied output gap is 0.92, which is smaller than 0.96, the first order autocorrelation of linearly de-trended output. The Phillips curve coefficients found in previous studies reflect the weak link between de-trended output and inflation in the data and the large difference in persistence between these two variables. In our model, even though  $\kappa$  is rather large, the relationship between inflation and the output gap is still not strongly positive because the inflation target also moves the AS-curve. When the variability of the inflation target is reduced and fixed in estimation, we obtain larger  $\kappa$ 's, a decrease in the autocorrelation of the output gap and strong positive cross-correlations between inflation and the output gap. In sum, the presence of both the inflation target and the natural rate of output in the AS equation implies a significantly positive conditional co-movement between the output gap and inflation, even though the unconditional correlation between them remains low as it is in the data. The unobservables are also critical in fitting the relative persistence of the output gap and inflation. Similarly, the  $\phi$  parameter still fits the dependence of the output gap to the real interest rate, but the real interest rate is now an implicit function of all the state variables, including the natural rate of output.

The estimates of the policy rule parameters are similar to those found in the literature. For all models, the estimated long-run response to expected inflation is larger than 1. The response to the output gap is always close to 0 and insignificantly different from 0. Finally, the smoothing parameter,  $\rho$ , is estimated to be 0.72 for model [CR,EI,N] and lies mostly in a [0.7-0.9] range, similar to previous studies.

The two unobservables are quite persistent, but clearly stationary processes. The

natural rate of output's persistence is close to 0.96, while the weight on the past inflation target in the inflation target equation is 0.88. Furthermore, the weight on current inflation in the construction of the long-run inflation target is close to 0.15. These parameters are reasonably robust across specifications. Finally, the five shock standard deviations are significant, with the monetary policy shock standard deviation larger than the others. There has been some evidence pointing towards a structural break in the  $\beta$  parameter (see, for instance, Clarida et al. (1999) or Lubik and Schorfheide (2004)). The large estimate of  $\sigma_{MP}$  may reflect the absence of such a break in our model. However, different models can yield quite different standard deviations for the monetary policy shocks and the shocks for the latent variables.

## 6.2 Output Gap and Inflation Target

One important feature of our analysis is that we can extract two economically important unobservable variables from the observable macro and term structure variables. The output gap is of special interest to the monetary authority, as it plays a crucial role in the monetary transmission mechanism of most macro models. Smets and Wouters (2003) and Laubach and Williams (2003) also extract the natural rate of output for the European and US economies from theoretical and empirical models respectively. An important difference between our work and theirs is that we use term structure information to filter out the natural rate, whereas they back it out of pure macro models through Kalman filter techniques. The dynamics of the inflation target are particularly important for the private sector, as the Federal Reserve has never announced targets for inflation and knowledge of the inflation target would be useful for both real and financial investment decisions.

The top Panel in Figure 2 shows the evolution of the output gap implied by Model

[CR,EI,N] together with the average of the output gap across our seven models.<sup>8</sup> The two series are very similar. It also displays a band of two cross-model standard deviations around the average. Several facts are worth noting. Before 1980, the output gap stayed above zero for most of the time. A positive output gap is typically interpreted as a proxy for excess demand. A popular view is that a high output gap made inflation rise through the second half of the 70s. Our output gap graph is consistent with that view. However, right before 1980, the output gap becomes negative. The aggressive monetary policy response to the high inflation rate is probably responsible for this sharp decline. After this, the output gap remains negative for most of the time up to 1995. This negative output gap was mainly caused by a surge in the natural rate of output, which remains above trend well into the mid-90s. Finally the output gap grows during the mid-1990s and starts to fall around 2000, coinciding with the latest recession.

The bottom Panel in Figure 2 analogously presents the natural rate of output implied by Model [CR,EI,N], the average of the natural rate across our seven models, together with a confidence band. Note that, first, there is a steady upward trend in the natural rate throughout the 60s. While it is possible that the natural rate did increase during that period, we think that the linear filtering of output overstates this growth. As the confidence bands show, the uncertainty is larger precisely at the beginning and the end of the sample. Second, the natural rate falls around 1973 and the late 70s. While the natural rate is exogenous in our setting, this may reflect the side-effects of the productivity slow-down brought about by oil price increases. Third, the natural rate stayed high throughout most of the 80s. Fourth, the natural rate did fall coinciding with the recession of the early 80s, but it remained above trend during

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<sup>8</sup>The output gap is measured as the percentage deviation of de-trended output with respect to the natural rate of output. Both de-trended output and the natural rate of output are measured as percentage deviations with respect to a linear trend. Therefore, the means of the output gap, de-trended output and the natural rate of output are 0.

the rest of the 80s. In the early nineties it fell below trend and has stayed close to trend since the mid nineties.

Figure 3 focusses on the inflation target. The top Panel shows the filtered inflation target from model [CR,EI,N], the average of the target across our seven models together with the confidence bands.<sup>9</sup> The bottom Panel shows the CPI inflation series for comparison. Three well differentiated sections can be identified along the sample. In the first one, the inflation target grows steadily up to the early 80s. Private sector expectations seem to have built up through the 60s and 70s contributing to the progressive increase in inflation. In the second one, the inflation target remains high for about 5 years. Finally, since the mid-eighties, the inflation target declines and remains low for the rest of the sample, tracking inflation closely. Much like in the case of the output gap, uncertainty about the inflation target is higher at the beginning and the end of the sample. Uncertainty is also high during the early 80s, when the inflation target was high.

### 6.3 Implied Macro Dynamics

In this section, we characterize the dynamics implied by the structural model using standard impulse response and variance decomposition analysis. Figure 4 shows the impulse response functions of the five macro variables to the structural shocks for model [CR,EI,N]. The AS shock is a negative technology or supply shock which decreases the productivity of firms. A typical example of an AS shock is an oil shock, as it raises overall marginal costs. As expected, the AS shock pushes inflation almost 2 percentage points above its steady state, but it soon returns to its original level, given the highly forward-looking nature of our AS equation. The monetary authority

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<sup>9</sup>Because we estimate the model with demeaned data, we add the mean of inflation back to the actual inflation target. This procedure is consistent with our model, where the mean of the inflation target coincides with that of inflation.



increases the interest rate following the supply shock. Because of the strong reaction of the Fed to the AS shock (the Taylor principle holds), the real rate increases and output exhibits a hump-shaped decline for several quarters. The inflation target initially increases after the AS shock but then decreases and stays below steady state due to the decline in inflation.

Our IS shock is a demand shock, which can also be interpreted as a preference shock (see Appendix A). Consistent with economic intuition and the results in the empirical VARs of Evans and Marshall (2004), the IS shock increases output, inflation, the interest rate and the inflation target for several quarters.

The monetary policy shock reflects shifts to the interest rate unexplained by the state of the economy. Given our strong monetary transmission mechanism, a contractionary monetary policy shock yields a strong decline of both output and inflation. The inflation target also declines, reinforcing the contractionary effect of the monetary policy shock on inflation and output. The interest rate increases following the monetary policy shock, but after three quarters it undershoots its steady-state level. This undershooting is related to the strong endogenous decrease of output and inflation to the monetary policy shock. As we show below, this reaction of the short-term interest rate to the monetary policy shock has implications for the reaction of the entire term structure to the monetary policy shock.

The microeconomic mechanism for our natural rate shock is an increase in the number of firms, which decreases the wedge between prices and marginal costs (a negative markup shock) and increases output. In other words, a natural rate shock shifts the AS curve down and not surprisingly, we see that an expansive natural rate shock increases output and lowers inflation. Through the monetary policy rule, the interest rate follows initially a similar path to inflation, decreasing substantially. Eventually, inflation rises above steady state again and so does the interest rate,

both overshooting their steady state during several periods. As a result, the inflation target, which partially reflects expected inflation, raises above steady-state almost immediately. Notice how output converges towards its natural level after 10 quarters following the natural rate shock and moves in parallel with it from then onwards.

An expansionary inflation target shock is an exogenous shift in the preferences of the Fed regarding its monetary policy goal. Because the inflation target is a long-term policy objective, a positive inflation target shock is akin to a persistent expansionary monetary policy shock. As a result, output and inflation exhibit a strong hump-shaped increase in response to the target shock, making the interest rate increase. Notice that in our setup, the strong response of inflation to a target shock is due to the relation between the inflation target, inflation expectations and inflation.

Figure 5 shows the variance decompositions at different horizons for the five macro variables in terms of the five structural shocks. The variance decompositions show the contribution of each macroeconomic shock to the overall forecast variance of each of the variables at different horizons. Inflation is mostly explained by the AS shock at short horizons. However, at medium and long-run horizons inflation dynamics are mostly driven by the monetary policy shock and the inflation target shock. Short-run output dynamics are mostly due to the IS and monetary policy shocks. The natural rate shock has a growing influence on output dynamics as the time horizon advances, reflecting the fact that in the long-run output tends to its natural level. Interest rate dynamics are dominated by the monetary policy shock at short horizons whereas in the long-run the inflation target shock has more influence. The inflation target shock dominates inflation target dynamics at all horizons, whereas the natural rate of output is always explained by its own shock due to our AR(1) specification. Given that the monetary authority is responsible for both the monetary policy and inflation target shocks, our results reveal monetary policy to be a key driver of macro dynamics. Smets and Wouters (2003) also find monetary policy shocks to play a key

role in explaining macro dynamics in the Euro area.

## 6.4 Why do term spreads predict output and inflation?

A substantial empirical literature, including Mishkin (1990), Estrella and Mishkin (1998) and Ang et al. ((forthcoming)) has demonstrated the relevance of term structure dynamics in predicting future output and inflation. In a recent paper, Estrella (2004) explores the structural sources of this predictability, including the monetary policy regime.

While the simple model in Estrella (2004) has the advantage of transparency, it is useful to re-examine this issue in a more empirically relevant model. To do so, we consider our reduced form model solution, which implies that (in demeaned form):

$$E_t z_{t+1} = \Omega_z z_t$$

Let  $\omega_{\pi,sp1}$ ,  $\omega_{\pi,sp2}$ ,  $\omega_{y,sp1}$ , and  $\omega_{y,sp2}$  the 1-4, 1-5, 2-4 and 2-5 elements of  $\Omega_z$ . We study the relation between term spreads and macro variables by adding up the spread coefficients of each equation. So, for instance, the overall influence of spreads on inflation will be measured by the sum of  $\omega_{\pi,sp1}$  and  $\omega_{\pi,sp2}$ . Both our model and the one in Estrella (2004) impose the expectations hypothesis. Hence, term spreads predict future output and inflation because output and inflation are interest rate factors. However, the degree of predictability (and even its sign) will naturally depend on the structural parameters.

We examine the effect of the monetary's authority anti-inflationary stance as captured by the  $\beta$  parameter. We perform a sensitivity analysis for different values of  $\beta$ , the monetary policy response to the gap between expected inflation and the target. Table 9 shows the projection coefficients of the macro variables on the spreads implied

by Model [CR,EI,N]. Except in one case, the coefficients on the spreads are positive for all  $\beta$ 's. In the case of inflation, this sum increases with  $\beta$ , but the opposite is true for output. One important qualification is that the reduced-form coefficients of inflation on the spreads implied by the model were not in agreement with the data (see Table 5), unlike their counterparts in the output equation. The reduced-form coefficients of inflation on the interest rate are an increasing function of  $\beta$ . However, the overall effect of the short-term rate on inflation and output is found by subtracting the spread coefficient from the one on the interest rate. The sign is then negative, as one would expect given our structural model. The fourth and last columns of table 9 show that the contractionary effect of the interest rate on inflation and output gets smaller as  $\beta$  increases. This result is also reported by Cho and Moreno (2004) in the context of a standard New-Keynesian model. The reason is that under large  $\beta$ 's, the Fed tries to aggressively offset any departure of inflation and output from their steady-states. As a result, the effects of all structural shocks –including the monetary policy shock– on macro dynamics will be smaller.

## 7 Term Structure Implications

### 7.1 Model Fit for Yields

Our model represents a 5-factor term structure model with three observed and two unobserved variables. Dai and Singleton (2000) claim that a model with three latent factors provides an adequate fit with the data. A quick test to see how well our model fits the complete term structure is to see how well it fits the yields not used in the estimation. We compute the difference between the actual and model-predicted yields. This difference can be viewed as measurement error and if the model fits the data well, this measurement error should not be too variable. We find that

the measurement error for the one and ten-year yields is 45 and 54 basis points (annualized) respectively.<sup>10</sup> While this is significantly different from zero, the fit can be considered reasonable given the parsimonious structural nature of our model.

While our model imposes the Expectations Hypothesis, it nevertheless fits these one year and 10 year yields much better than pure macro models, which have been the norm in the macro literature. Let's consider the macro VAR(3) model, which was shown before to explain the dynamics of macro variables extraordinarily well. In Figure 6, the left hand side graphs plot the one year and 10 year yields and their predicted values from the structural model. On the right hand side, we compute the one-year and 10 year yields using the VAR(3) model. The fit is visibly much worse for the 10-year yields, although similar for the one-year yield. Indeed, the implied measurement error of the 1 and 10 year yields are 45 and 169 basis points, respectively. The "shifting endpoint" specification of Kozicki and Tinsley (2001) essentially circumvents this poor fit by embedding term structure information in the drift of the factor dynamics.

## 7.2 Structural Term Structure Factors

It is standard to label the three factors that are necessary to fit term structure dynamics as the level, the slope and the curvature factors. We measure the level as the equally weighted average of the three month rate, 1 year and 5 year yields, the slope as the 5 year spread and the curvature as the sum of the three month rate and 5 year rate minus twice the one year rate. Figure 7 shows the impulse responses of the level, slope and curvature factors to the five structural shocks. The AS shock initially raises the level, but then it undershoots the levels steady state for several

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<sup>10</sup>The 10 year zero-coupon yield was constructed splicing two series. We use the McCulloch and Kwon series up to the 3rd quarter of 1987; from the 4th quarter of 1987 to the end of the sample, we use the 10 year zero-coupon yield estimated using the method of Svensson (2004).

quarters. As explained in section 6.3, the interest rate undershooting is related to the strong endogenous response of the monetary policy authority to inflation, which ends up lowering inflation beyond its steady-state during some quarters. The Expectation Hypothesis implies that the initial rise in yields is strongest at short maturities. Consequently, it is no surprise that the AS shock initially lowers the slope, then, when the level effect turns negative, raises the slope above its steady-state. The curvature effect follows the slope effect closely.

Similarly to Evans and Marshall (2004), the IS or demand shock raises the level factor during several years. The IS shock also lowers the slope and curvature during some quarters. These two responses are again related to the hump-shaped response of the short-term interest rate to the IS shock and the fact that the expectation hypothesis holds in our setup. Essentially, a positive IS shock causes a flattening upward shift in the yield curve. The monetary policy shock initially raises the level, but then it produces a strong hump-shaped negative response on the level. This is again related to the undershooting of the short-term interest rate after a monetary policy shock. The slope initially decreases after the monetary policy shock but then it increases during several quarters. The initial slope decline happens because a monetary policy shock naturally shifts up the short-end of the yield curve, while it lowers the medium and long part of the yield curve, through its effect on inflationary expectations. The subsequent slope increase arises because the short rate undershoots after a few quarters. Finally, the curvature of the yield curve increases for 10 quarters after the monetary policy shock.

The natural rate, or essentially a productivity shock, not surprisingly induces an initial decline in the level of the yield curve. After 4 quarters the level exhibits a persistent increase, mimicking the response of the short-term rate to the natural rate shock. Both the slope and the curvature factors increase after the natural rate shock during ten quarters. As Figure 4 shows, the natural rate shock raises the

future expected short-term rates whereas it lowers the current short rate. Since the expectations hypothesis holds in our setup, that implies that the spread increases. Figure 9 below corroborates this intuition.

Finally, the inflation target shock has a very pronounced positive effect on the level of the yield curve. This has to do with the strong persistent hump-shaped response of the interest rate to the target shock. It also makes the slope and the curvature decline during several periods, since the target shock has a stronger positive effect on short term rates than on long rates.<sup>11</sup>

To complement the impulse response functions, Figure 8 shows the variance decompositions of the three factors at different time horizons. The inflation target shock explains more than 50% of the variation in the level of the term structure at all time horizons and over 75% at short horizons. After the 5th quarter, the monetary policy shock explains around 25% of the level dynamics. In the short-run, the IS shock explains around 15%, whereas in the long-run, it is the natural rate shock which explains around 15% of the variation in the level factor.

The monetary policy shock is the dominant factor behind the slope dynamics at all horizons, as it primarily affects the short end of the yield curve.<sup>12</sup> This fact is especially evident at short horizons, where almost 90% of the slope variance is explained by the monetary policy shock. The inflation target shock, which has a dominant effect at the long end of the yield curve gains importance at longer horizons. The IS shock and the natural rate shock explain each around 10% of the slope dynamics at virtually all horizons.

The variance decomposition of the curvature factor yields similar results to the

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<sup>11</sup>Rudebusch and Wu (2004) obtain the opposite reaction of spreads to their level shock. This is probably related to the fact that they incorporate a time-varying risk premium in the term structure whereas we maintain the expectation hypothesis throughout.

<sup>12</sup>In the context of a business-cycle model with adjustment costs, Wu (2001) also finds that monetary policy shocks explain most of the slope fluctuations.

slope factor, with the monetary policy shock being the dominant factor again, explaining around 60% of the curvature factor dynamics at all horizons. Finally, it is worthwhile noting that the AS shock influence on the dynamics of the term structure is overall very small.

An implication of our study is that the inflation target shock is the level factor whereas the monetary policy shock drives both the slope and curvature factors. Our results are consistent with the framework in Rudebusch and Wu (2004), where the unobservable variables are directly labelled level and slope.

### 7.3 “Endogenous” Excess Sensitivity

Our model can shed light on two empirical regularities that have received much attention in recent work : The excess sensitivity and the excess volatility of long term interest rates. Gurkaynak, Sack and Swanson ((forthcoming)) recently show a particularly intriguing empirical failure of standard structural models: they fail to generate significant responses of forward interest rates to any macro economic and monetary policy shocks. However, in the data, US long-term forward interest rates react considerably to surprises in macroeconomics data releases and monetary policy announcements. They use a model with a slow-moving inflation target to better match these empirical facts. Now we show that our model yields a strong contemporaneous response of the term structure to several shocks in our model.

Figure 9 shows the contemporaneous responses of the entire term structure to our five structural shocks. The AS shock shifts the short end of the yield curve but has virtually no effect on yields of maturities beyond ten quarters. Our model-predicts a long-lasting response of the bond to the IS shock and the shocks to the unobservable macro variables. The IS shock produces an upward shift in the entire term structure, but affects more strongly the yields of maturities close to one year, leading to a hump-



shaped response. The IS and natural rate shocks have inverse but symmetric effects on the term structure. While the IS shock shifts the term structure upwards, the natural rate shifts the term structure down for maturities down up to 5 years. This is to be expected as the IS shock is a demand shock, whereas the natural rate shock is essentially a supply shock.

The monetary policy shock shifts the short end of the curve upward but it has a negative, if small, contemporaneous effect on yields of maturities of five quarters and higher. Gurkaynak et al. ((forthcoming)) also show this pattern, but in their exercise, the monetary policy shock starts having a negative effect on bond rates at a longer maturities. Our result is again due to the interest rate

undershooting in response to the monetary policy shock. Since the expectations hypothesis holds, future expected decreases in short-term rates imply declines of medium and long-term rates. Finally, the inflation target shock produces a very persistent, strong and hump-shaped positive response of the entire term structure. As agents perceive a change in the monetary authority's stance, they adjust their inflation expectations upwards so that interest rates increase at all maturities.

Note that the sensitivity of long rates to the inflation target, the natural rate and the IS shocks remains very strong even at maturities of 10 years. Ellingsen and Soderstrom (2004) and Gurkaynak et al. ((forthcoming)) show that their structural macro models can explain the sensitivity of long-rates to structural macro shocks. While their models use several lags of the macro variables to generate persistence with a slow decaying component, our model can account for the variability of the long rates with a parsimonious VAR(1) specification. More importantly, whereas Ellingsen and Soderstrom (2004) stress the importance of the monetary policy shock, in our model the IS shock and the shocks to the unobservable macro variables are much more important in explaining the sensitivity of the long rates than the monetary

policy shock is.

Ellingsen and Soderstrom (2004) provide a simple direct test of the excess sensitivity of long interest rates. They regress changes in long rates on changes in short rates. It is straightforward to show that these projection coefficients are quite sensitive to the persistence generated by the model and the magnitude of the reaction of the term structure to the shocks. We create a simple test statistic where we weight the deviations of the model-implied projection coefficients from the empirical projection coefficients using the inverse of the data covariance matrix. We perform this test for all of our models, finding that the microfounded model matches this evidence perfectly. The test does reject for the other models but the [CR,EI,N] model (the one we have focused most of our discussion on) and the [CR,CI,N] model still perform significantly better than the other models.

## 8 Conclusions

The first contribution of our paper is methodological. We show how to use a no-arbitrage term structure model to help identify a standard New-Keynesian macro model with additional unobservable factors. Whereas there are many possible implementations of our framework, in this article we introduce the natural rate of output and time-varying inflation target in an otherwise standard model.

From a macroeconomic perspective our contribution is that we use term structure information to help identify structural macroeconomic and monetary policy parameters and at the same time impart additional persistence to the macro system. From a finance perspective, our contribution is that we derive a no-arbitrage tractable term structure model where all the factors obey New-Keynesian structural relations.

Our key findings are as follows. First, our structural estimation robustly identifies

a large Phillips curve parameter and a large response of output to the real interest rate. Second, the model mimics much of the salient empirical macro dynamics uncovered through long-lagged VARs in previous work, even though our reduced-form model is simply a first-order VAR. Third, the unobservable inflation target process is the most important level factor whereas the monetary policy shocks dominate the variation in slope and curvature factors.

There are a number of avenues for future work. First, the finance literature has stressed the importance of stochastic risk aversion in helping to explain salient features of asset returns (see Campbell and Cochrane (1999) and Bekaert, Engstrom and Grenadier (2004)). Dai and Singleton (2002) show how time-varying prices of risk play a critical role in explaining deviations of the Expectations Hypothesis for the U.S. term structure. However, their model has no structural interpretation. Piazzesi and Swanson (2004) find risk premiums in federal funds futures rates which appear counter-cyclical. A follow-up paper will explore the effect of stochastic risk aversion on our findings. Second, New-Keynesian models continue to be plagued by the poor measurement of the output gap. The fact that most measures impose deterministic trends is one potential reason for the poor performance of the output gap based estimates in these models. We intend to propose a model with stochastic trends where the output gap is filtered through the model and output growth data, rather than measured directly from the data.

# Appendix

In this appendix we provide the microfoundations of the New-Keynesian macro model presented in the main text. Since the model is fairly standard, we briefly explain how the IS and AS equation exhibit endogenous persistence.

## A IS Equation under External Habit Formation

The representative agent's utility function has the following form:

$$U(C_t, N_t; F_t) = \frac{F_t C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\chi}}{1+\chi} \quad (47)$$

where  $C_t$  is the level of real consumption,  $\sigma$  is the local risk aversion coefficient and the inverse of the intertemporal elasticity of substitution,  $N_t$  is the labor supply,  $\chi$  is the inverse of the intertemporal labor supply elasticity.  $F_t$  represents an aggregate demand shifting factor. Specifically,  $F_t$  is composed out of two parts:

$$F_t = H_t G_t \quad (48)$$

where  $H_t$  is an external habit level, that is, the agent takes  $H_t$  as exogenously given even though it may depend on past consumption, and  $G_t$  is an exogenous aggregate demand shock that can also be interpreted as a preference shock. Analogously to Fuhrer (2000), we assume that  $H_t = C_{t-1}^\eta$  where  $\eta$  measures the degree of habit dependence on the past consumption level. The maximization of equation (1) is subject to the following budget constraint:

$$P_t C_t + d'_t Q_t = W_t N_t + (D_t + Q_t)' d_{t-1} \quad (49)$$

where  $P_t$  is the price level,  $d_t$  is a vector of portfolio weights,  $Q_t$  a vector of asset prices and  $D_t$  is a vector of dividends. We do not consider the government sector explicitly.

Because of the assumption of the additive separability of the preference specification over consumption and labor supply, the marginal disutility of work does not affect the optimal consumption decision. Consequently, the marginal utility of consumption does not depend on the labor supply and we can construct the nominal intertemporal marginal rate of substitution in consumption as follows:

$$M_{t+1} = \psi \frac{U_C(C_{t+1}, N_{t+1}; F_{t+1})P_t}{U_C(C_t, N_t; F_t)P_{t+1}} = \psi \frac{C_{t+1}^{-\sigma} F_{t+1} P_t}{C_t^{-\sigma} F_t P_{t+1}}$$

The variable  $M_{t+1}$  can also be viewed as a nominal stochastic discount factor and will price all financial assets in this economy. In particular, the nominal interest rate,  $i_t$ , satisfies:

$$E_t [M_{t+1}(1 + i_t)] = 1$$

After imposing the resource constraint,  $C_t = Y_t$ , we apply logs to obtain:

$$m_{t+1} = \ln \psi - \sigma y_{t+1} + (\sigma + \eta)y_t - \eta y_{t-1} + (g_{t+1} - g_t) - \pi_{t+1} \quad (50)$$

where  $g_t = \ln G_t$  is an *i.i.d* shock with mean zero. By assuming that  $y_t$  has a deterministic trend, equation (50) also holds for de-trended output. From now on,  $y_t$  refers to de-trended output. Under the assumption of log-normality, the Euler equation is given by:

$$E_t m_{t+1} + 0.5V_t(m_{t+1}) = -i_t \quad (51)$$

Using the definition of  $m_{t+1}$ , we get the IS equation:

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1 - \mu)y_t - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \quad (52)$$

where  $\mu = \frac{\sigma}{\sigma+\eta}$ ,  $\phi = \frac{1}{\sigma+\eta}$ ,  $\alpha_{IS} = -\phi(\ln \psi + 0.5V_t(m_{t+1}))$  and  $\epsilon_{IS,t} = \phi g_t$ . Note that  $V_t(m_{t+1})$  depends on the state variable dynamics of the model. If the innovations of the model are homoskedastic, the pricing kernel variance is constant.

## B AS Equation with inflation indexation

In order to set up an explicit price optimization problem, Calvo (1983) and the subsequent literature have assumed monopolistic competition in the intermediate product markets. A retail distributor combines the differentiated output of a continuum of monopolistically competitive firms,  $Y_t(i)$ , into a composite product,  $Y_t$ , with elasticity of substitution between goods  $\varepsilon > 1$ :

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{1-\varepsilon}} \quad (53)$$

The demand for the product of each firm  $i$  is obtained by the usual expression (see Blanchard and Kiyotaki (1987)):

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t \quad (54)$$

where  $P_t(i)$  is the price for the product of firm  $i$  and  $P_t$  is the aggregate index defined as:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \quad (55)$$

In the Calvo (1983) pricing framework, a subset of firms maximize their profits according to an arrival rate  $(1 - \theta)$ . Thus, each firm resets prices every period with probability  $(1 - \theta)$ . We further assume that the price-setters who do not adjust optimally, index their prices taking into account previous inflation, enabling inflation

to exhibit endogenous persistence. Hence,

$$P_t(i) = P_{t-1}(i) \left[ \frac{P_{t-1}}{P_{t-2}} \right]^\tau \quad (56)$$

where  $\tau$  is the degree of indexation to previous inflation and it is between 0 and 1.

Using the law of large numbers, the price index becomes:

$$P_t = \left[ (1 - \theta) \bar{P}_t(i)^{1-\varepsilon} + \theta \left( P_{t-1} \left( \frac{P_{t-1}}{P_{t-2}} \right)^\tau \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (57)$$

where  $\bar{P}_t(i)$  is the optimal reset price. Log-linearizing this expression yields:

$$\pi_t - \tau \pi_{t-1} = \frac{1 - \theta}{\theta} \hat{p}_t(i) \quad (58)$$

where  $\hat{p}_t(i) = \log \left( \frac{\bar{P}_t(i)}{P_t} \right)$ . The optimal dynamic price-setting problem becomes:

$$\max_{P_t(i)} E_t \left[ \sum_{T=t}^{\infty} \theta^{T-t} M_{t,T} \Pi_T(i) \right] \quad (59)$$

where  $M_{t,T}$  is the nominal stochastic discount factor or pricing kernel for contingent claims and  $\Pi_T(i)$  is the nominal profit of firm  $i$  at time  $T$ .  $M_{t,T}$  is given by (see also Appendix A):

$$M_{t,T} = \frac{\psi^{T-t} U_c(C_T; F_T) P_t}{U_c(C_t; F_t) P_T} \quad (60)$$

and  $\Pi_T(i)$  is given by:

$$\Pi_T(i) = \left[ P_t(i) \left( \frac{P_{T-1}}{P_{t-1}} \right)^\tau - S_T(i) P_T \right] \left[ \frac{P_t(i)}{P_T} \left( \frac{P_{T-1}}{P_{t-1}} \right)^\tau \right]^{-\varepsilon} Y_T \quad (61)$$

where  $S_T(i)$  is the real marginal cost, defined as  $S_T(i) = \left( \frac{w_T}{P_T} \right) / \left( \frac{\partial Y_T(i)}{\partial N_T} \right)$ . Using equations (56), (60) and (61), the first order condition associated with the maximization

problem in (59) can be expressed as:

$$E_t \left\{ \sum_{T=t}^{\infty} (\theta\psi)^{T-t} \left[ U_c(C_T; F_T) Y_T \bar{P}_t(i)^{-\varepsilon} P_T^\varepsilon \left( \frac{P_{T-1}}{P_{t-1}} \right)^{-\tau\varepsilon} \right] \left[ \frac{\bar{P}_t(i)}{P_T} \left( \frac{P_{T-1}}{P_{t-1}} \right)^\tau - \frac{\varepsilon}{\varepsilon-1} S_T(i) \right] \right\} = 0 \quad (62)$$

Log-linearizing (62) around the steady state and solving for  $\hat{p}_t(i) = \log(\frac{\bar{P}_t(i)}{P_t})$ , we obtain:

$$\hat{p}_t(i) = (1 - \theta\psi) \sum_{T=t}^{\infty} (\theta\psi)^{T-t} E_t \hat{s}_T(i) + \sum_{T=t+1}^{\infty} (\theta\psi)^{T-t} E_t [\pi_T - \tau\pi_{T-1}] \quad (63)$$

where  $\hat{s}_T(i)$  is the percentage deviation from steady state of the log of the real marginal cost of producing  $Y_t(i)$ . From now on, we will focus on the average real marginal cost deviation from its steady state value, i.e. the real marginal cost for a good with output  $Y_t(i) = Y_t$ . As a result, we will drop the indexing of real marginal costs. Subtracting  $\theta\psi\hat{p}_{t+1}(i)$  from both sides of the last equation and using (58) yields a relation describing the inflation dynamics:

$$\pi_t - \tau\pi_{t-1} = \psi(E_t\pi_{t+1} - \tau\pi_t) + \frac{(1-\theta)(1-\theta\psi)}{\theta} \hat{s}_t \quad (64)$$

Notice the key role of the nominal rigidities linking the real sector of the economy with inflation. We then obtain the following AS equation:

$$\pi_t = \delta_1 E_t \pi_{t+1} + \delta_2 \pi_{t-1} + \varpi \hat{s}_t \quad (65)$$

where  $\delta_1 = \frac{\psi}{1+\psi\tau}$ ,  $\delta_2 = \frac{\tau}{1+\psi\tau}$  and  $\varpi = \frac{(1-\theta)(1-\theta\psi)}{\theta(1+\psi\tau)}$ . Notice that for  $\psi$  arbitrarily close to one, the sum of  $\delta_1$  and  $\delta_2$  is approximately one.



## C Natural Rate of Output and Real Marginal Cost

We assume that the production technology of the average firm is specified as:

$$Y_t = \xi_t N_t \quad (66)$$

where  $\xi_t$  is the firm-independent technology shock. The labor demand can then be simplified as:

$$S_t = \xi_t^{-1} \frac{W_t}{P_t} \quad (67)$$

The optimality condition for the labor supply is given (indirectly) by:

$$\frac{W_t}{P_t} = - \frac{U_N(N_t)}{U_C(C_t; F_t)} \quad (68)$$

Using the utility function in equation (47) and replacing  $F_t$  with  $C_{t-1}^\eta G_t$ , we find:

$$\frac{W_t}{P_t} = N_t^\chi C_t^\sigma C_{t-1}^{-\eta} G_t^{-1} \quad (69)$$

By equating (67) and (69), the real marginal cost is given by:

$$S_t = N_t^\chi C_t^\sigma C_{t-1}^{-\eta} G_t^{-1} \xi_t^{-1} \quad (70)$$

Finally, using Equation (66) and the resource constraint, we obtain that

$$S_t = Y_t^{\chi+\sigma} Y_{t-1}^{-\eta} G_t^{-1} \xi_t^{-1-\chi} \quad (71)$$

The natural rate of output is the level of output,  $Y_t^n$ , that would prevail if prices were fully flexible and exogenous innovations are in their steady state. In the case of

monopolistic competition, this implies that  $Y_t^n$  is the level of output satisfying:

$$S_t^n \equiv S_t(Y_t = Y_t^n; G_t = 1, \xi_t = 1) = (Y_t^n)^{\chi+\sigma} (Y_{t-1}^n)^{-\eta} = f^{-1} \quad (72)$$

where  $f = \frac{\varepsilon}{\varepsilon-1}$  is the price markup. Hence, taking logs, a process for the natural rate of output results:

$$y_t^n = \alpha_{y^n} + \lambda y_{t-1}^n \quad (73)$$

where  $\alpha_{y^n} = -\frac{1}{\chi+\sigma} \ln f$  and  $\lambda = \frac{\eta}{\chi+\sigma}$ . We exogenously augment the natural rate equation with an innovation,  $\epsilon_{y^n,t}$ . One possible interpretation of  $\epsilon_{y^n,t}$  would be a shock to the markup. The natural rate equation can then be expressed as:

$$y_t^n = \alpha_{y^n} + \lambda y_{t-1}^n + \epsilon_{y^n,t} \quad (74)$$

Note that the external habit not only generates endogenous persistence of output but it also endogenously determines the persistence of the natural rate of output. Consequently the dynamic path of the real marginal cost deviation from its steady state is also governed by the degree of habit persistence. Specifically, combining (71) and (72), the real marginal cost is then given by:

$$\hat{s}_t = (\chi + \sigma)(y_t - y_t^n - \lambda(y_{t-1} - y_{t-1}^n)) - g_t - (1 + \chi) \ln \xi_t. \quad (75)$$

We can derive the final AS equation that links inflation and the output gap by plugging the real marginal cost equation into equation (65):

$$\pi_t = \delta_1 E_t \pi_{t+1} + \delta_2 \pi_{t-1} + \kappa \left( (y_t - y_t^n) - \lambda(y_{t-1} - y_{t-1}^n) \right) - \zeta \epsilon_{IS,t} + \epsilon_{AS,t} \quad (76)$$

where  $\kappa = \varpi(\chi + \sigma)$ ,  $\zeta = \varpi(\sigma + \eta)$  and  $\epsilon_{AS,t} = -\varpi(1 + \chi) \ln \xi_t$ . Note that both the IS and AS shocks enter now into the AS equation.

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Table 1: **The Relationship between the Term Spreads and Macro Variables**

Panel A: Term Spreads Affecting Macro Variables

Variable	CPI Inflation		GDPD Inflation	
	<i>GC</i>	<i>CC</i>	<i>GC</i>	<i>CC</i>
$\pi_t$	0.050	0.012	0.358	0.609
$y_t$	0.717	0.333	0.725	0.322
$i_t$	0.170	0.000	0.160	0.000

Panel B: Macro Variables Affecting the Term Spreads

Variable	CPI Inflation		GDPD Inflation	
	<i>GC</i>	<i>CC</i>	<i>GC</i>	<i>CC</i>
$sp_{12,t}$	0.006	0.021	0.007	0.041
$sp_{20,t}$	0.005	0.001	0.006	0.003

This Table shows the results of Granger Causality (GC) and contemporaneous correlation (CC) tests. The first column of Panel A shows the p-value of GC test statistics that test whether the one and five year spreads Granger-cause the three macro variables. The second column shows the p-value of CC test statistics, testing whether the spreads affect the three macro variables contemporaneously. The first column of Panel B shows the p-value for the GC test statistics of the three macro variables Granger-causing the spreads. The second column shows the p-value for the CC test statistics testing whether the three macro variables affect the spreads contemporaneously. The third and fourth columns show the same results when inflation is constructed by the GDP deflator. The GC test of Panel A (Panel B) is performed by regressing each macro variable on one lag of each of the variables and testing the null hypothesis that coefficients on the term spreads (macro variables) are jointly 0. The CC test of Panel A (Panel B) is performed by regressing each variable on other variables and testing the null hypothesis that coefficients on the term spreads (macro variables) are jointly zero. The tests in Panel A are  $\chi^2(2)$  and the tests in Panel B are  $\chi^2(3)$  distributed.



Table 2: **Persistence**

Panel A: Autocorrelogram of the Raw data					
Lag	$\pi_t$	$y_t$	$i_t$	$sp_{12,t}$	$sp_{20,t}$
1	0.760 (0.086)	0.934 (0.028)	0.922 (0.048)	0.693 (0.086)	0.746 (0.080)
2	0.716 (0.108)	0.849 (0.050)	0.878 (0.064)	0.513 (0.087)	0.603 (0.089)
3	0.712 (0.094)	0.749 (0.069)	0.864 (0.081)	0.492 (0.100)	0.589 (0.081)
5	0.573 (0.140)	0.540 (0.091)	0.728 (0.114)	0.198 (0.127)	0.299 (0.120)
8	0.410 (0.128)	0.296 (0.102)	0.544 (0.114)	0.047 (0.112)	0.083 (0.118)
10	0.379 (0.130)	0.199 (0.109)	0.453 (0.010)	-0.046 (0.119)	-0.049 (0.117)
$\hat{\rho}^{10}$	0.064	0.505	0.444	0.026	0.053
Panel B: Autocorrelogram implied by a VAR (1)					
Lag	$\pi_t$	$y_t$	$i_t$	$sp_{12,t}$	$sp_{20,t}$
1	0.702 (0.091)	0.953 (0.021)	0.874 (0.056)	0.395 (0.099)	0.731 (0.087)
2	0.509 (0.129)	0.901 (0.041)	0.767 (0.097)	0.193 (0.097)	0.554 (0.131)
3	0.382 (0.139)	0.847 (0.060)	0.676 (0.126)	0.112 (0.083)	0.426 (0.150)
5	0.231 (0.131)	0.735 (0.093)	0.529 (0.163)	0.048 (0.055)	0.253 (0.146)
8	0.115 (0.108)	0.577 (0.126)	0.370 (0.197)	0.010 (0.032)	0.114 (0.110)
10	0.069 (0.095)	0.482 (0.139)	0.292 (0.212)	-0.001 (0.028)	0.065 (0.094)

Panel A shows empirical autocorrelograms for all 5 variables, whereas Panel B shows the autocorrelograms implied by an unrestricted first-order VAR. The last line in Panel A reports the first-order autocorrelation coefficient to the 10-th power. Panel A standard errors are GMM-based and are in parentheses. Panel B standard errors are constructed using the delta method and are in parentheses.

Table 3: **Model Specifications**

Taylor Rule	Inflation Target Specification	
Desired Nominal Rate	New	Standard
Constant real rate, Expected Inflation	CR,EI,N	CR,EI,S
Time-varying real rate, Expected Inflation	TVR,EI,N	TVR,EI,S
Constant	CR,CI,N	CR,CI,S

This Table presents the model taxonomy which will be employed throughout the paper. There are 6 possible model combinations according to the Taylor Rule and Inflation Target specifications.

Table 4: **Autocorrelation Goodness of Fit**

Model	$\pi_t$ (P-val)	$y_t$ (P-val)	$i_t$ (P-val)	$sp_{12,t}$ (P-val)	$sp_{20,t}$ (P-val)
MF	18.10(0.000)	4.63(0.031)	3.05(0.081)	17.36(0.000)	21.15(0.000)
CR,EI,N	9.66(0.002)	3.13(0.077)	0.27(0.606)	1.20(0.274)	0.05(0.828)
TVR,EI,N	12.56(0.000)	3.05(0.081)	0.02(0.888)	2.40(0.121)	3.96(0.047)
CR,CI,N	8.39(0.004)	6.67(0.010)	0.01(0.908)	0.31(0.580)	0.23(0.634)
CR,EI,S	13.49(0.000)	7.57(0.006)	2.08(0.149)	7.43(0.006)	8.67(0.003)
TVR,EI,S	19.14(0.000)	12.67(0.000)	2.42(0.120)	8.53(0.003)	12.80(0.000)
CR,CI,S	13.79(0.000)	7.33(0.007)	2.28(0.131)	7.24(0.007)	8.98(0.003)

Consider  $\theta = (\rho_1 \ \rho_4 \ \rho_8)'$  and  $\alpha = (0.5 \ 0.3 \ 0.2)'$ . Our loss function is a quadratic form in  $\alpha'(\hat{\theta} - \theta_{mod})$  weighted by the inverse of an estimate of the standard error of the statistic. To produce such an approximate standard error, we assume that each variable follows an AR(1) process and calculate the higher order ARs using the delta method. For any variable, suppose that  $x_t = \rho x_{t-1} + \epsilon_t$ . Then  $\sqrt{T}(\hat{\rho} - \rho_0) \sim N(0, (1 - \rho_0^2))$ . Note that  $(\hat{\rho}^4 - \rho_0^4) \approx 4\rho_0^3(\hat{\rho} - \rho_0)$  and  $(\hat{\rho}^8 - \rho_0^8) \approx 8\rho_0^7(\hat{\rho} - \rho_0)$ . Let  $g = (1 \ 4\rho^3 \ 8\rho^7)'$ . Then  $\sqrt{T}(\hat{\theta} - \theta_0) \sim N(0, gg'(1 - \rho_0^2))$ . Define  $A = gg'(1 - \rho_0^2)$ . Then  $\sqrt{T}\alpha'(\hat{\theta} - \theta_0) \sim N(0, \alpha'gg'\alpha(1 - \rho_0^2))$ . Note that  $\alpha'(\hat{\theta} - \theta_0)$  and its variance  $\alpha'gg'\alpha(1 - \rho_0^2) = \alpha'A\alpha$  are scalars. Therefore, a natural statistic is  $T(\alpha'(\hat{\theta} - \theta_{mod}))^2(\alpha'A\alpha)^{-1}$ . We report this statistic for the 5 variables of interest for all the models. Even though this is not a formal test statistic, we also produce p-values from a  $\chi^2(1)$  distribution.

Table 5: **The Dynamics of Macro Variables and Term Spreads**

Panel A: Feedback Parameters implied by the Model					
Variable	$\pi_{t-1}$	$y_{t-1}$	$i_{t-1}$	$sp_{12,t-1}$	$sp_{20,t-1}$
$\pi_t$	0.550 ( 0.056)	0.014 ( 0.027)	0.308 (0.098)	2.545 ( 0.390)	-1.977 ( 0.411)
$y_t$	0.010 ( 0.004)	0.913 ( 0.017)	0.104 (0.039)	0.396 ( 0.225)	0.073 ( 0.191)
$i_t$	0.109 ( 0.021)	0.053 ( 0.025)	0.755 (0.059)	1.314 ( 0.296)	-1.087 ( 0.282)
$sp_{12,t}$	-0.112 ( 0.022)	-0.056 ( 0.026)	0.258 (0.061)	-0.426 ( 0.300)	1.298 ( 0.289)
$sp_{20,t}$	-0.109 ( 0.021)	-0.051 ( 0.024)	0.236 (0.058)	-1.384 ( 0.296)	2.199 ( 0.281)
Panel B: Feedback Parameters implied by an Unrestricted VAR(1)					
Variable	$\pi_{t-1}$	$y_{t-1}$	$i_{t-1}$	$sp_{12,t-1}$	$sp_{20,t-1}$
$\pi_t$	0.492 ( 0.076)	0.144 ( 0.052)	0.294 ( 0.124)	-1.467 ( 0.611)	1.007 (0.611)
$y_t$	-0.058 ( 0.037)	0.972 ( 0.028)	-0.031 ( 0.064)	0.037 ( 0.328)	0.005 (0.328)
$i_t$	-0.049 ( 0.045)	0.077 ( 0.032)	0.960 ( 0.074)	-0.267 ( 0.357)	0.365 (0.359)
$sp_{12,t}$	0.037 ( 0.031)	-0.045 ( 0.023)	0.026 ( 0.052)	-0.093 ( 0.257)	0.649 (0.265)
$sp_{20,t}$	0.045 ( 0.033)	-0.057 ( 0.025)	0.024 ( 0.056)	-0.853 ( 0.271)	1.418 (0.278)

Panel A shows the feedback parameters implied by the model. Panel B shows the unconstrained VAR(1) feedback coefficients. Standard errors were constructed using the bootstrap procedure described in the text.

Table 6: **Correlations of Model and VAR-based forecasts**

Model	$k$	$\pi_t$	$y_t$	$i_t$	$sp_{12,t}$	$sp_{20,t}$
MF	1-3	0.716	0.937	0.979	0.832	0.884
	4-7	0.557	0.724	0.911	0.500	0.474
	8-10	0.486	0.538	0.851	0.038	-0.006
CR,EI,N	1-3	0.836	0.891	0.973	0.914	0.932
	4-7	0.560	0.367	0.908	0.611	0.398
	8-10	0.420	0.022	0.818	-0.613	-0.668
TVR,EI,N	1-3	0.855	0.909	0.969	0.898	0.922
	4-7	0.503	0.385	0.906	0.535	0.466
	8-10	0.411	0.202	0.812	-0.404	-0.495
CR,CI,N	1-3	0.813	0.905	0.973	0.877	0.899
	4-7	0.540	-0.014	0.907	0.619	0.433
	8-10	0.399	-0.446	0.808	-0.491	-0.583
CR,EI,S	1-3	0.839	0.901	0.936	0.844	0.869
	4-7	0.554	0.524	0.896	0.503	0.416
	8-10	0.428	0.382	0.803	-0.575	-0.599
TVR,EI,S	1-3	0.843	0.922	0.959	0.872	0.894
	4-7	0.525	0.465	0.893	0.479	0.446
	8-10	0.411	0.387	0.783	-0.448	-0.477
CR,CI,S	1-3	0.829	0.902	0.959	0.851	0.878
	4-7	0.547	0.519	0.894	0.498	0.417
	8-10	0.424	0.395	0.797	-0.570	-0.591

We calculate  $E_t Z_{t+k}$  implied by the model and an unconstrained VAR(1) model for  $k = 1, 2, 3, \dots, 10$ . After calculating the correlation coefficient of the two series for different  $k$ 's, we calculate the mean correlations for  $k = 1-3$ ,  $k = 4-7$  and  $k = 8-10$  periods.

Table 7: **Correlation of Innovations**

Panel A: Correlation of Innovations implied by the Model

Variable	$y$	$i$	$sp_{12}$	$sp_{20}$
$\pi$	0.636 ( 0.109)	-0.130 ( 0.124)	0.495 ( 0.153)	0.424 ( 0.148)
$y$		-0.399 ( 0.109)	0.759 ( 0.090)	0.692 ( 0.094)
$i$			-0.770 ( 0.068)	-0.824 ( 0.054)
$sp_{12}$				0.990 ( 0.004)

Panel B: Correlation of Innovations implied by an Unrestricted VAR(1)

Variable	$y$	$i$	$sp_{12}$	$sp_{20}$
$\pi$	0.024 ( 0.121)	0.340 ( 0.128)	-0.207 ( 0.142)	-0.229 ( 0.139)
$y$		0.236 ( 0.126)	-0.033 ( 0.120)	-0.066 ( 0.130)
$i$			-0.680 ( 0.066)	-0.775 ( 0.054)
$sp_{12}$				0.974 ( 0.005)

This Table shows the correlation of the innovations to inflation, the output gap, interest rate, 3 year term spread and 5 year term spread. Panel A shows the correlations implied by the structural model and Panel B shows the sample correlation of VAR(1) innovations. Standard errors were constructed using the bootstrap procedure described in the text.

Table 8: GMM Estimates of the Structural Parameters

Parameter	CR,EI,N	Std Error		Cross-Model			
	Estimate	GMM	Bootstrap	Average	Std Error	Min	Max
$\delta$	0.611	( 0.010)	( 0.031)	0.572	( 0.038)	0.525	0.626
$\kappa$	0.064	( 0.007)	( 0.022)	0.075	( 0.044)	0.041	0.174
$\sigma$	3.156	( 0.466)	( 1.632)	3.205	( 0.322)	2.727	4.952
$\eta$	4.294	( 0.470)	( 1.383)	4.780	( 1.024)	3.469	6.796
$\rho$	0.723	( 0.028)	( 0.083)	0.799	( 0.068)	0.705	0.915
$\beta_\pi$	1.525	( 0.148)	( 0.251)	2.381	( 0.954)	1.525	2.999
$\gamma$	0.001	( 0.047)	( 0.020)	0.013	( 0.020)	0.001	0.057
$\lambda$	0.958	( 0.006)	( 0.026)	0.945	( 0.020)	0.910	0.975
$\omega$	0.877	( 0.013)	( 0.031)	0.924	( 0.040)	0.810	0.979
d	0.866	( 0.014)	( 0.041)	0.959	( 0.062)	0.757	0.990
$\sigma_{AS}$	1.249	( 0.053)	( 0.123)	1.326	( 0.162)	1.191	1.690
$\sigma_{IS}$	0.671	( 0.033)	( 0.055)	0.682	( 0.027)	0.658	0.739
$\sigma_{MP}$	2.177	( 0.119)	( 0.287)	2.538	( 1.105)	1.222	4.18
$\sigma_{y^n}$	1.380	( 0.115)	( 0.817)	1.691	( 0.878)	0.601	2.947
$\sigma_{\pi^*}$	0.730	( 0.059)	( 0.723)	0.854	( 0.698)	0.370	2.388
Implied	CR,EI,N	Std Error		Cross-Model			
Parameter	Estimate	GMM	Bootstrap	Average	Std Error	Min	Max
$\mu$	0.424	( 0.013)	( 0.026)	0.420	( 0.013)	0.401	0.445
$\phi$	0.134	( 0.017)	( 0.029)	0.126	( 0.023)	0.085	0.160
$\varphi_1$	0.500	( 0.003)	( 0.007)	0.395	( 0.229)	0.492	0.533
$\varphi_2$	0.492	( 0.003)	( 0.005)	0.377	( 0.219)	0.466	0.497
$\varphi_3$	0.008	( 0.002)	( 0.003)	0.003	( 0.004)	0.000	0.009
$\tilde{\varphi}_2$				0.934	( 0.006)	0.929	0.941
$\tilde{\varphi}_3$				0.048	( 0.012)	0.034	0.057
$\hat{\chi}$				2.022			
$\hat{\omega}$				0.025			
$\hat{\tau}$				0.703			
$\hat{\theta}$				0.818			
$\hat{\zeta}$				0.293			
$\hat{\vartheta}$				1.685			

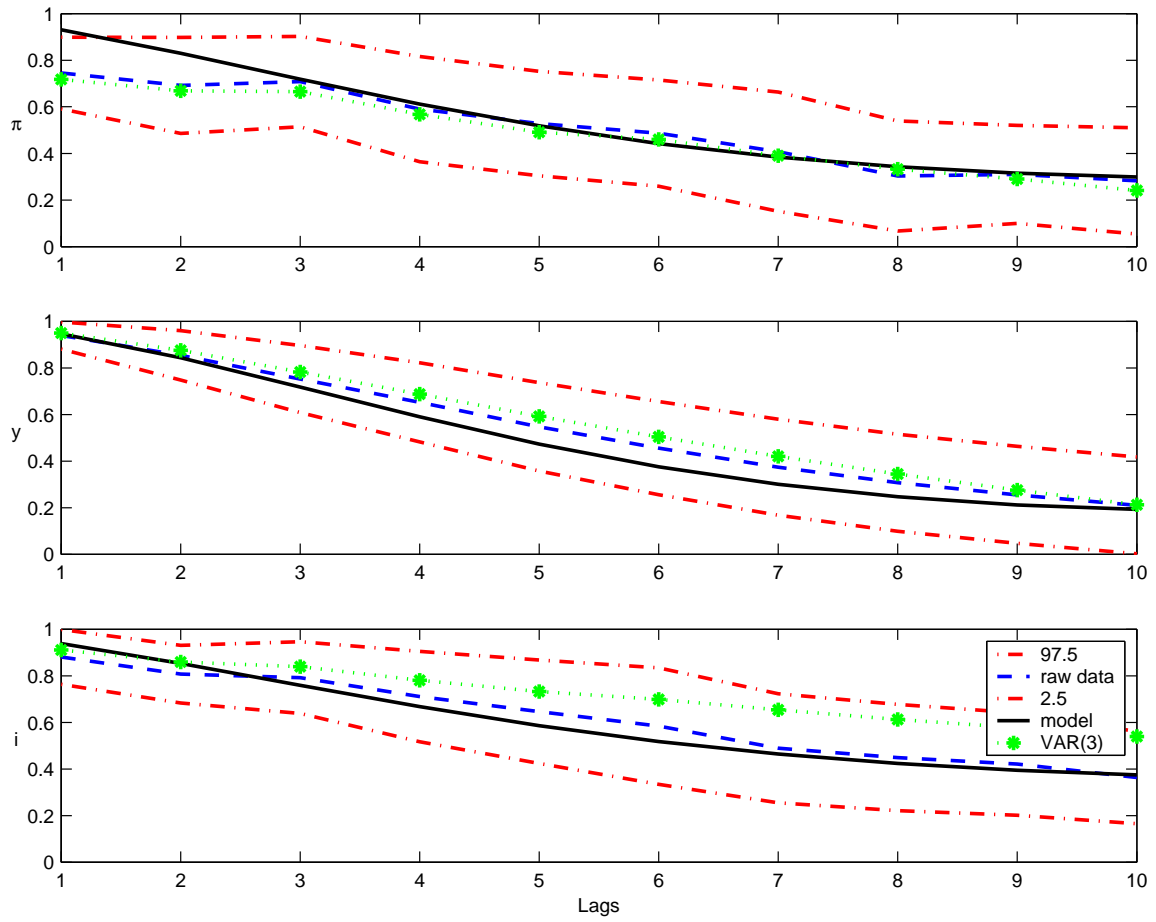
The second column reports the parameter estimates for model [CR,EI,N]. The third and fourth columns list the GMM standard errors of the structural parameters and those obtained through the bootstrap procedure. The fifth to eighth columns show the mean, standard deviation, minimum and maximum of the 7 models' estimates. The parameters with a tilde are those implied by the standard inflation target specification i.e., where  $\varphi_1=0$ . The hatted parameters are those implied by the microfounded model. To facilitate the interpretation of the interest rate response to expected inflation,  $\beta_\pi$  denotes the long-run response of the interest rate to expected inflation. For the EI models,  $\beta_\pi = 1 + \beta$ . For the remaining models,  $\beta_\pi = \beta$ .

Table 9: **Term Spreads and Macroeconomic Prediction: Projection Coefficients**

$\beta_\pi$	$\omega_{\pi,sp}$	$\omega_{\pi,i}$	$\omega_{\pi,i-sp}$	$\omega_{y,sp}$	$\omega_{y,i}$	$\omega_{y,i-sp}$
0.5	-0.256	-0.706	-0.450	0.529	-0.123	-0.652
1.1	0.369	0.082	-0.287	0.529	0.108	-0.435
1.5	0.563	0.300	-0.263	0.475	0.106	-0.369

$\beta_\pi$  is the long-run response of the interest rate to expected inflation (in our [CR,EI,N] model it is  $1 + \beta$ ).  $\omega_{j,sp}$  is the sum of the equation  $j$  reduced-form coefficients on the spreads ( $j = \pi, y$ , i.e. inflation and output equations, respectively).  $\omega_{j,i}$  is the equation  $j$  coefficient on the interest rate. Finally  $\omega_{j,i-sp} = \omega_{j,i} - \omega_{j,sp}$ ; it represents the overall reduced-form impact of the interest rate on inflation.

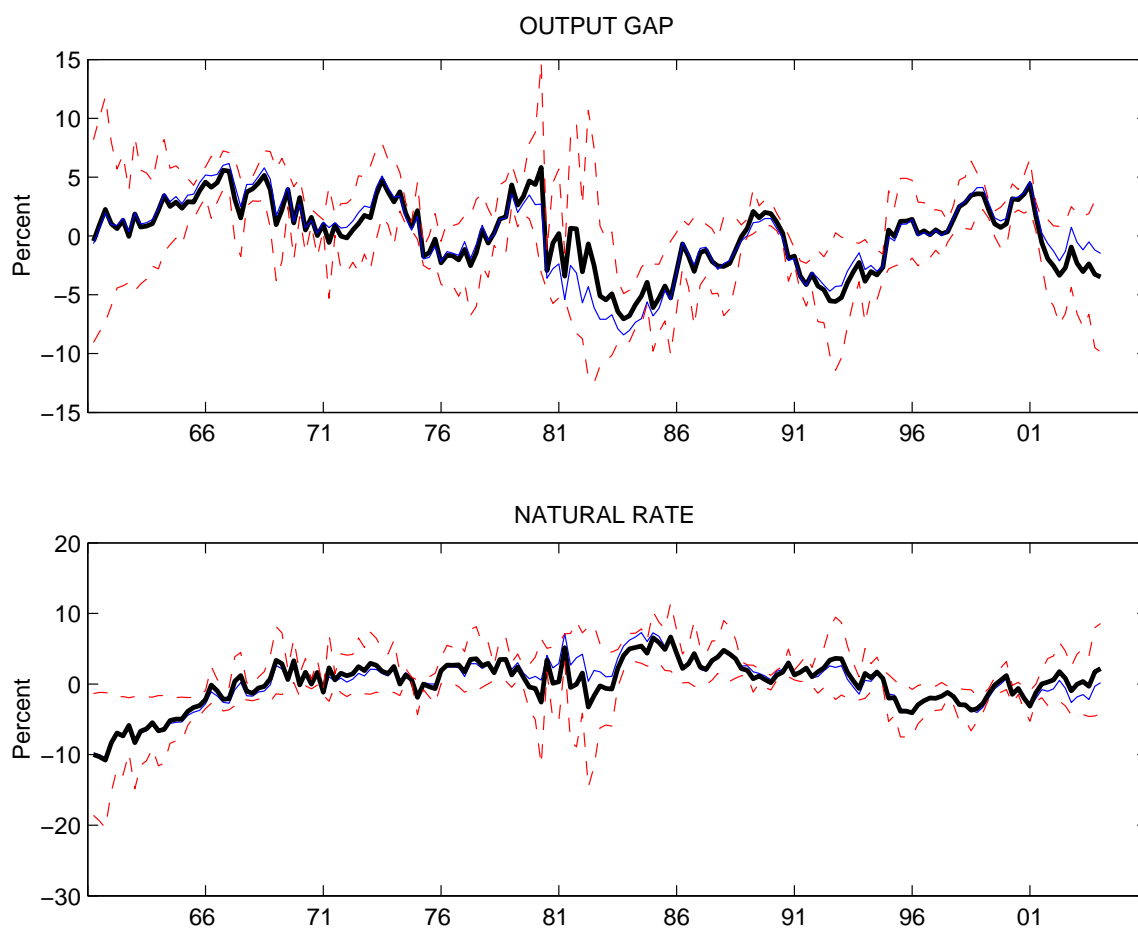
Figure 1: Autocorrelograms of the Raw data, the Model and a VAR(3)



This figure compares the empirical autocorrelograms of inflation, the output gap and the interest rate with those implied by the model and unrestricted VAR(3). The dash-dotted lines represent the 95% confidence interval constructed by a GMM estimation of the empirical autocorrelation.

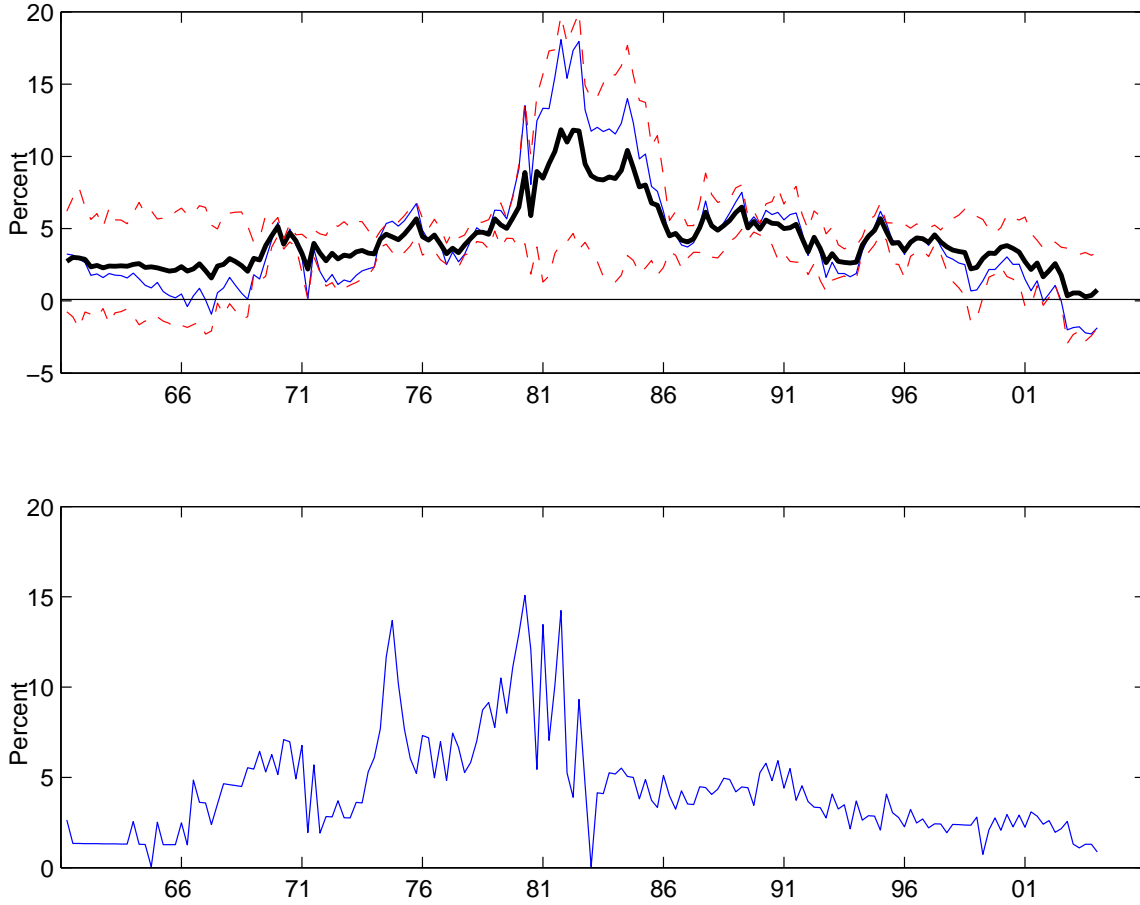


Figure 2: Output Gap and Natural Rate of Output



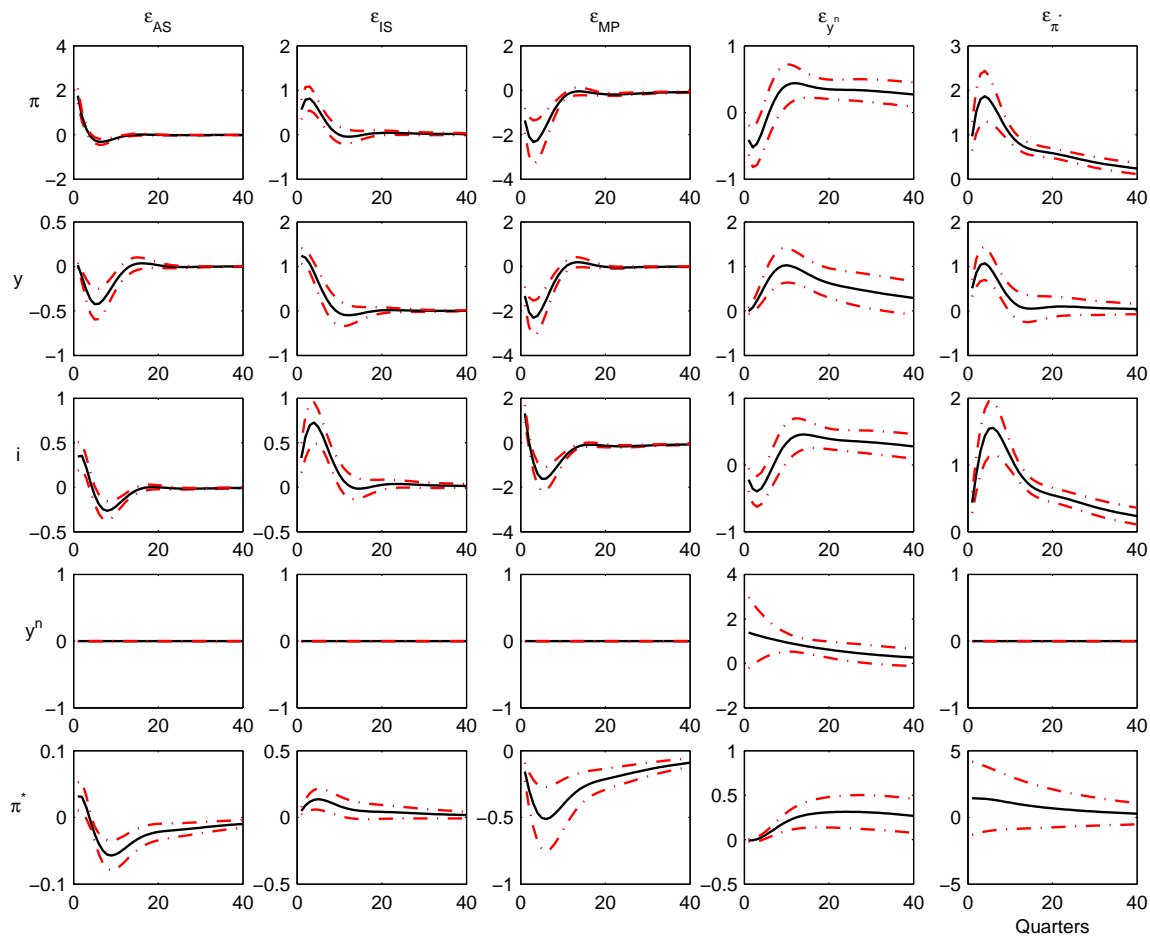
The top Panel shows the average output gap across the 7 models we estimate (thick line) and the model [CR,EI,N] output gap (thin line) for our sample period: 1961:1Q-2003:4Q. The bottom Panel shows the average natural rate across the 7 models we estimate (thick line) and the model [CR,EI,N] natural rate (thin line). Both panels also graph confidence bands in dashed lines. The confidence bands were constructed adding and subtracting 2 cross-model standard deviations to the average values.

Figure 3: Inflation Target and Inflation



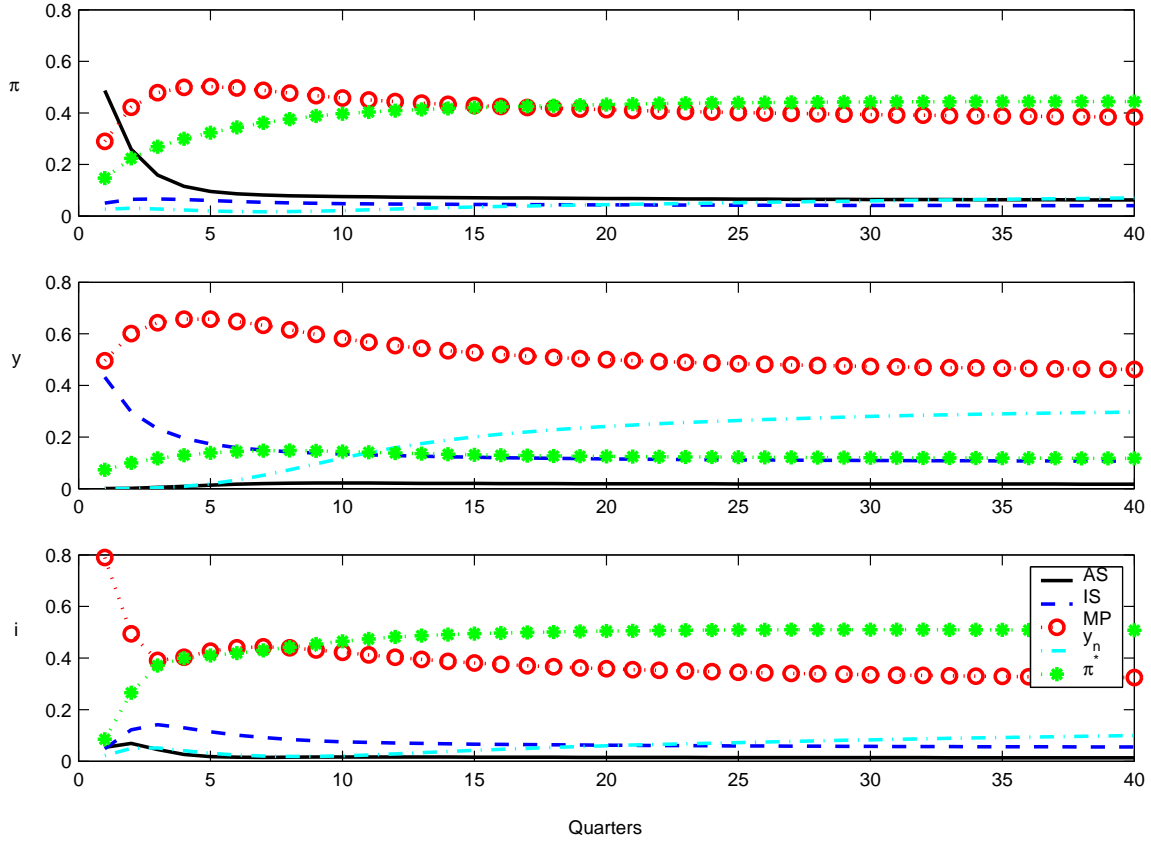
The top Panel shows the average inflation target across the 7 models we estimate (thick line) and the model [CR,EI,N] inflation target (thin line) for our sample period: 1961:1Q-2003:4Q. It also graphs the confidence bands in dashed lines. The confidence bands were constructed adding and subtracting 2 cross-model standard deviations to the average inflation target. The bottom Panel shows the CPI inflation rate.

Figure 4: Impulse Response Functions of Macro Variables



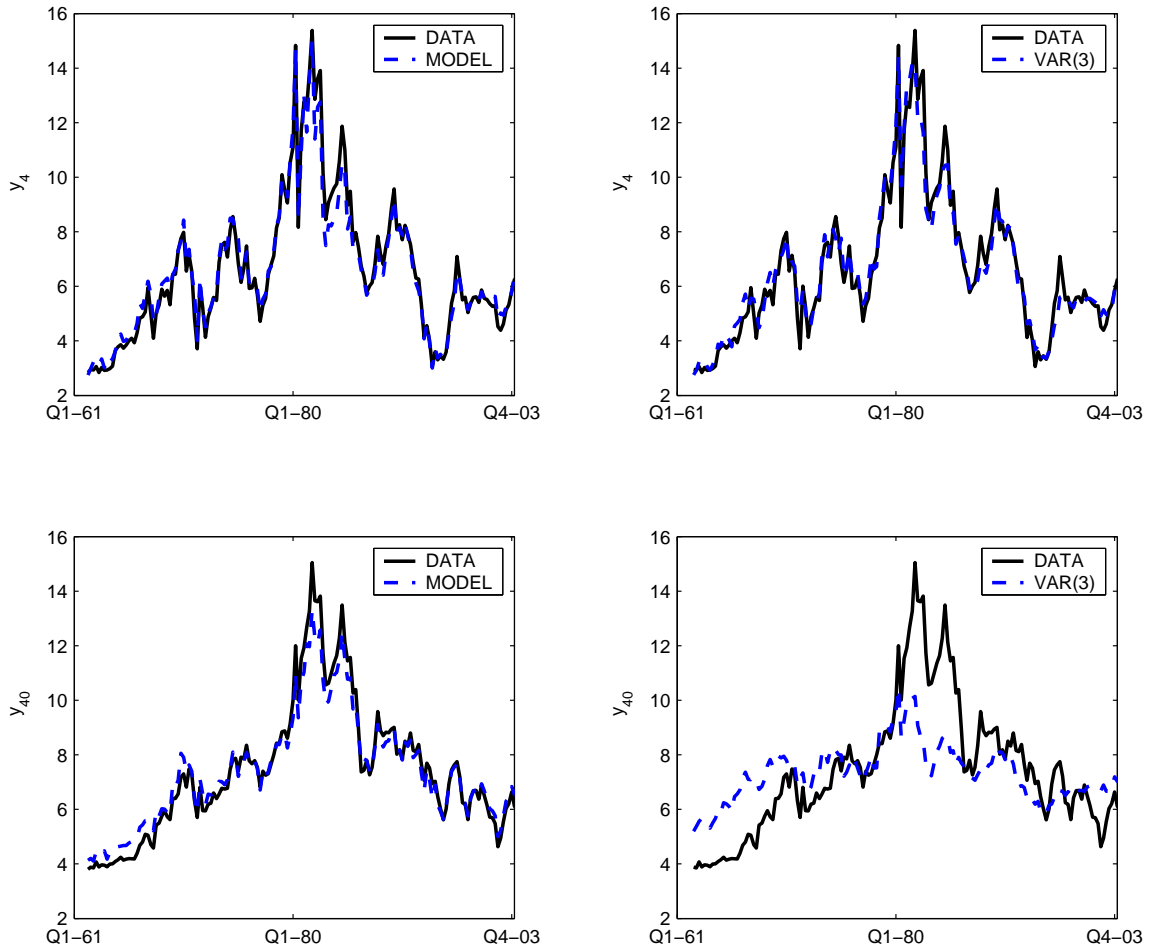
This figure shows the impulse response functions (in percentage deviations from steady state) of the five macro variables to the structural shocks. 95% confidence intervals appear in dashed lines and were constructed using the bootstrap procedure described in the text.

Figure 5: Variance Decompositions for the Macro Variables



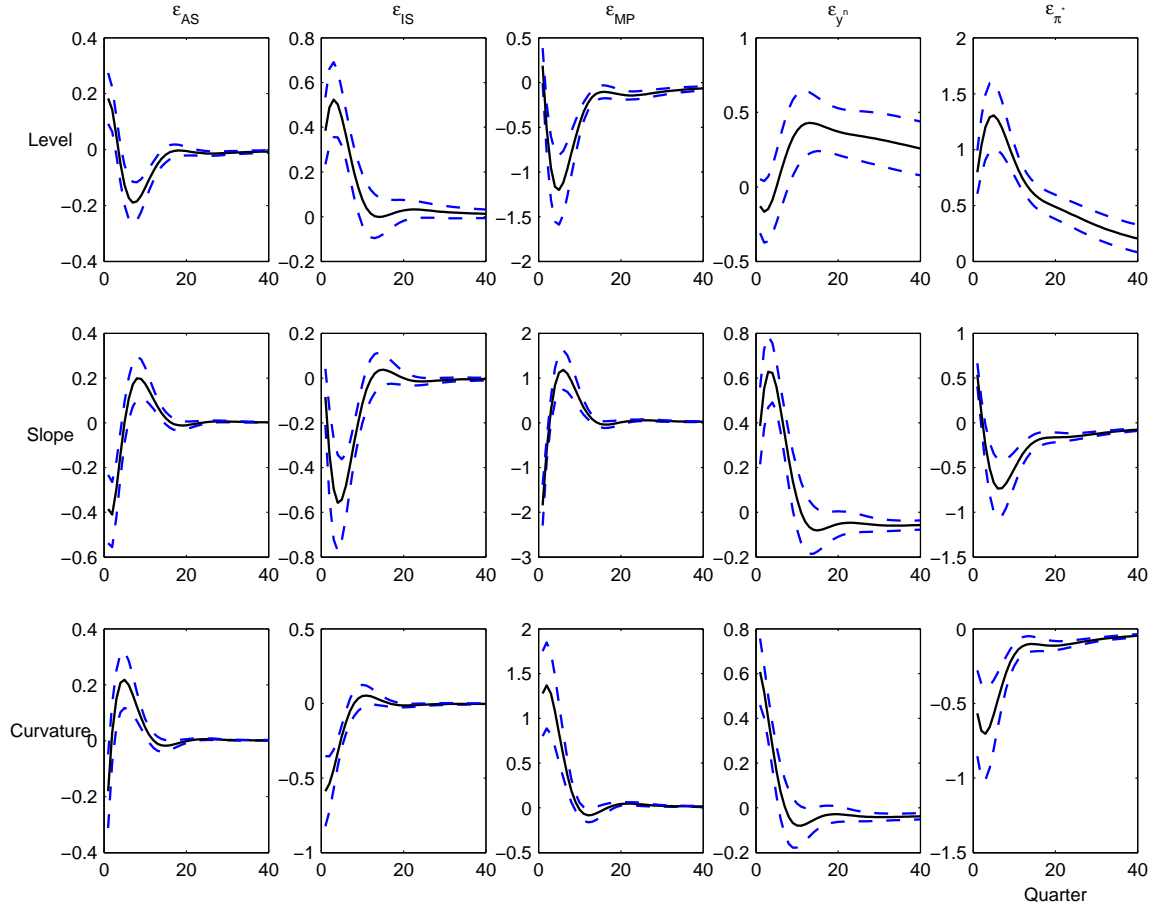
This Figure shows the variance decomposition at different time horizons for the macro variables in terms of the five structural macro shocks. The variance decomposition of a variable at quarter  $h$  represents the percentage of the  $h$ -step forecast variance explained by each shock. Given our model reduced-form:  $z_t = \Omega_z z_{t-1} + \Gamma_z \epsilon_t$   $\epsilon_t \sim N(0, D)$ , where inflation, de-trended output and the interest rate are the first three elements in the vector  $z_t$ . The  $h$ -step ahead mean square error is given by:  $MSE(h) = \sum_{i=0}^{h-1} \Omega_z^i \Gamma_z D \Gamma_z' (\Omega_z')^i$ , where the  $h$ -step ahead  $k$ -th equation MSE ( $mse(h,k)$ ) is the  $(k, k)$  element in the matrix  $MSE(h)$ . Let  $I(h,j) = \sum_{i=0}^{h-1} \Omega_z^i \Gamma_z F \Gamma_z' (\Omega_z')^i$ , where  $F$  is a matrix with zeros everywhere except in its  $(j,j)$  entry, where it coincides with matrix  $D$ . The impact of the  $j$ -th shock in the MSE of a variable  $k$  is given by the  $(k,k)$  element in  $I(h,j)$ . The contribution of a shock  $j$  to the  $h$ -step ahead variance of variable  $k$  is then given by  $VD(j,k,h) = \frac{I(h,j)}{mse(h,k)}$ .

Figure 6: Fit for Yields



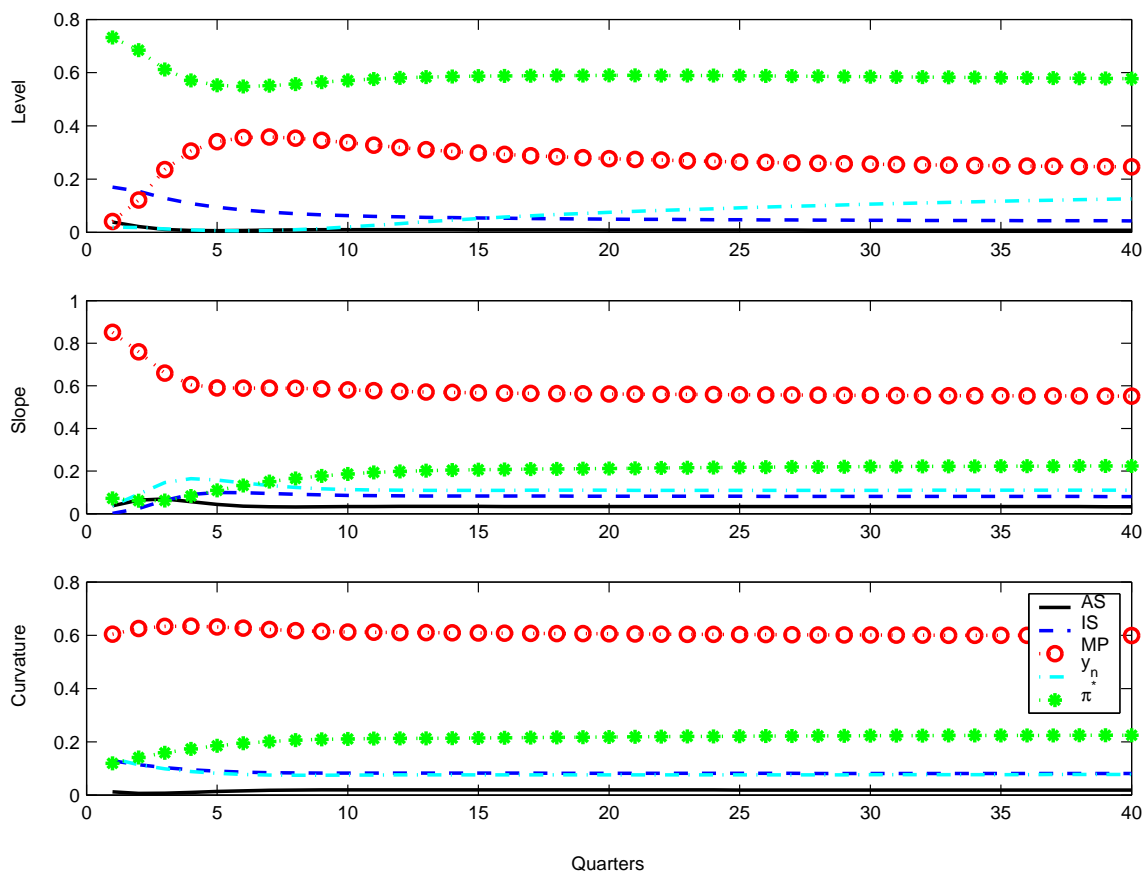
This Figure compares the fit of the 1 and 10 year yields provided by model [CR,EI,N] with that implied by a VAR(3) for our sample period: 1961:1Q-2003:4Q.

Figure 7: Impulse Responses of Term Structure Factors to Macro Shocks



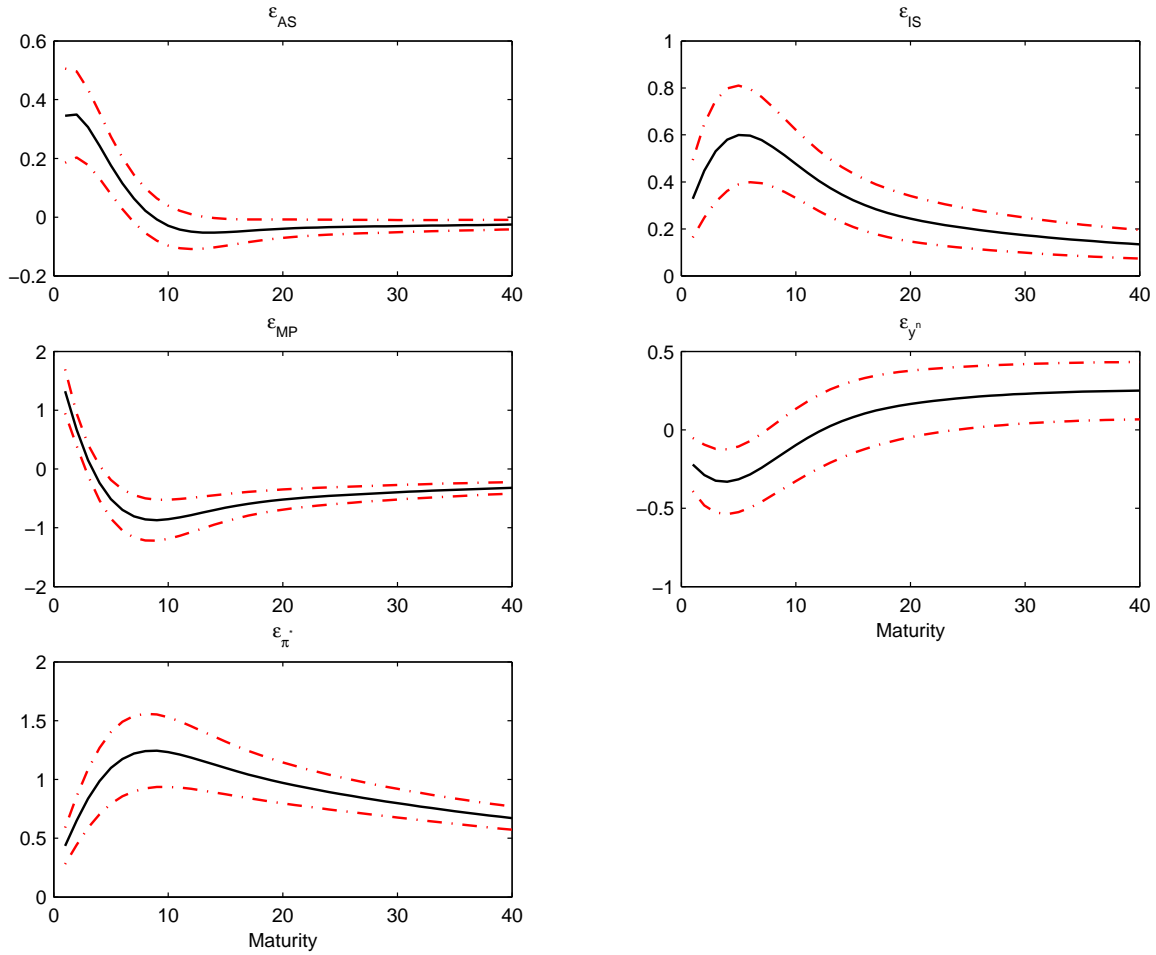
This figure shows the impulse response functions (in percentage deviations from steady state) of the three term structure factors - level, slope and curvature - to the structural shocks. The slope is computed as the 5 year spread. The curvature factor is computed as the sum of the interest rate and 5 year rate minus twice the one year rate. 95% confidence intervals appear in dashed lines and were constructed using the bootstrap procedure described in the text.

Figure 8: Variance Decompositions for the Term Structure Factors



This Figure shows the variance decomposition of the term structure factors in terms of the five structural macro shocks. The variance decomposition of a variable at quarter  $h$  represents the percentage of the  $h$ -step forecast variance explained by each shock. They were constructed as explained in the note to Figure 5, with the appropriate transformations of the variable in  $z_t$  into level, slope and curvature.

Figure 9: Contemporaneous Responses of Yields to Macro Shocks



This figure shows the contemporaneous responses of yields of different maturities to the five structural macro shocks. 95% confidence bands appear in dashed lines and were constructed using the bootstrap procedure described in the text.