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Uncovering the U.S. Term Premium: An Alternative Route

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Uncovering the U.S. Term Premium: An Alternative Route*

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October 24, 2007

Abstract

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JEL Classification: E4, G1, C5

Keywords: Interest Rates, Term Premium, Fractional Integration

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1 Introduction

What is the current term premium on U.S. long-term securities? In a recent article, Cochrane and Piazessi (2006) show that the estimates of the term premium on long-term treasury bonds crucially depend upon the order of integration assumed for short-term interest rates. So, for instance, during the early 2000s the term premium implied by an I(1) model for the short-rates was more than two percentage points above that implied by an I(0) model. As a result, these two alternative models give rise to very different implications for public policy (see Bernanke (2006)), government finances and long-term savers and investors.

In this paper we estimate a very stylized univariate fractionally integrated model which simultaneously determines the order of integration of the short-rate and the term premium. This approach thus avoids the controversial ex-ante choice between an I(0) and an I(1) model for the order of integration. It directly estimates the order of integration from the data and it permits the order of integration to be of a fractional value, allowing for alternative dynamics to those offered by standard I(0)/I(1) models.

The estimated order of integration of the short-term rate at the monthly frequency is 0.84. Thus the short-rate exhibits long-memory and is mean-reverting. An important difference between the dynamics of this process and those implied by a standard stationary I(0) process is that the mean reversion of our estimated model takes place at a much slower rate so that the long-distance expectations of the short-rate display a great deal of volatility, comparable to that of an I(1) process. In this way, the expectations of the short-rate implied by the fractionally integrated process are able to better fit the dynamics of both long-term yields and forward rates.

We find that the term premium has remained positive for most of our sample period

(1971-2006). In the early 2000s, the term premium increased and hovered above 2.5% until the beginning of 2004, when it began a steady and steep decline. At the end of 2006, the term premium was essentially zero. This decline is thus the main reason behind the decrease of the long-rates during this period, even if, as we show below, the expectations (at all horizons) of the short-rate were rapidly increasing following the interest rate hikes engineered by the Fed from 2004 to 2006. Our economic analysis shows that the term premium is significantly and positively related with unemployment. In contrast to the term premium implied by the I(0) model, the I(d)-implied premium significantly declines with both inflation and the real rate.

The discussion on the order of integration of the short-term nominal interest rates is not a new one. The interest-rate model employed, for instance, by Cox, Ingersoll, and Ross (1985), is stationary and mean-reverting I(0), whereas Campbell and Shiller (1987) assume a unit-root. The drawback of I(0) models is that they imply long-rates which are not volatile enough (see Shiller (1979)) whereas the problem with I(1) models is that they imply that the term premium necessarily increases (or decreases) forever as the bond maturity rises (see Campbell, Law, and MacKinlay (1997)). In alternative richer frameworks, Kozicki and Tinsley (2001) propose a model with shifting endpoints for the short-rate which is able to fit the term structure, whereas Ang and Bekaert (2002) propose regime-switching models for the short-rates.

Our paper is also related to a growing body of literature aiming at identifying the term premium in U.S. long-term securities. Kim and Wright (2005), Bernanke, Reinhart, and Sack (2005) and Backus and Wright (2007) also identify a significant decline of the term premium in rich reduced-form contexts as the culprit of the recent low long-term rates. They however assume an I(0) framework for the short-rate in their models and their implied estimates of the historical term premiums are quite different to ours. Other

papers which back out the term premium in a structural macro-finance model context are Ravenna and Seppala (2006) and Rudebusch, Swanson, and Wu (2006). While these works have the goal of determining the mechanics behind term premium dynamics, our approach directly focuses on identifying the correct term premium. Once we identify the term premium with our stylized model, we relate it to macro, financial and monetary policy variables.

There is a number of papers which use fractional integration techniques to estimate interest rates. Backus and Zin (1993) observed that the volatility of bond yields does not decline exponentially when the maturity of the bond increases. In fact, they noticed that the decline was hyperbolic and consistent with the fractionally integrated specification. Diebold and Lindner (1996) and Gil-Alana and Robinson (1997) are two examples of univariate estimations of interest rates allowing for long-memory. Phillips (1998) provided evidence based on semi-parametric methods that both ex-ante and ex-post U.S. real interest rates are fractionally integrated. Diebold and Inoue (2001) show that it may be difficult to distinguish between long-memory and regime-switching. In a recent article, Connolly, Gner, and Hightower (2007) find that long-memory persists even after controlling for regime switches in the mean of interest rates. None of these papers however examines the term premium implications of a fractionally integrated process for the short-term interest rate.

This article is structured as follows. Section 2 illustrates our approach to identify the term premium in a fractionally integrated framework. It shows the estimates of the term premium identified with the fractionally integrated model for the short-rate and discusses the implications of this model from economic and financial perspectives. Section 3 provides some economic intuition for the sources of term premium dynamics in our fractionally integrated framework. Section 4 concludes.

2 Identifying the Term Premium: An Alternative Approach

We start this section by describing our statistical approach to identify the term premium without taking an ex-ante decision on the order of integration of the short-rate. We then estimate the fractional integration model and compare the resulting term premium with those implied by standard I(0) and I(1) processes. After that, we examine the unusually low long interest rates of the last years in the context of our I(d) model for the short-rate. Finally we show that our results are robust to the use of multivariate specifications and interest rate decompositions.

2.1 A Fractional Integration Approach

As Backus and Wright (2007) point out, there is an identity relation between the long-term (ten-year) rate (i_t^n) and the sum of current and expected short-term (one-year) rates (i_t) plus a time-varying term premium ($tp_{t,n}$). A standard characterization of this relation can be expressed as follows:

$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}. \tag{1}$$

Therefore, the key element for the identification of the term premium are the expectations of the short-term rate. Researchers have employed a wide variety of models to characterize the short-rate. Among the two simplest models are the standard I(0) and I(1) time series processes:

$$(1-L)^k(i_t-\mu) = \epsilon_t \tag{2}$$

$$\epsilon_t = \rho\epsilon_{t-1}+\xi_t \tag{3}$$

where ξ_t is assumed to be i.i.d. If $k=0$, this model is a stationary AR(1) process, whereas if $k=1$, we have an ARIMA(1,1,0) process. We estimated these two models with the zero-coupon bond one-year rates retrieved from the Gurkaynak, Sack, and Wright (2006) database.[1] The sample period goes between August 1971 and December 2006 and the frequency is monthly. As it turns out, the implied term premiums by these two models are significantly different. Figure 1 shows that there are important divergences across term premiums in the last four decades. For instance, at the beginning of 2004 the I(1) model predicts a term premium more than two percentage points above the I(0) model. Additionally, the I(0)-implied premium has been negative during 2005 and 2006, while the I(1) premium has remained positive during most of these two years. This is the point which Cochrane and Piazessi (2006) illustrate in a multivariate context: There are potentially huge bond pricing model specification errors.

The main contribution of this article is to use a simple fractional integration model to endogenously determine both the order of integration of the short-term process and the associated term premium. In this alternative way, we avoid the controversial ex-ante decision on what the order of integration of the short-term process *is*. We simply let

[1]Results with the Fama and Bliss (1987) database did not reveal any significant difference.

the data speak and estimate the following ARFIMA(1,d,0) process for the short-rate:

$$(1-L)^d(i_t-\mu) = \epsilon_t \tag{4}$$

$$\epsilon_t = \rho\epsilon_{t-1}+\xi_t \tag{5}$$

where d is the (possibly fractional) order of integration of the short-rate. This process can also be expressed in a single equation as:

$$(1-\rho L)(1-L)^d(i_t-\mu)=\xi_t. \tag{6}$$

The above expression can be represented in terms of an infinite autoregressive process of the form:

$$\psi(L;\rho,d)(i_t-\mu)=\xi_t \tag{7}$$

where $\psi(L;\rho,d)=\sum_{j=0}^{\infty}\psi_j L^j$, and $(i_t-\mu)=0$, for $t\leq 0$.[2] As it is well-known, the dynamics of this process depend on the values of d and ρ, being crucial the value of d. If $d=0$, the series is a covariance stationary process and possesses 'short memory', with the autocorrelations decaying fairly rapid. If d belongs to the interval $(0,0.50)$, i_t is still covariance stationary, but both the autocorrelations and the response of a variable to a shock take much longer to disappear than in a standard ($d=0$) stationary case.[3] If $d\in[0.50,1)$, the series is no longer covariance stationary, but is still mean-reverting, with the effect of the shocks dying away in the long-run. If $d=1$ the series is a non-stationary, non-mean-reverting $I(1)$ process.

[2]This is a standard assumption in the empirical work with fractional integration. For further details, see Gil-Alana and Hualde (2007), dealing with the Type I and Type II I(d) processes.

[3]If $d=0$ and ϵ_t follows an AR process, the decay in the autocorrelations is exponentially rapid compared with the I($d, d>0$) case, where the decay is hyperbolic.

2.2 Empirical Results

We employ the econometric method devised by Robinson (1994) which allows for testing (possibly) fractionally integrated processes. Appendix A shows all the details related with this method, which is based on the Lagrange Multiplier (LM) principle in a parametric framework. We should note here that even though Robinson's (1994) method is a testing procedure, we can approximate the estimation of d by choosing the value that produces the lowest statistic in absolute value. Note that this method is based on the Whittle function, which is an approximation to the likelihood function. An advantage of this method is that it is valid for any real value of d, therefore encompassing stationary ($d < 0.5$) and non-stationary ($d \geq 0.5$) hypotheses, unlike other methods that require first differencing to render the series stationary prior to the estimation of d. Nevertheless, we also employed the maximum likelihood approach in the time domain by Sowell (1992) and the results were almost exactly the same as those reported in the paper.

The estimates for the same data, frequency and period than the I(0)/I(1) models illustrated above, are reported in table 1. It shows that $\hat{d} = 0.84$ (and significantly different from 1 and 0), $\hat{\rho} = 0.47$ and $\hat{\mu} = 5.53$ (and significantly different from 0). Therefore, the interest rate exhibits long-memory, is non-stationary but mean-reverting. table 1 also shows that the long-rate exhibits similar dynamics, with $\hat{d} = 0.89$.

Figure 2 compares the term premium implied by our fractional integration model with those implied by the I(0) and I(1) models. The dynamics of the I(d) term premium are quite similar to those implied by the I(1) model. This is sensible because its order of integration (0.84) is quite similar to 1. Indeed, the correlation between the I(d)-implied premium and both the I(1) and I(0)-implied premiums are 0.79 and 0.56 respectively.[4]

[4]We estimate the correlation between a variable x_t and y_t and the respective standard error using a generalized method of moments (GMM) estimation. The following moment condition is estimated:

Figure 2 shows that there is a great deal of variation in all three term premiums but, which one displays higher persistence? We can answer this question by estimating the order of integration of the three estimated term premiums. This is what we do and report on table 2. All three term premiums display long-memory, with the d parameter being in the [0.5, 1) region. The I(d) implied premium displays the least memory of the three models, with $d = 0.69$, followed closely by the I(1)-implied premium (0.71) and by the I(0)-implied (0.86). Several conclusions can be reached from these estimates. First, the $I(d)$ model is the one which explains a larger part of the long-rates through the expectations part of our general decomposition. Interestingly, the term premium order of integration is statistically lower than the order of integration of the long-term rate. In other words, the expectations of the short-term rate implied by our model explain a significant part of the dynamics of the long-rate. Moreover, the standard deviation of the long-rate implied by the I(d) model and the implied expectations of future short-rates is 2.48, very close to the historical one (2.34) and statistically indistinguishable from it. Additionally, the correlation between theoretical and historical ten-year spreads is 79%, and statistically significant. Second, the I(1)-implied premium has a very similar order of integration to the I(d)-premium and also its associated expectations also help explain the variations of the long rate. The opposite is true for the I(0) model, whose implied term premium has a significantly higher order of integration, very close to that of the long-term rate. Third, there is still substantial time-variation in the term premium. Therefore, the expectations hypothesis is clearly rejected by the data in our stylized setting.

$e_t = (x_t - \bar{x})(y_t - \bar{y}) - \rho(y_t - \bar{y})^2$, where ρ is the correlation coefficient, $\bar{x}$ is the sample mean of x_t and $\bar{y}$ is the sample mean of y_t. The weighting matrix is constructed using three Newey and West (1987) lags.

2.3 Long-distance Expectations and The "Conundrum"

Cochrane and Piazessi (2006) note that while long-distance expectations of short-rate in the I(0) model are very close to the sample mean, those in the I(1) model are very close to the current values of the short-rate. In other words, the I(1) expectations display much more volatility than the I(0) expectations. Figure 3 plots the expectations of the short-rates at different horizons for the three models. The top and bottom panels show the expectations for the I(0) and I(1) models, respectively, whereas the middle panel plots the analogous expectations associated with our I(d) model. As can be gleaned from this graph, even though the expectations of the short-rate at long-horizons exhibit mean-reversion, they are very volatile, similarly to the I(1) model. Figure 4 makes this point transparent, as it plots the expectations of the one-year rate ten years out for the three models. While the expectations of the I(0) model are close to the sample mean throughout the sample span, those implied by the I(1) model are very close to the current short-rate dynamics, as should be expected from a unit root model. The I(d) model yields expectations which exhibit *very* slow mean-reversion and are thus quite volatile. As Backus and Wright (2007) point out, this is a desirable feature of realistic term structure models.[5] Finally, we would like to emphasize that even though the expectations of the I(d) model are quite close to those of the I(1) model, the nature of the two models is very different. The first is mean-reverting, the second is not.

Our I(d)-implied premium (as well as the I(1)-implied) is very similar to one financial variable observable in real time, the term spread. Figure 5 shows that the two variables

[5]Gurkaynak, Sack, and Swanson (2005) and Bekaert, Cho, and Moreno (2005) manage to produce volatile long-distance short-rate expectations through the introduction of very persistent unobservable macro variables -such as the inflation target or the natural rate of output- which in their structural models are filtered through the term structure. These works, in turn, provide a macroeconomic interpretation to the important level factor discussed in the affine term structure models of the finance literature, such as Dai and Singleton (2000).

display a very similar co-movement. Its correlation is 0.75 and statistically significant. The reason is very simple. Since the expectations of the short-rate display long-memory, long-distance expectations are very similar to the current short-rate. As a result, the weighted sum of expectations of the short-rate is very similar to the short-rate itself, so that the term premium is very close to the ten-year term spread.

Policy-makers, bond-watchers and academics have been analyzing over the last two and a half years the remarks made by Alan Greenspan in front of the U.S. Congress in February 2005, when he dubbed "conundrum" the long-rate dynamics during 2004 and 2005. At that time the Fed was pursuing a contractionary monetary policy but, contrary to several episodes of previous decades, long-term rates were slightly decreasing. The simple, if general, decomposition of the long-rate in equation (1) shows that the dynamics of the long-rate are completely determined by those of the expectations and of the term premium. Of course, the I(0) and the I(1) explanations of the long-rate dynamics must be different, since they imply very different distant expectations for the short-rate. As Figure 4 showed, during 2004 and 2005, the ten-year expectations of the I(1) model were much lower than those of the I(0) model, although they were rapidly converging. The expectations of the I(d) model were hovering in between both models, but were closer to the I(1) model. Figure 6 plots the term premiums implied by the three models since the beginning of 2000. It shows clear divergences between the I(0) and I(1) models. The I(d)-implied premium lies again in between both, but closer to that implied by the I(1) premium, especially since the beginning of 2004. Thus, both the I(d) and I(1) models explain the conundrum in terms of a steep decline of the term premium in 2004, 2005 and 2006, coexisting with a rapid increase in the long-distance expectations of the short-rate. By August 2006, the term premium implied by the I(d) model was essentially zero, more than 2.5 percentage points lower than in the Spring of 2004.

Figure 7 plots the expectations of the short-rate at different horizons across models. There are six graphs which correspond to four important episodes occurred over the last 35 years (the beginning of the 1973 oil crisis, the 1980, 1981 and 2001 recessions) and two recent times (2003 and 2006). It is evident from the graphs that the predictions of the I(1) model are almost constant across horizons. In contrast, the I(0) model always predicts a fast convergence towards the sample mean so that the predictions of the short-rate 10 years out are very similar to this mean. The I(d) model exhibits very slow mean-reversion. For instance, in January 1980, at historically high interest rates, the prediction of the one-year rate one-year ahead is very close to the current rate (around 11%), whereas ten years ahead is slightly below 10%. Very similar dynamics are found in December 2001 and December 2006, but then the interest rates were at historically low levels, so that the predictions of the short-rate ten years out were 3.5% and 2.5%, respectively. Interestingly, the June 1981 and December 2006 episodes show that the reversion of the I(d) expectations can be non-monotonic, with the expectations path exhibiting a hump-shape.

2.4 Robustness

The results in the previous sections crucially depend on the estimate of the fractional integration parameter of the short-rate, d. In order to gauge the robustness of this parameter estimate to the short-run dynamics of the series, we employ the technique devised by Bloomfield (1973). This model is described in Appendix B and the essential idea is that it approximates ARMA structures with a reduced number of parameters. Moreover, this model can be flexibly accommodated in the context of Robinson's (1994) tests, and the researcher does not have to specify parametrically the process for the short-rate dynamics, as the procedure approximates the dynamics of alternative ARMA(p, q)

processes. We employ the model of Bloomfield (1973) with one and two parameters, which differ on the persistence degree of the I(0) dynamics of the interest rate. As table 3 shows, the estimates of the order of integration of the short-rate are 0.87 and 0.88, very close and statistically insignificantly different from the value originally estimated (0.84). Table 3 shows that the estimates of the order of integration of the long-rate (0.92 and 0.91) are also very similar to those of the original model (0.89).

Our framework contains two departures with respect to that in Cochrane and Piazessi (2006). We now show that our results are not affected by these differences. Firstly, they computed the term premium with the standard formula for forward interest rates:

$$f_t^n = E_t i_{t+n-1} + \hat{tp}_{t,n}. \tag{8}$$

That is, they compute the expectations of the one-year rate nine years out and subtract it from the ten-year forward rate to obtain the term premium. Figure 8 compares the term premiums obtained with this forward identity and those obtained with the decomposition of equation (1) across interest rate models. They display essentially the same dynamics, with significant correlations well above 95%. One slight difference between the two term premiums is that the one computed with the forward formula is typically above the one computed with our initial identity. This is especially the case in the first part of the sample for the I(0) model, when the short-rate was at historical high levels and the expectations of the short-rate nine years out were much lower than expectations of the short-rate one or two years out. Curiously, for the I(d) and I(1) models, the term premium computed with the forward prices is higher for the second part of the sample. This happens because the 10-year forward rate has remained regularly higher than the 10-year yield since the mid-80s, as Figure 9 shows.

Secondly, our short-rate forecast model is univariate, whereas the setting employed by Cochrane and Piazessi (2006) and other authors is multivariate. While we agree that there is predictability of future short-term rates based on current macroeconomic and financial variables, it is also true that much of that information is contained in the current value of the short-rate itself, for instance through monetary policy actions (see Taylor (1993) and Clarida, Galí, and Gertler (1999)). Thus, the short-term interest rate contains a great deal of information regarding past, current and expected values of macro and financial variables. Moreover, since macro and financial variables often display autocorrelation, this is in effect captured by the infinite autoregressive dynamic structure of our fractionally integrated model.

We also directly assess wether the results of univariate models hold in the multivariate setup employed by Cochrane and Piazessi (2006).[6] To this end, we compare the term premiums implied by univariate models (AR(1) and ARIMA(1,1,0)) and those implied by their multivariate counterparts (I(0) Vector Autoregressive (VAR(1)) in levels and I(1) Vector Error Correction (VECM) in spreads), respectively. The VAR and VECM models are computed with one, two, three, four and five-year yields. Figure 10 indeed shows that the univariate and multivariate term premiums are essentially the same across integration orders, with significant correlations above 95%. In other words, there is not significant additional information in the two-to-five-year yields in order to identify the ten-year premium.

[6]Working with yields and forwards results in almost identical implications.

3 Economic Analysis

In this section we try to develop some economic intuition on the sources of the I(d)-implied term premium dynamics. We first examine the impact of some important macro variables on the term premium in a linear regression context and compare the results with those implied by the I(0) and I(1) term premium. Then, we gauge the historical response of the monetary policy authority to the term premiums computed under the I(0), I(d) and I(1) models.

3.1 Macroeconomic Analysis

What are the factors behind the variations in the term premium? A straightforward approach to this question is to regress the I(d)-implied term premium on a standard set of macroeconomic variables. We choose monthly inflation, the unemployment rate, the ex-post real interest rate, and the output (industrial production) gap computed through with the Hodrick and Prescott (1997) filter. All series were retrieved from the Federal Reserve Bank of St. Louis Economic Research Database. We compare the results of our I(d)-implied premium with those under I(0) and I(1)-implied premiums.

We perform two sets of OLS regressions. First we try to uncover the individual effect of each variable on the term premium:

$$tp_{t,n} = \alpha_i + \beta_i x_{t,i} + \eta_{t,i}, \tag{9}$$

where $x_{i,t}$ is the i-th macro variable and $\eta_{t,i}$ is assumed to be i.i.d. Alternatively, we regress the term premium on the full set of observable macro variables (X_t) in order to

obtain the conditional effects of each time series:

$$tp_{t,n} = \alpha + \beta' X_t + \eta_t. \tag{10}$$

Table 4 shows the slope coefficients of both sets of regressions for the three premiums.[7] We first discuss the results for the I(d)-implied term premium regressions. There are significant relations between the four variables (inflation, the unemployment rate, the real rate and the output gap) and the term premium. We also tried including output (industrial production) growth in the regressions but it was never significant. Monthly inflation predicts a lower term premium. Thus our I(d) model implies that increases in inflation increase the average of current and expected short-term rates more than the long-rate. A higher unemployment rate is associated with a higher term premium. It could be that higher unemployment signals uncertainty about the state of the economy, which, in turn, is reflected in higher term premiums. Below, we further elaborate on this relation. Higher real rates predict lower term premiums. A plausible interpretation of this result is that a tougher monetary policy stance anchors down inflation expectations, lowering the required term premium. In the joint regression, a higher output gap predicts higher future inflation through excess demand, so that the term premium increases. Except for the output gap, the results of the individual regressions are consistent with the joint regression. Moreover, the size of the coefficients in the joint regression are larger than those in the individual ones for inflation, the real rate and the unemployment rate. Note also that the adjusted R^2 of the joint regression is very close to 57%, indicating a strong explanatory power of our set of variables on the time-varying term premium.

The results with the I(1)-implied premiums are very similar to those under the I(d) premiums. Their coefficients have the same signs, very similar statistical significance but

[7] Regressions on lagged values of X_t did not reveal significant differences.

an overall larger size than their I(d)-implied counterparts. This is not the case however for the regressions with the I(0)-implied premium. The regression of the term premium on inflation yields a positive, although not significant coefficient estimate, whereas the regression on the real rate is also positive and significant. For unemployment and the output gap, the signs are in agreement with the I(d) premium. The joint regression yields similar results to the individual regressions, except for the sign of the inflation coefficient (still non-significant) and the output gap, which now becomes positive.

Across term premium specifications, the unemployment rate appears as the strongest predictor of term premium dynamics. This is interesting because the unemployment rate is a real variable and no other variables apart from nominal interest rates are used in identifying the term premiums. To develop more intuition about the positive relation between the term premium and the unemployment rate, we plot the demeaned series of the unemployment rate and the term premiums in Figure 11. The relation among the term premiums and the unemployment rate appears striking, especially since the late 80s and at medium and long frequencies, when the dynamics of the unemployment rate almost mimic those of the term premiums. Indeed, the correlations between the three term premiums and the unemployment rate since 1988 range from 74% to 79%, and are statistically significant. Our stylized analysis thus suggests to closely follow the unemployment rate as a driver of term premium dynamics.

3.2 Monetary Policy

In a widely commented speech already as Federal Reserve Chairman, Bernanke (2006) said on March 20, 2006:

"What does the historically unusual behavior of long-term yields imply for the conduct of monetary policy? The answer, it turns out, depends critically on the source of that

behavior. To the extent that the decline in forward rates can be traced to a decline in the term premium, perhaps for one or more of the reasons I have just suggested, the effect is financially stimulative and argues for greater monetary policy restraint, all else being equal... However, if the behavior of long-term yields reflects current or prospective economic conditions, the implications for policy may be quite different indeed, quite the opposite."

A close read of Bernanke's statement reveals that this reference to the term premium as a monetary policy driver is related to its predictive power on future output. Therefore, it would be interesting to measure whether the monetary policy authority has reacted to term premiums historically We test this hypothesis in a simple monetary policy rule framework and estimate the following Taylor-type rule with smoothing, where the Fed reacts to contemporaneous values of inflation, the output gap and also the term premium, i.e.,

$$i_t^p = \rho i_{t-1}^p + (1-\rho)[\beta\pi_t + \gamma(y_t - \bar{y}_t) + \lambda tp_{t,n}], \tag{11}$$

where i_t^p is the monetary policy rate, in our case, the Federal funds rate. Taking Bernanke's comments literally, λ should be negative. But, has it been? We consider the possibility that the Federal Reserve reacted to the I(0) or I(1) premiums instead of the estimated I(d) term premium. We perform simple OLS regression analysis for different sample periods corresponding to different decades and Federal Reserve chairmen.

Table 5 shows the estimates of λ. Several results are worth highlighting. First, all the coefficient estimates across Fed chairmen and term premiums are found to be negative. Second, there is only one significant estimate, the response to the I(0)-implied premium in the Burns era.[8] This hints that the Fed may have been responding to (the wrong) term premium during the 70s. Third, the responses during the Burns era to the term premiums are quantitatively larger than those of the other two Fed chiefs across premiums. Fourth, the Greenspan responses are the smallest, especially for the (more empirically relevant)

[8]This result is robust to the inclusion of the unemployment rate as a regressor.

I(d) and I(1) premiums, where they are almost zero.

4 Conclusions

A clear advantage of our fractional integration approach is that it introduces a route to avoid the specification error present in many models of the term structure of interest rates. Our framework allows for the endogenous determination of both the order of integration of the interest rates and the associated term premiums. Cochrane and Piazessi (2006) ended up preferring the predictions and implications of their I(1) model over those of the I(0) model. According to our econometric tests, the I(1) model is indeed closer to our model.

While our approach can be a solution to the model specification errors, the sampling error of both forecast and term premium calculations continues to be a concern for researchers (see Rudebusch (2007)). In this paper, we have shown that the asymptotic confidence interval for the short-rate order of integration parameter is quite tight. Nevertheless, treating the order of integration as a parameter estimate adds another source of uncertainty which could translate into uncertain forecasts. We intend to measure these errors in future research. The advantage of the fractional integration framework is that researchers can focus on only one source of uncertainty instead of two sources.

In future work we also intend to distinguish between nominal and real term premiums. In this respect, articles by Kim and Wright (2005) and Ang, Bekaert, and Wei (2007) are significant contributions. In a fractional integration framework, it would be interesting to elucidate where the long-memory for short-rates is coming from. At a theoretical level, an argument employed to justify fractional integration is the aggregation of heterogeneous AR processes (see, for instance, Robinson (1978) Granger (1980)). It could be that

an important component of interest rate dynamics is coming, for instance, from the diverse formation of inflation expectations. Finally, we intend to investigate the term premium in a multivariate framework, including relevant macro and financial variables. In section 3 we actually identified significant relations between the term premium and macro variables. In this context, the application of the emerging fractional cointegration techniques (see Robinson and Hualde (2003) and Johansen (2007) among others) may prove a fruitful avenue of research.

References

Ang, Andrew, and Geert Bekaert, 2002, Regime Switches in Interest Rates, *Journal of Business and Economic Statistics* 20, 163–182.

Ang, Andrew, Geert Bekaert, and Min Wei, 2007, The Term Structure of Real Rates and Expected Inflation, *Forthcoming Journal of Finance*.

Backus, David K., and Jonathan H. Wright, 2007, Cracking the Conundrum, *Brooking Papers on Economic Activity* 1, 293–316.

Backus, David K., and Stanley E. Zin, 1993, Long-Memory Inflation Uncertainty. Evidence from the Term Structure of Interest Rates, *Journal of Money, Credit and Banking* 25, 681–700.

Bekaert, Geert, Seonghoon Cho, and Antonio Moreno, 2005, New-Keynesian Macroeconomics and the Term Structure, *NBER Working Paper No 11340*.

Bernanke, Ben S., 2006, Remarks before the Economic Club of New York, New York March 20, 2006, .

Bernanke, Ben S., Vincent Reinhart, and Brian Sack, 2005, Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment, *Brooking Papers on Economic Activity* 2, 1–78.

Bloomfield, Peter, 1973, An Exponential Model in the Spectrum of a Scalar Time Series, *Biometrika* 60, 217–226.

Campbell, John Y., Andrew W. Law, and A. Craig MacKinlay, 1997, *The Econometrics of the Financial Markets*. (Princeton University Press Princeton, NJ).

Campbell, John Y, and Robert J Shiller, 1987, Cointegration and Tests of Present Value Models, *Journal of Political Economy* 95, 1062–88.

Clarida, Richard H., Jordi Galí, and Mark Gertler, 1999, The Science of Monetary Policy: A New Keynesian Perspective, *Journal of Economic Literature* 37, 1661–707.

Cochrane, John, and Monika Piazessi, 2006, Decomposing the Yield Curve, *Mimeo, University of Chicago.*

Connolly, Robert A., Z. Nuray Gner, and Kenneth N. Hightower, 2007, Evidence on the Extent and Potential Sources of Long Memory in U.S. Treasury Security Returns and Yields, *Journal of Money, Credit and Banking* 39, 689–702.

Cox, John, Jonathan Ingersoll, and Stephen Ross, 1985, A theory of term structure of interest rates, *Econometrica* 53, 385–408.

Dai, Qiang, and Kenneth Singleton, 2000, Specification Analysis of Affine Term Structure Models, *Journal of Finance* 55, 531–552.

Diebold, Francis X., and Atsushi Inoue, 2001, Long Memory and Regime Switching, *Journal of Econometrics* 105, 131–159.

Diebold, Francis X., and Peter Lindner, 1996, Fractional Integration and Interval Prediction, *Economics Letters* 50, 305–313.

Fama, Eugene F., and Robert R. Bliss, 1987, The Information in Long-Maturity Forward Rates, *American Economic Review* 77, 680–692.

Gil-Alana, Luis, and Javier Hualde, 2007, Fractional Integration and Cointegration, *Forthcoming, Palgrave Handbook of Applied Econometrics.*

Gil-Alana, Luis A., and Peter M. Robinson, 1997, Testing of Unit Roots and Other Nonstationary Hypotheses in Macroeconomic Time Series, *Journal of Econometrics* 80, 241–268.

Granger, Clive W. J., 1980, Long Memory Relationships and the Aggregation of Dynamic Models, *Journal of Econometrics* 14, 227–238.

Gurkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2006, The U.S. Treasury Yield Curve: 1961 to the Present, *Forthcoming Journal of Monetary Economics*.

Gurkaynak, Refet T., Brian Sack, and Eric Swanson, 2005, The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models, *American Economic Review* 95, 425–436.

Hodrick, Robert J, and Edward C Prescott, 1997, Postwar U.S. Business Cycles: An Empirical Investigation, *Journal of Money, Credit and Banking* 29, 1–16.

Johansen, Soren, 2007, A Representation Theory for a Class of Vector Autoregressive Models for Fractional Processes, *Mimeo, University of Copenhagen*.

Kim, Don H., and Jonathan H. Wright, 2005, An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates, *Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series No 2005-33*.

Kozicki, Sharon, and Peter A. Tinsley, 2001, Shifting Endpoints in the Term Structure of Interest Rates, *Journal of Monetary Economics* 47, 613–652.

Newey, Whitney K, and Kenneth D West, 1987, A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703–708.

Phillips, Peter C.B., 1998, Econometric Analysis of Fisher's Equation, *Yale University, Cowles Foundation Discussion Paper 1180*.

Ravenna, Federico, and Juha Seppala, 2006, Monetary policy and rejections of the expectations hypothesis, Research Discussion Papers 25/2006 Bank of Finland.

Robinson, Peter M., 1978, Statistical Inference for a Random Coefficient Autoregressive Model, *Scandinavian Journal of Statistics* 5, 163–168.

Robinson, Peter M., 1994, Efficient Tests of Nonstationary Hypotheses, *Journal of the American Statistical Association* 89, 1420–1437.

Robinson, P. M., and J. Hualde, 2003, Cointegration in Fractional Systems with Unknown Integration Orders, *Econometrica* 71, 1727–1766.

Rudebusch, Glenn D., 2007, Comment to "Cracking the Conundrum" by David K. Backus and Jonathan H. Wright, *Brooking Papers on Economic Activity* 1, 317–335.

Rudebusch, Glenn D., Eric T. Swanson, and Tao Wu, 2006, The bond yield "conundrum" from a macro-finance perspective, *Federal Reserve Bank of San Francisco Working Paper Series 16*.

Shiller, Robert J., 1979, The Volatility of Long-Term interest Rates and Expectations Models of the Term Structure, *Journal of Political Economy* 87, 1190–1219.

Sowell, Fallaw, 1992, Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models, *Journal of Econometrics* 53, 165–188.

Taylor, John B., 1993, Discretion versus Policy Rules in Practice, *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.

Appendix

A Robinson's (1994) Univariate Parametric Fractional Integration Test

Robinson (1994) proposes the following parametric test statistic in order to test for the order of integration in the model outlined in equations (4)-(5) under the null hypothesis that $d = d_0$ for any given real value d_0. It is based on the Lagrange Multiplier (LM) principle in the frequency domain, and is given by:

$$\hat{r} = \frac{T^{\frac{1}{2}}}{\hat{\sigma}^2} \hat{A}^{-\frac{1}{2}} \hat{a}, \tag{12}$$

where T is the sample size and

$$\hat{a} = -\frac{2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j)\, g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j) \tag{13}$$

$$\hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j) \tag{14}$$

$$\hat{A} = \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j)\, \hat{\epsilon}(\lambda_j)' \times \left(\sum_{j+1}^{T-1} \hat{\epsilon}(\lambda_j)\, \hat{\epsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\epsilon}(\lambda_j)\, \psi(\lambda_j) \right) \tag{15}$$

$$\psi(\lambda_j) = \log \left| 2\sin \frac{\lambda_j}{2} \right| \tag{16}$$

$$\hat{\epsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log(g(\lambda_j; \hat{\tau})) \tag{17}$$

$$\lambda_j = \frac{2\pi j}{T} \tag{18}$$

$$\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau) \tag{19}$$

where T^* is a compact subset of the R^q Euclidean space. $I(\lambda_j)$ is the periodogram of ϵ_t evaluated under the null, and g is a known function coming from the spectral density of ϵ_t, $f = \left(\frac{\sigma^2}{2\pi}\right) g$. In our case, since ϵ_t follows an AR(1) process, $g = |1 - \rho e^{i\lambda}|^{-2}$.

Based on H_0 ($d = d_0$), under very mild regularity conditions, Robinson (1994) showed that:

$$\hat{r} \rightarrow_d N(0,1) \quad \text{as} \quad T \rightarrow \infty. \tag{20}$$

It is also shown in Robinson (1994) that this test is the most efficient in the Pitman sense against local departures from the null.

B Bloomfield's (1973) Univariate Non-Parametric Method

The approach devised by Bloomfield (1973) allows to capture the potential ARMA(p, q) structures of the residuals of a process (ϵ_t) in a non-parametric form. The model of Bloomfield (1973) is exclusively described in terms of its spectral density function, which is given by:

$$f(\lambda_j; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} \exp\{2 \sum_{l=0}^{m} \tau_l \cos(\lambda_j l).\} \tag{21}$$

Suppose that ϵ_t follows an ARMA(p, q) process of form:

$$\epsilon_t = \sum_{r=1}^{p} \phi_r \epsilon_{t-r} + \sum_{r=1}^{q} \theta_r \xi_{t-r} + \xi_t \tag{22}$$

where ξ_t is a white noise process, all zeros of $\Phi(L) = (1 - \phi_1 L - \ldots - \phi_p L^p)$ lying outside the unit circle and all zeros of $\Theta(L) = (1 - \theta_1 L - \ldots - \theta_q L^q)$ lying outside or on the unit circle. Clearly, the spectral density function of this process is then given by:

$$f(\lambda_j; \tau) = \frac{\sigma^2}{2\pi} \left| \frac{1 + \sum_{r=1}^{q} \theta_r \mathrm{e}^{i\lambda r}}{1 - \sum_{r=1}^{p} \phi_r \mathrm{e}^{i\lambda r}} \right|^2 . \tag{23}$$

Bloomfield (1973) showed that the logarithm of the above function is a fairly well behaved function and can thus be approximated by a truncated Fourier series. He showed that (21) approximates (23) well with small values of m and for alternative values of p and q. Similarly to the the stationary AR(p) case, this model has exponentially decaying autocorrelations and thus, using this specification we do not need to rely on so many parameters as in the ARMA processes. Moreover, this approximation remains valid even if the roots of the AR polynomial are close to the unit circle, the Bloomfield model being stationary across all values of τ. Further, in the context of the tests of Robinson (1994), $\hat{\epsilon}(\lambda_j)$ reduces to $2\cos\lambda$ in (17) and $\hat{A}$ in (15) becomes simply $\sum_{l=m+1}^{\infty} \left(\frac{1}{l^2}\right)$.

Table 1: **Short and Long-Term Interest Rates: Fractional Integration Estimates**

	i_t^{short}		i_t^{long}	
	Param.	95% C.I.	Param.	95% C.I.
d	0.84	[0.77, 0.93]	0.89	[0.82, 0.98]
ρ	0.47	[0.40, 0.51]	0.40	[0.32, 0.47]
μ	5.53	[4.82, 6.24]	6.26	[5.80, 6.70]

This table shows the estimates and 95% confidence intervals of the ARFIMA(1,d,0) model for both the short-term (one-year) and long-term interest (ten-year) rates:

$$(1-\rho L)(1-L)^d(i_t^j-\mu)=\xi_t,$$

where j can be the short or long-term interest rate. The model was estimated through the parametric technique derived in Robinson (1994).

Table 2: **Order of Integration of the Term Premiums**

	Param.	95% C.I.
I(0)	0.86	[0.75, 0.98]
I(d)	0.69	[0.60, 0.79]
I(1)	0.71	[0.60, 0.82]

This table shows the estimates and 95% confidence intervals of the ARFIMA(1,d,0) model for the term premiums implied by the three short-rate models:

$$(1-\rho L)(1-L)^d(tp_{t,n}(I(j))-\mu)=\xi_t,$$

where j can be 0, d or 1. The model was estimated through the parametric technique derived in Robinson (1994).

Table 3: **Short and Long-Term Interest Rates: Fractional Integration Estimates (Robustness)**

	i_t^{short}		i_t^{long}	
	Param.	95% C.I.	Param.	95% C.I.
$d(m=1)$	0.87	[0.79, 0.94]	0.92	[0.85, 1.00]
$d(m=2)$	0.88	[0.80, 0.98]	0.91	[0.79, 1.02]

This table shows the estimates and 95% confidence intervals of the fractional integration model for the short-rate and long-rate employing (in the context of Robinson's (1994) testing procedure) the method of Bloomfield (1973), which approximates non-parametrically the ARMA part of the interest rate process. Two models ($m = 1, 2$) which imply alternative ARMA structures for the short-run dynamics are estimated.

Table 4: **Term Premium Regressions on Macro Variables**

I(0)-implied term premium			
π_t	u_t	rr_t	$y_t - \bar{y}_t$
0.01			
	0.59*		
		0.13*	
			−0.05*
−0.02	0.62*	0.06*	0.16*

I(d)-implied term premium			
π_t	u_t	rr_t	$y_t - \bar{y}_t$
−0.09*			
	0.21*		
		−0.04*	
			−0.08*
−0.18*	0.39*	−0.17*	0.17*

I(1)-implied term premium			
π_t	u_t	rr_t	$y_t - \bar{y}_t$
−0.21*			
	0.23*		
		−0.09*	
			−0.27*
−0.36*	0.52*	−0.31*	0.14*

Note: This table shows the slope coefficients of term premium regressions on the macro variables. The table shows the coefficients of the regressions on the individual macro predictors as well as the slope coefficients of the regression with the full set of macro variables. The analysis is performed under the three premiums derived in the paper. The starred coefficients are statistically significant at the 1% confidence level. π_t is the inflation rate, u_t is the unemployment rate, rr_t is the ex-post real interest rate and $y_t - \bar{y}_t$ is the output gap (H-P detrended). All regressions included an additional constant.

Table 5: **Monetary Policy and Term Premiums**

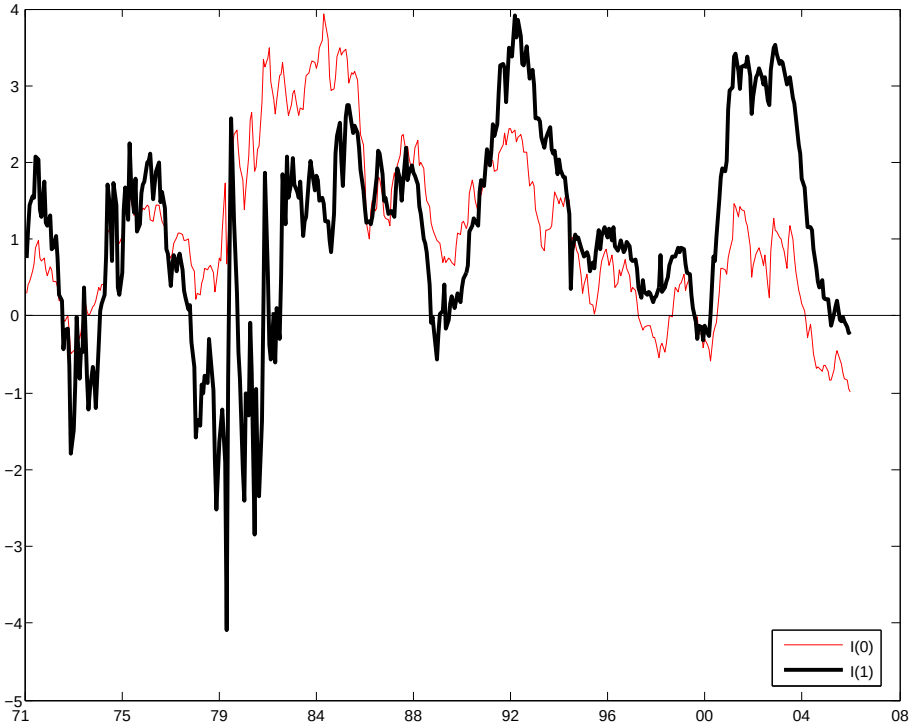
	Burns	Volcker	Greenspan
I(0)-tp	-4.51^*	-2.32	-1.75
I(d)-tp	-4.09	-0.59	-0.06
I(1)-tp	-4.84	-0.47	-0.05

Note: This table shows the response of the monetary policy rate (Federal funds rate) to the term premiums identified with the three estimated models (I(0), I(d) and I(1)). The monetary policy equation estimated at the monthly frequency is the following Taylor rule:

$$i_t^p = \rho i_{t-1}^p + (1-\rho)[\beta\pi_t + \gamma(y_t - \bar{y}_t) + \lambda tp_{t,n}].$$

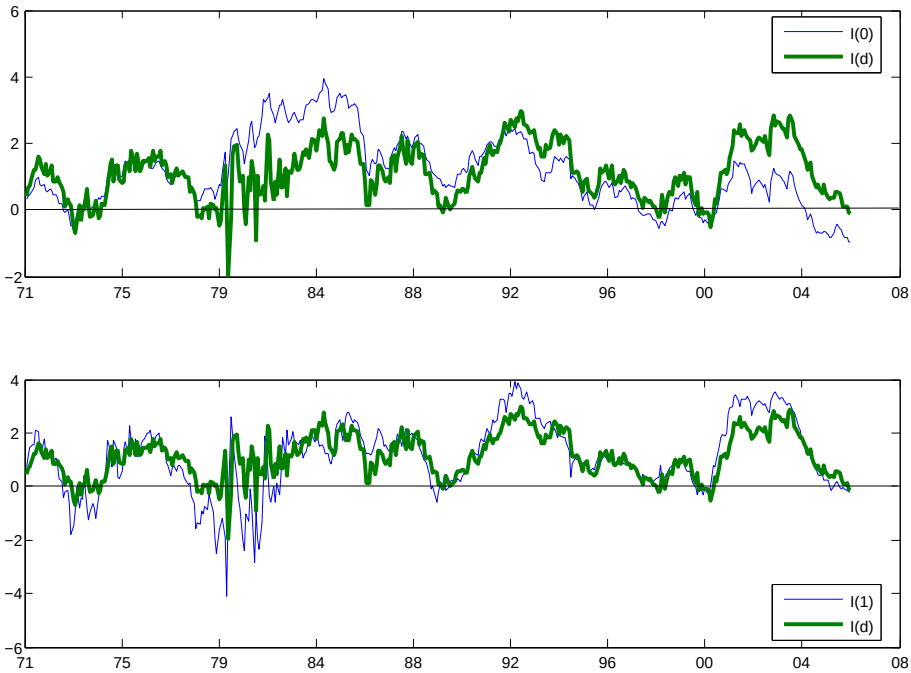
i_t^p is the monetary policy rate, π_t is inflation, $(y_t - \bar{y}_t)$ is the H-P detrended output and $tp_{t,n}$ is the ten-year term premium. The table lists the OLS estimate of λ for the three subsamples associated with the tenure of three Federal Reserve chairmen. The starred coefficients are statistically significant at the 1% confidence level. The standard errors of the coefficients are computed via the delta-method. All variables were demeaned previously to the estimation.

Figure 1: **I(0) Versus I(1) Term Premium**



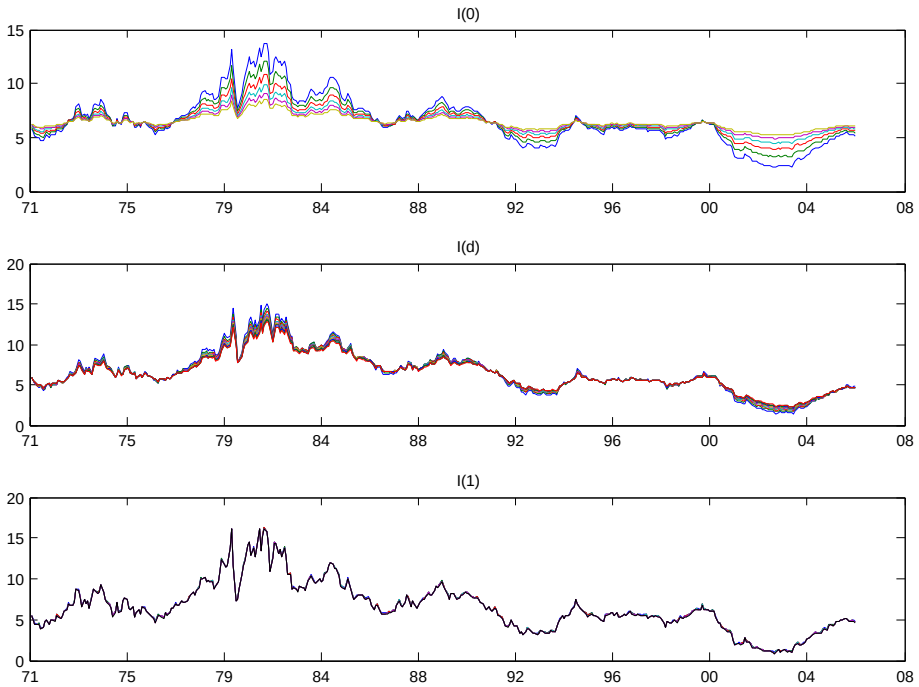
Note: This figure compares the term premiums implied by the AR(1) and ARIMA(1,1,0) models for the short-term rate. The sample period is August 1971-December 2006.

Figure 2: **I(d) Versus I(0)/I(1) Term Premiums**



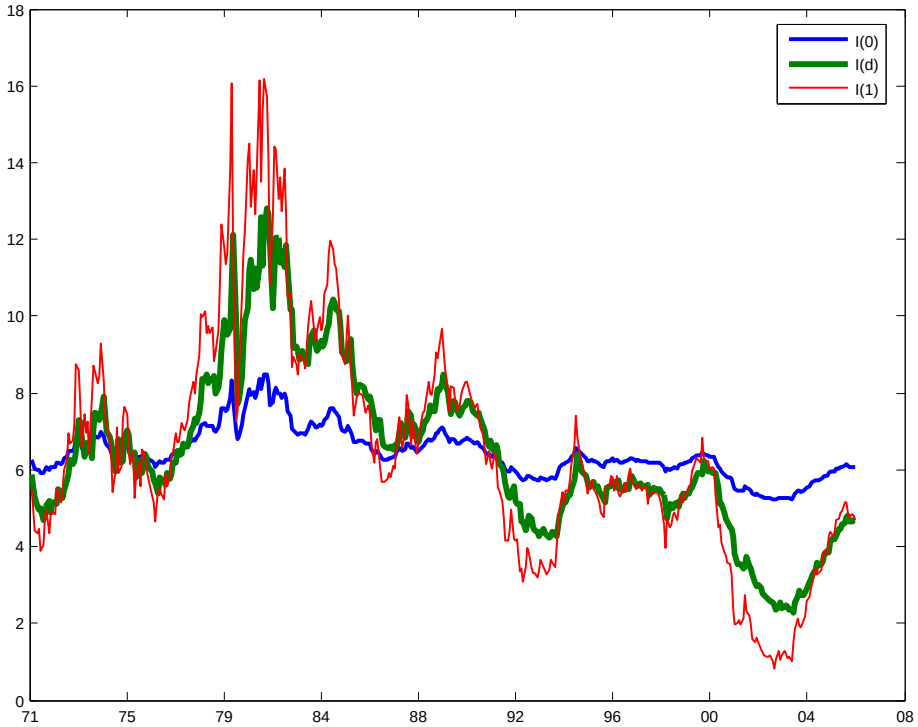
Note: The top figure compares the I(d)-implied term premium with the I(0)-implied term premium. The bottom figure compares the I(d)-implied term premium with the I(1)-implied term premium. The sample period is August 1971-December 2006.

Figure 3: **Expectations of Short-Rates across Horizons and Models**



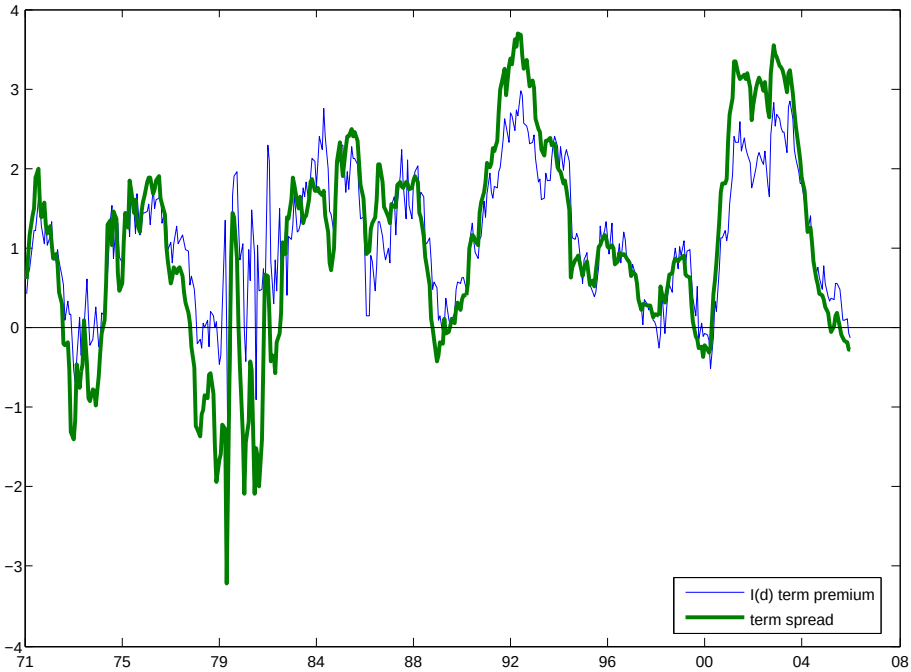
Note: This figure compares the expectations of the short-rate one, three, five, seven, nine and ten years out for the I(0), I(d) and I(1) models. The sample period is August 1971-December 2006.

Figure 4: **Expectations of the Short-Rate Ten years out across Models**



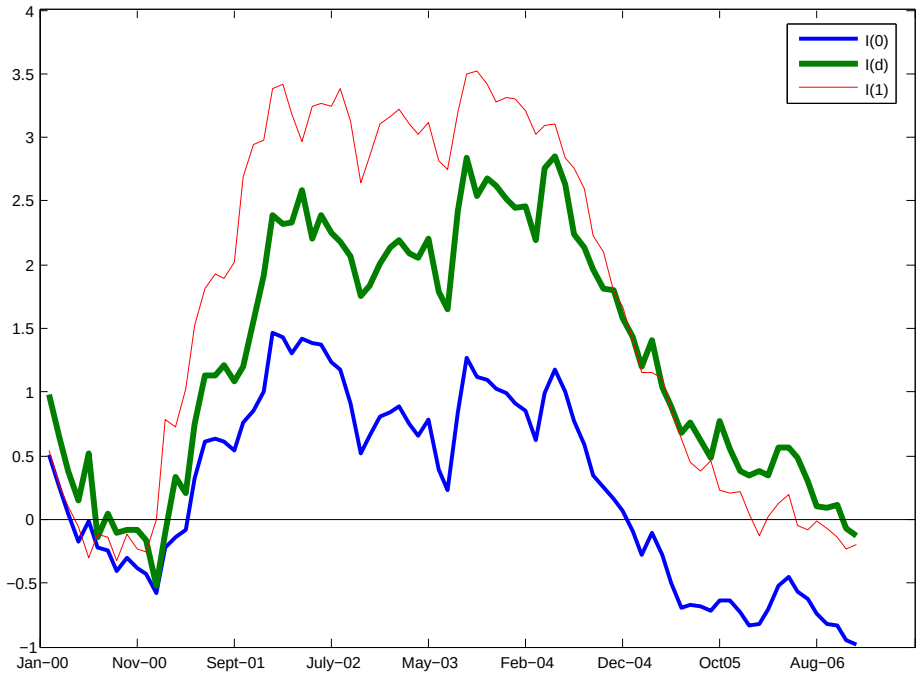
Note: This figure compares the expectations of the short-rate ten years out across univariate models. The sample period is August 1971-December 2006.

Figure 5: **I(d) Term Premium and Term Spread**



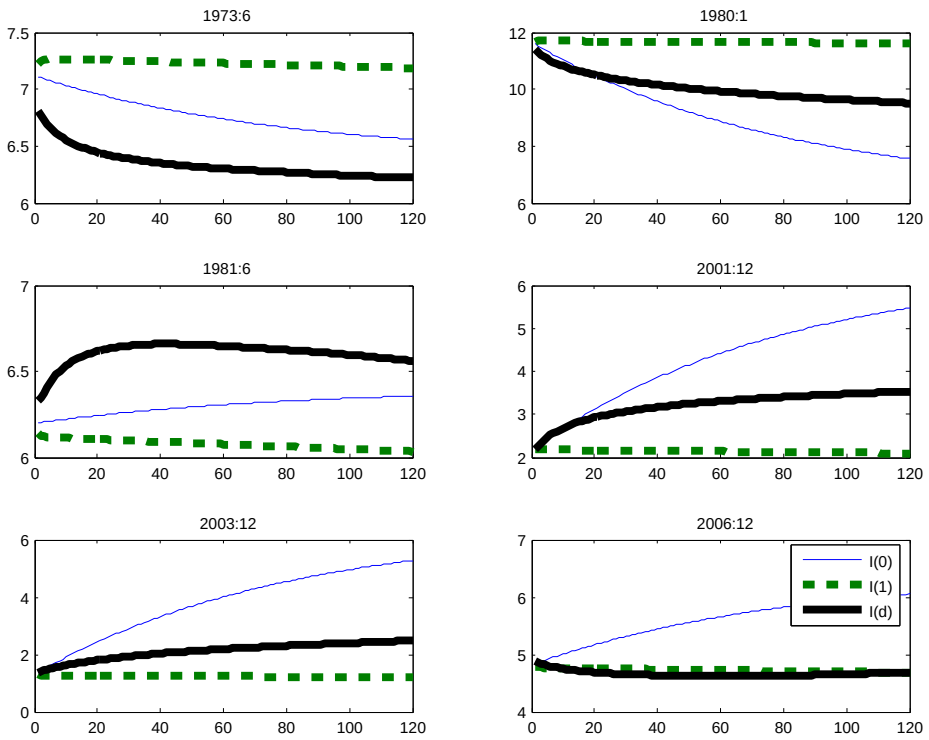
Note: This figure compares the I(d) term premium and the term spread (ten-year minus one-year yield). The sample period is August 1971-December 2006.

Figure 6: **Comparison of the Term Premium Across Models: 2000-2006**



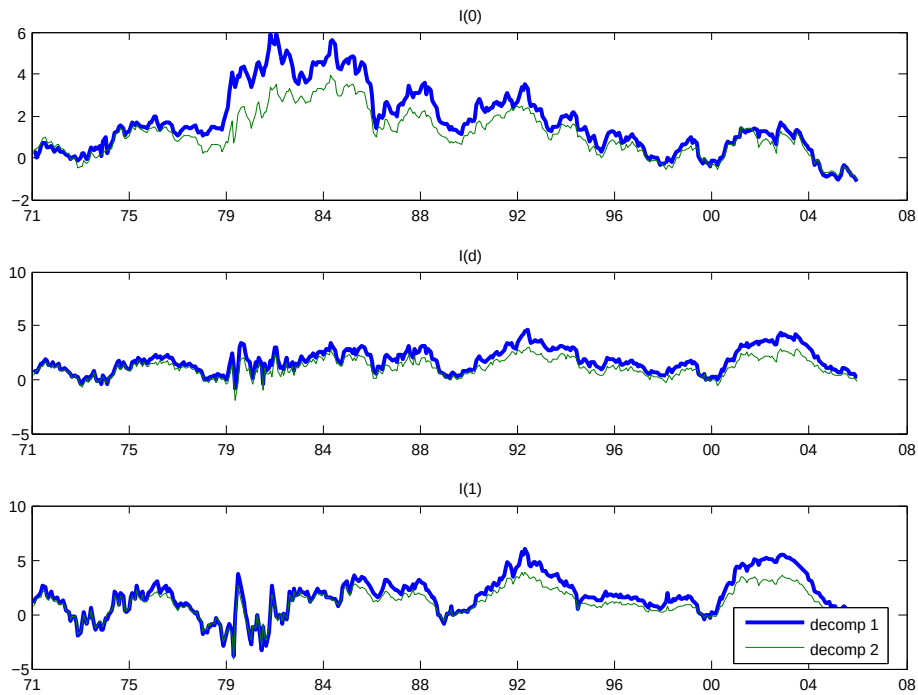
Note: This figure compares the term premiums implied by the I(0), I(d) and I(1) models from January 2000 to December 2006.

Figure 7: **Prediction of the Short-Rate Across Models at Different Times**



Note: This figure plots the predictions of the short-rate at different horizons (one-month to ten-year) and times across interest rate models.

Figure 8: **Term Premiums Across Models and Decompositions**



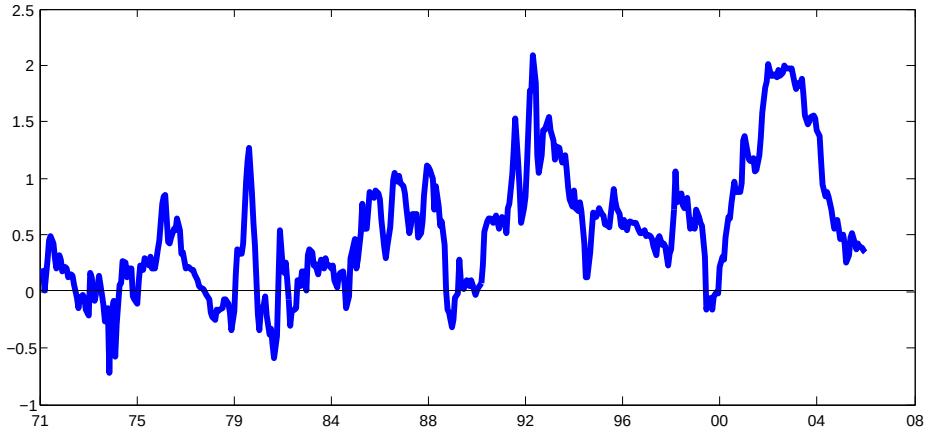
Note: This figure compares the term premiums across models and decompositions. The sample period is August 1971-December 2006. Decomposition 1 is:

$$f_t^n = E_t i_{t+n-1} + \hat{tp}_{t,n}$$

Decomposition 2 is:

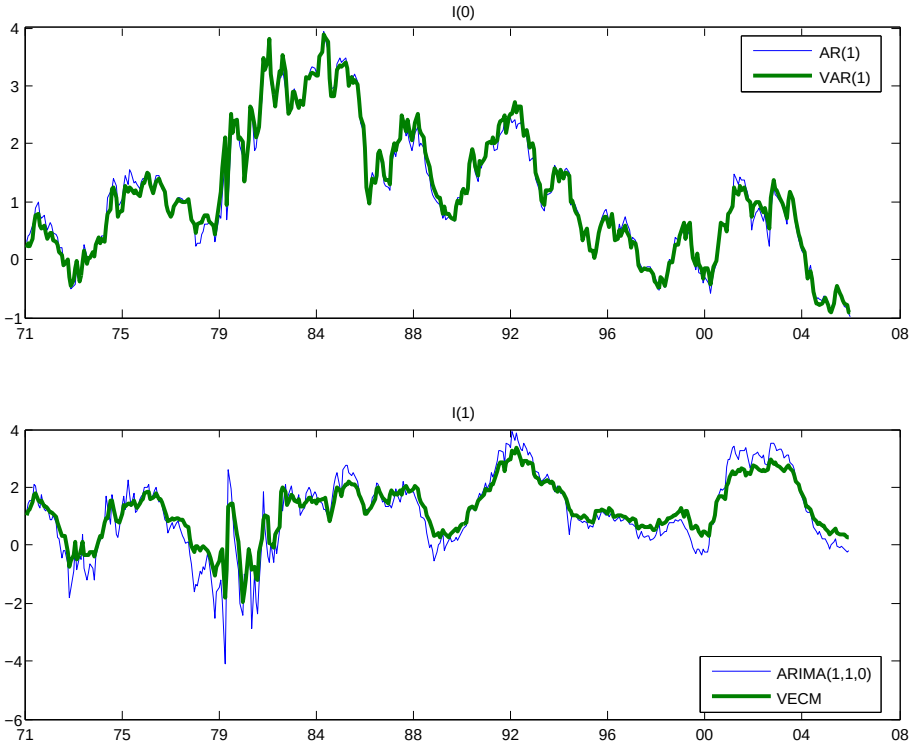
$$i_t^n = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} + tp_{t,n}$$

Figure 9: **Ten-year Forward minus Ten-year Yield**



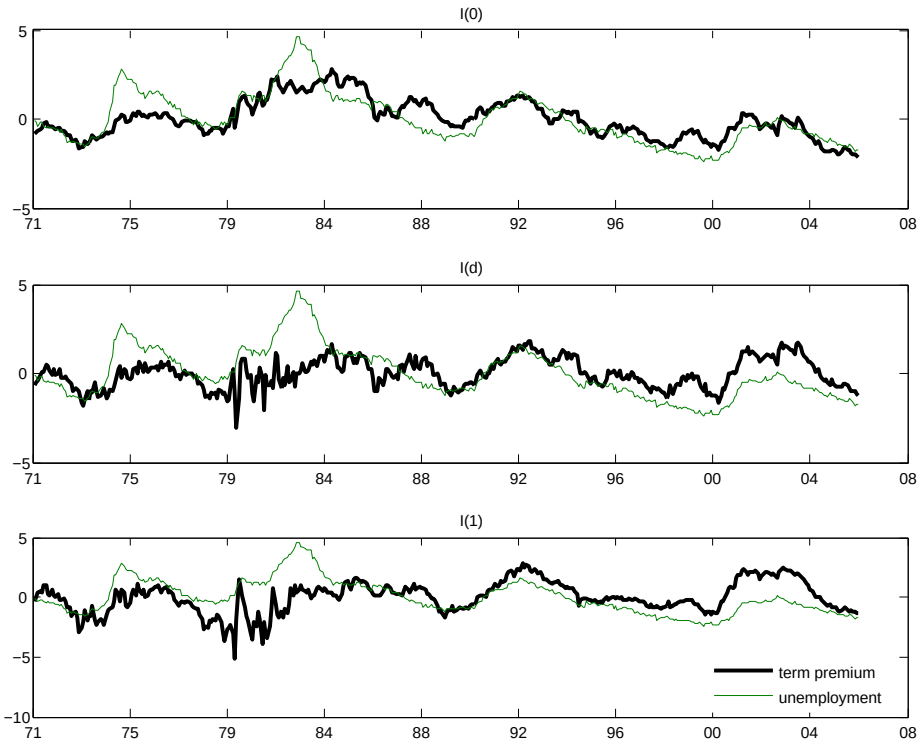
Note: This figure plots the difference between the ten-year forward rate and the ten-year yield. The sample period is August 1971-December 2006.

Figure 10: **I(0)/I(1) Term Premiums: Univariate Versus Multivariate Models**



Note: The top figure compares the term premiums implied by the univariate AR(1) and multivariate vector autoregressive (VAR(1)) models. The bottom figure compares the term premiums implied by the univariate ARIMA(1,1,0) and multivariate Vector Error Correction (VECM) models. The VAR and VECM models are computed with one, two, three, four and five-year yields. The sample period is August 1971-December 2006.

Figure 11: **Term Premiums and Unemployment Rates**



Note: The top figure compares the term premium implied by the I(0) model for short-rates with the unemployment rate. The graph in the middle compares the term premium implied by the I(d) model for short-rates with the unemployment rate. The bottom graph compares the term premium implied by the I(1) model for short-rates with the unemployment rate. All variables across graphs are demeaned.