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Retail sales. Persistence in the short term and long  
term dynamics

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#### ABSTRACT

The management of retail sales is of paramount importance to retail organisations and retail policy makers. This study examines the degrees of time persistence and seasonality of various retail sectors using innovative seasonal and non-seasonal fractional integration and autoregressions models. Adapting data from both the Australian and US retail sectors, the results indicate that the impacts of seasonality and persistence are not consistent across the various retail sectors. It also clear that retail sales forecasts are better explained in terms of a long memory model that incorporates both persistence and seasonal components.

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## **1. Introduction**

Developing a strong understanding of the persistence, seasonality and forecasting behaviour of retail sales is directly linked to the success and future policy formulations of any retail business (DeConinck and Bachmann, 2005). Persistence is a measure of the extent to which short term shocks in current market conditions lead to permanent future changes (Zhou et al., 2003). In a shock we mean an event which takes place at a particular point in the series, and it is not confined to the point at which it occurs. A shock is known to have a temporary or short term effect, if after a number of periods the series returns back to its original performance level (for example, retail sales might increase due to advertising or price promotion, but drop back after the marketing stimulus is withdrawn). On the other hand a shock is known to have a persistent or long term impact if its short run impact is carried over forward to set a new trend in performance (for example, a persistence drop in retail sales might result from an economic downturn, inflation, or change in exchange rate).

Dekimpe and Hanssens (1995a, b) and Ouyang et al. (2002) have provided a good summary on the importance of persistence analysis, especially in terms of its direct impact on policy implications. In fact, when retail businesses have a prior knowledge of the persistence behaviour of retail sales they can reap the benefit of positive effects, or avoid the drawbacks of a negative effect. Depending on the degree of persistence, different policy measures can also be adopted.

For instance, in the case of a unit root, shocks will be permanent and the series will be very persistent. On the other hand, if the series is stationary, shocks will be temporary and the series will be mean reverting and less persistent than in the previous case. In the context when the shock is positive and the series is mean reverting, strong policy measures must be adopted to maintain the series at the higher level. In the same way, if a shock is negative and the series contains, for instance, a unit root, the effect of that shock will be permanent, and again strong measures should be adopted to bring the series back to its original trend. On the other hand, if the series is mean reverting and the shock is negative, there is no need of strong policy measures since the series will return to its original trend sometimes in the future.

In order to obtain accurate measurement of persistence of retail sales, it is also essential to take into account the seasonality characteristics of the series. Traditionally, seasonal fluctuations have been considered as a nuisance that shadows the most important components of the series. If seasonality is not correctly handled, then the persistence of shocks are also not correctly determined, leading to misperception in the consequences of retail policies. Seasonality should be modelled according to the specific characteristics of the data (Bandyopadhyay, 2009). However, there is little consensus on how seasonality should be treated in empirical applications. In fact, as the statistical properties of different seasonal models are distinct, the imposition of one kind when another is present can result in serious bias or loss of information, and it is thus useful to establish what kind of seasonality is present in the data. Seasonality can be modelled deterministically or stochastically. In the former case, seasonal dummy variables are employed and the seasonal component is supposed to be fixed across time. Stochastic seasonality is the one

that usually occurs in economic data, including retailing data, and this can be stationary or nonstationary. If it is nonstationary, seasonal unit roots are generally adopted and they are based on the assumption that the seasonal component is changing across time. (Luis, mention a bit here somewhere about the disadvantage of seasonally adjusted data)

Forecasts have also important implications for retail companies, especially those which have a large share in the market. Peterson (1993), for instance, showed that larger retailers are more likely to use time-series methods and prepare industry forecasts, while smaller retailers emphasize judgmental methods and company forecasts. Better forecasts of aggregate retail sales can improve the forecasts of individual retailers because changes in their sales levels are often (quasi-)systematic. So far, different models have been proposed in the literature to forecast retail sales, but none has taken into account the simultaneous impact of seasonality and persistence on retail sales. If seasonality and persistence have direct impact on retail sales, it is logical to assume that their inclusion in a forecasting model can lead to more accurate and comprehensive results.

In the present study, we were driven by all the above mentioned factors, and our aim was to provide a more advanced assessment of the persistence, seasonality and forecasting behaviour of retail sales. We extend the existing literature by adapting a fractional integration and autoregressions model to analyze the behaviour in retail sales previously analysed by standard methods such as AR(I)MA models. Our model also incorporates both seasonal and non-seasonal structures in a unified treatment. While previous key studies in the area (Dekimpe and Hanssens, 1995a, b) focus on integer degrees of differentiation (usually 0 or 1), we permit here fractional values, allowing thus for a much

richer degree of flexibility in the dynamic specification of the series. The study also introduces and tests a forecasting model that allows for both persistence and seasonality in forecasting retail data.

The study also improves on existing studies by extending the persistence, seasonality analysis to cover multiple sectors. Our interest is to determine whether different retail sectors experience heterogeneous seasonality and persistence patterns. This is crucial for policy formulation, as in case of a heterogeneous behaviour, future policies need also to take into account this heterogeneity. The paper focuses on data from the Australian retail sector, but also provides supporting evidences from the the US retail sector. Specifically, we proceed as follows: Firstly, we analyze the persistent behaviour of retail sales. We distinguish between short term and long term by means of the duration of the shocks, which is specified in terms of short memory and long memory processes. Secondly, we examine the univariate behaviour of the series in terms of both fractional integration and autoregressions in order to assess whether the series present a persistent pattern over time. Using fractional integration we identify persistence in a continuous range between zero and one and not in the dichotomic range of zero and one as is the case in the standard time series methods. Thirdly, the seasonality of the series is also investigated, for each of the retail series separately, using again here short term and long term dynamics. Finally, a forecasting experiment is conducted to check which of the different approaches adopted better describes the data.

The outline of the paper is as follows: Section 2 presents an overview of both the Australian and U.S. retail industries. Section 3 presents the literature revision. Section 4

briefly describes the methodology employed in the paper. Section 5 is devoted to the empirical results, also dealing with the forecasting abilities of the selected models, while Section 6 contains some concluding comments.

## **2. The Australian and the U.S. Retail Industry**

The retail industry in both Australia and the U.S. constitutes a major part of the national economy. In Australia, for instance, the retail industry accounts on average for around 5.7% of total GDP (Australian Year Book, 2008), and in the U.S., the industry provides more than 11% of total employment opportunities.

**[Table 1 near here]**

In both countries, the demand for the retail industry has traditionally been driven by changes in consumers' disposable income, level of employment, wages, taxes and interest rates. Recently, the Australian retail sales have contracted by 0.2% in 2008-09, mainly due to the low economic growth and decrease in consumer confidence and high unemployment. Similar trends also occurred in the U.S., where the total retail sales declined by 0.1% overall in 2008, in comparison to 2007 (US Census Bureau, 2008). Factors which have affected the industry include the rise in interest rates, higher fuel prices, increasing grocery costs and an overall expansion in the cost of living. Other negative factors included the increase in the unemployment rate, fluctuations in the

household disposable income, decrease in consumers' confidence level and the slow progress of the economy (IBISWorld, 2009, 2010).

Thus, it can be said that the retail industry in both countries is going through a critical and uncertain period. Recently, the Australian and U.S. governments tried to stimulate consumers' spending through some stimulus package, but customers are still extremely cautious due to the economic downturn and the accelerated feelings of job insecurity and financial instability (IBISWorld, 2010). In other words, price substitution still seems to take number one priority when spending.

As this study focuses on analysing the behaviour of retail sales across various retail sectors, the results can thus directly assist in future policy formulation towards improving or revitalising the retail industry at this critical time. Our analysis starts with the Australian retail sector, and then provides supportive evidence from the U.S. retail sector. In this way the scope of our findings is thus extended to assist policy makers in both countries. The study is also innovative in terms of adapting more accurate methodologies based on fractional integration, which permit more flexibility in the dynamic specification of the series, and which aim to improve the reliability and robustness of the results reported. In the next section, we present a review of the literature before describing in more detail the methodology used in the study.



### 3. Literature Review

The literature is rich with studies which have focused on several aspects of retail sales such as the relationship between sales and employee satisfaction (Arndt et al., 2006), relationship between sales and employee performance (Ramaseshan, 1997). Studies addressing the persistence and seasonality of retail sales are however rare in the literature. More in line with the present research, Dekimpe and Hanssens (1995a,b, 1999) investigated the persistence of marketing effect on retail sales, using the Dickey-Fuller unit root test and Vector Autoregressive (VAR) models. The authors concluded that a home improvement chain's price-oriented print advertising had a high short-run impact with limited sales persistence (mainly short-run benefits), while TV spending had a low short-run impact with substantial sales persistence (mainly long-run benefits). From the overall conclusion, it was **clear that marketing** can indeed have persistent performance effects on retail sales. Other studies on persistence model aimed to determine the short-run and long-run effects of various marketing activities on market performance with some examples include the sales impact of price promotions (Dekimpe et al., 1999), distribution changes (Bronnenberg et al., 2000), channel additions (Deleersnyder et al., 2002). Some recent studies have also examined the impact of marketing persistence on the consumer durables market (Ouyang et al., 2002; Irvine, 2007), concluding that temporary shocks can create a long-lasting effect on a firm's sales and production performance.

Studies on forecasting of retail sales are also relatively limited in the literature. Some key studies in the area include Alon et al. (2001) and Chu and Zhang (2003), which investigated the forecasting properties of various methods (artificial neural networks

(ANN), ARIMA models and multivariate regression, applied to aggregate retail sales. The results suggested that the ANN methods produce the best results. Similar findings are obtained in Chu and Zhang (2003) comparing linear and non-linear models. In an earlier study, Alon (1997) also found that the Winters' exponential smoothing model forecasts aggregate retail sales more accurately than the simple exponential and Holt's models. The Winters' model was shown to be a robust model that can accurately forecast individual product sales, company sales, income statement items, and aggregate retail sales.

Other studies on forecasting have focused on issues such as market response forecasting (van Wezel and Baets, 1995; Agrawal and Schorling, 1996), consumer choice forecasting (West et al., 1997; Davies et al., 1999), tourism marketing (Mazanec, 1999), and market segmentation analysis (Fish et al., 1995; Natter, 1999). Most of the models used in these studies have focused on methods such as the ANN and multinomial logit model. Though the ANN methods have been widely employed in retailing as a competitive model to the logistic regressions and it has been proven to be a good forecasting method compared with other approaches, it has several drawbacks in the context of time series models such as its "black box" nature, the greater computational burden, the proneness to overfitting and the empirical nature of the model itself. In this context, the parametric statistical models employed in this work can be considered as plausible alternative ways to describe the retail time series data.

From the review of the above literature, it was clear to us that the issues of seasonality, time persistence and forecasting have not been analysed together in retailing. This is despite the direct link between the three concepts. For instance, the available studies on

persistence discussed above have ignored in most cases the simultaneous impact of seasonality on persistence. Note that with modelling seasonality either as a short memory (AR) process or using a long memory (fractionally integrated) model, persistence plays a crucial role, with the autocorrelations decaying exponentially in the short memory case and hyperbolically in the long memory case. The issue of persistence has also been ignored in most papers dealing with forecasting in retail sales data. In the following points, we describe in more detail the current gaps in the literature and how the present study addresses these gaps.

### **3. 1. Persistence and Seasonality heterogeneity**

The paper has a major focus to check whether the degree of persistence in retail sales is heterogeneous and varies among different retail sectors (e.g. food retailing, department stores, clothing and soft good retailing, household retailing, other retailing, cafés, restaurants and takeaway services). As mentioned before, this is crucial for policy formulation, as in the case of heterogeneous before, improvement policies might also need to be specific to each sector (i.e. not homogenous across all retail sectors).

An innovation of this paper is that in measuring persistence we simultaneously account for the seasonality and the dependence in the data using short memory and long memory processes. In this way, the study thus also reflects the seasonality behaviour of various retail sectors while most previous works have focused on aggregate retail sales (Alon et al., 2007). There is general agreement in the literature that like many other economic time series, retail sales have strong trends and seasonal patterns. Previous persistence studies in

the literature have accounted for seasonality using seasonally adjusted data. However, here seasonality is treated as one of the feature to be explained within our specific modelling approaches based on short and long memory processes. Note that the use of seasonal adjustment procedures has been strongly criticized by many authors in the belief that their statistical properties are difficult to assess from a theoretical viewpoint. In fact, authors such as Ghysels (1988), Barsky and Miron (1989), Braun and Evans (1995) among many others point out that seasonal adjustment might lead to mistaken inferences about economic relationships between time series data, also causing a significant loss of valuable information about the behaviour of the series.

Thus, with persistence analysis we can determine the nature of a shock in a particular sector. This is also essential for policy implication as the strength of a policy can be dependent on the persistence behaviour of a certain series. When a series is stationary and mean reverting, the effect of a given shock on it will have a transitory effect, and will disappear fairly rapidly; if the series is nonstationary but mean reverting (e.g., if it is fractionally integrated with an order of integration in the interval  $(0.5, 1)$ ), shocks will still be transitory though they will take longer time to disappear than in the previous case. If the series is nonstationary and mean reversion does not take place (e.g., the unit root model) persistence is a strong feature in the data with shocks having a permanent nature.

Thus, it seems intuitive that retailers take into consideration the time persistence and the seasonality of sales, as with a good understanding of these two phenomena, authorities can predict and take advantage of a positive effect in the industry, or, equally important,

avoid being victimized by a negative effect. Sales are the life line of any business survival and therefore of paramount importance for retail business.

### 3. 2. Forecasting Accuracy

As stated before, current forecasting models in the retail literature have ignored the combined impact of persistence and seasonality on retail sales forecasts. We aim here to check whether retail sales forecasts can be better explained in terms of a model that incorporates both long run persistence and seasonal components. This will be achieved by comparing the forecasting accuracy of our models based on fractional differencing to other competing models existing in the literature that use integer degrees of differentiation. Up to our knowledge, long memory models have not been implemented on retail sales despite the fact that they include as particular cases the standard AR(I)MA models widely employed in the literature.

We can provide two definitions of long memory, one in the time domain and the other in the frequency domain. The time domain definition of long memory states that given a covariance stationary process  $\{u_t, t = 0, \pm 1, \dots\}$ , with autocovariance function  $E[(u_t - Eu_t)(u_{t-j} - Eu_t)] = \gamma_j$ ,  $u_t$  displays the property of long memory if

$$\lim_{T \rightarrow \infty} \sum_{j=-T}^T |\gamma_j|$$

is infinite. A frequency domain definition may be as follows. Suppose that  $u_t$  has an absolutely continuous spectral distribution, so that it has a spectral density function, denoted by  $f(\lambda)$ , and defined as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda j, \quad -\pi < \lambda \leq \pi.$$

Then,  $u_t$  displays long memory if the spectral density function has a pole at some frequency  $\lambda$  in the interval  $[0, \pi]$ . Most of the empirical literature has concentrated on the case where the singularity or pole in the spectrum occurs at the zero frequency. This is the case of the standard I(d) models that will be first presented in Section 4. However, there might be situations where the singularity or pole in the spectrum takes place at other frequencies. This is the case of the seasonal fractional processes or seasonal I(d) models that will also be examined in this work.

Fractional differencing models have been found to outperform non-fractional ones in a number of papers including Diebold and Lindner (1996), Bos et al. (2002) and Man (2003). In seasonal context, Ray (1993) and Sutcliffe (1994) illustrated the advantages of seasonally fractionally differencing models for forecasting monthly data.

#### 4. Methodology

Retail sales time series may display nonstationarities that should be adequately modelled to make statistical inference. Traditionally, the two approaches employed in the literature are the “*trend stationarity*” and the “*stochastic difference*” representations. In the former (“*trend stationarity*”) the time series is described in terms of a deterministic function of time, usually of the form:

$$y_t = \alpha + \beta t + u_t, \quad t = 1, 2, \dots, \quad (1)$$

where  $y_t$  is the observed time series,  $\alpha$  and  $\beta$  are the coefficients corresponding to the intercept and the linear trend, and  $u_t$  is an  $I(0)$  process that may contain a weakly autocorrelated (e.g., ARMA) structure. In the second approach (the “*stochastic difference*” representation) the series is nonstationary  $I(1)$  and contains a unit root, such that first differences are then required to render the series stationary  $I(0)$ . In other words,

$$(1 - L)y_t = u_t, \quad t = 1, 2, \dots, \quad (2)$$

where  $L$  is the lag operator ( $Ly_t = y_{t-1}$ ), and  $u_t$  is again  $I(0)$ . This approach, widely employed in economic time series, also allows the inclusion of an intercept and a linear time trend, and many test statistics have been proposed in the last thirty years to check for the presence of unit roots in macroeconomic and financial time series data.<sup>1</sup>

To illustrate the difference between the two approaches in terms of the duration of the shocks we can consider the following model,

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t, \quad u_t = \varphi u_{t-1} + \varepsilon_t,$$

with  $|\varphi| < 1$ , where, if  $d = 0$ , we obtain the “*trend stationary*” representation with AR(1) errors, while, if  $d = 1$ , we have the “*stochastic difference*” or unit root model, specified in this case as an ARIMA(1, 1, 0) model with a linear trend. We can then compute in the two cases the impulse responses for the detrended series,  $x_t$ , by using the infinite moving average form of the processes, i.e.,

$$x_t = \sum_{j=0}^{\infty} \phi_j \varepsilon_{t-j},$$

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<sup>1</sup> Examples are the procedures of Dickey and Fuller (ADF, 1979); Phillips and Perron (PP, 1988); Kwiatkowski et al. (KPSS, 1992), Elliot et al. (ERS, 1996), Ng and Perron (NP, 2001), etc.

and, in case of the “trend stationary” representation ( $d = 0$ ),  $\phi_j = \varphi^j$ , and thus  $\phi_j \rightarrow 0$  as  $j \rightarrow \infty$ , i.e., decaying exponentially and relatively fast to zero. On the contrary, in case of the unit root model ( $d = 1$ ),  $\phi_j$  does not converge to zero and thus, the effect of the shock remains in the series forever.

Nevertheless, the two above-mentioned approaches are nested in a more general specification, permitting for fractional orders of integration. Thus, we can consider models of form:

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (3)$$

where  $d$  can be any real value. Thus, the parameter  $d$  might be 0 or 1, but it may also take values between these two numbers or even above 1. Note that the polynomial  $(1-L)^d$  in (3) can be expressed in terms of its Binomial expansion, such that, for all real  $d$ ,

$$(1 - L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - d L + \frac{d(d-1)}{2} L^2 - \dots ,$$

and thus

$$(1 - L)^d x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \dots .$$

In this context,  $d$  plays a crucial role since will be an indicator of the degree of dependence of the time series. Thus, the higher the value of  $d$  is, the higher the level of association will be between the observations. Processes with  $d > 0$  in (3) display the property of “*long memory*”, characterized because the spectral density function of the process is unbounded at the origin. The origin of these processes is in the 1960s, when Granger (1966) and Adelman (1965) pointed out that most aggregate economic time



series have a typical shape where the spectral density increases dramatically as the frequency approaches zero. However, differencing the data frequently leads to overdifferencing at the zero frequency. Fifteen years later, Robinson (1978) and Granger (1980) showed that aggregation could be a source of fractional integration. Since then, fractional processes have been widely employed to describe the dynamics of many time series (see, e.g. Diebold and Rudebusch, 1989; 1991a; Sowell, 1992; Baillie, 1996; Gil-Alana and Robinson, 1997; etc.).

The series that will be analyzed in this article present a trending behaviour that might be modelled in terms of a deterministic trend or using unit (or fractional) degrees of differentiation. However, they also present a strong seasonal pattern that is changing across time. Therefore, seasonal unit roots will also be considered, and again here we extend the model of integer differentiation to the fractional case, examining models of form:

$$y_t = \alpha + \beta t + x_t; \quad (1 - L^s)^{d_s} x_t = u_t, \quad t = 1, 2, \dots, \quad (4)$$

where  $s$  indicates the number of time periods per year ( $s = 4$  with quarterly data;  $s = 12$  with monthly data), and  $d_s$  is the seasonal fractional differencing parameter. Similarly to the non-seasonal case, the seasonal fractional polynomial in (4) can be expanded, for all real  $d_s$ , such that

$$(1 - L^s)^{d_s} = \sum_{j=0}^{\infty} \psi_j L^{js} = \sum_{j=0}^{\infty} \binom{d_s}{j} (-1)^j L^{js} = 1 - d_s L^s + \frac{d_s(d_s-1)}{2} L^{2s} - \dots,$$

and thus

$$(1 - L^s)^{d_s} x_t = x_t - d_s x_{t-s} + \frac{d_s(d_s-1)}{2} x_{t-2s} - \dots,$$

and  $d_s$  is therefore an indicator of the degree of seasonal long range dependence. Empirical applications using this approach include the papers of Porter-Hudak (1990), Ray (1993), Sutcliffe (1994) and Gil-Alana and Robinson (2001), and if  $d_s = 1$ , we have the seasonal unit root model advocated by Dickey, Hasza and Fuller (DHF, 1984); Hylleberg, Engle, Granger and Yoo (HEGY, 1990), and Beualieu and Miron (1993) among many others.

Finally, we combine the two approaches described in (3) and (4) in a single framework, and consider a model with two fractional differencing parameters, one referring to the long run evolution ( $d$ ) and the other affecting the seasonal structure ( $d_s$ ). In other words, we consider now a model of form:

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d (1 - L^{12})^{d_s} x_t = u_t, \quad (5)$$

assuming that  $u_t$  is  $I(0)$  and adapting the forms of white noise, AR(1) and seasonal AR(1) processes. Here, if  $d = d_s = 1$ , we obtain the classical “airline model” of Box and Jenkins (1976). Note that the three fractional structures in (3), (4) and (5) admit infinite moving average representations and therefore we can easily build up the impulse response functions for the selected models.

The methodology employed in this paper is based on the Whittle function in the frequency domain (Dahlhaus, 1989) along with a testing procedure developed by Robinson (1994) that permits us to test all the above specifications. The latter method has the advantage that it does not require preliminary differencing to render the series stationary since it is valid for any real value  $d$  (or  $d_s$ ), encompassing thus stationary ( $d < 0.5$ ) and nonstationary ( $d \geq 0.5$ ) hypotheses. Moreover, the limiting distribution is

standard (normal, in the cases of equations (3) and (4)) and chi-square in the case of (5)), and this limit behaviour holds independently of the inclusion or exclusion of deterministic terms in the model and the modelling approach for the  $I(0)$  disturbances. Moreover, Gaussianity is not a requirement, a moment condition of only 2 being necessary. This method is briefly described in Appendix 1.

## **5. Data**

The time series retail sales of Australian data included in the analysis covered various retail sectors, listed in more detail in Appendix 2. We work with the original time series data (seasonally unadjusted), for the time period April, 1982 – February, 2009. All data were collected from the Australian Bureau of Statistics (catalogue 8501.0). Special details on the retail sales for the different retail groups over the period 2002–03 to 2006–07 are presented in Table 1.

**[Table 1 and Figure 1 near here]**

Figure 1 displays the time series plots. We observe in all cases a strong seasonal pattern, with values increasing in the last part of the sample. Thus, there seems to be some degree of increased volatility in the last third of the data. Nevertheless this should not affect our results for the estimation of the differencing parameters since the methods employed are robust against conditional heteroskedastic errors (Robinson, 1994). Moreover, the fact that the seasonal component is changing across time confirms that seasonal dummy

variables are not required, and that the series are nonstationary with respect to the seasonal component.<sup>3</sup>

As stated above, we have also collected data on some US retail sectors to provide supportive evidence to our results. The sectors are also listed in Appendix 2. Due to space limitations we do not report here all the descriptive tables and figure of these sectors, which can be obtained from the author(s) upon request. All data were collected from the US Census Bureau. For space limitation we do not display the time series plots of the various sectors, but it was clear that they present similar patterns to the US case.

## **6. Results**

In this section we present the results of the study in line with the issues highlighted in Section 3. We first introduce the models to test for the persistence and seasonality in the data, and then we compare the forecasting accuracy of the competing models.

### **6.1. Persistence and Seasonality of Retail Sales**

Though not reported in the paper, we first conducted standard unit root testing procedures (Dickey and Fuller, ADF, 1979; Phillips and Perron, PP, 1988, and Kwiatkowski et al, KPSS, 1992) to determine if the series were stationary  $I(0)$  or nonstationary  $I(1)$  around a stationary seasonal structure. The results here were a bit ambiguous, finding different results depending on the methodology used. On the other hand, employing seasonal unit

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<sup>3</sup> Seasonal dummy variables were employed in the regression models below and the values were found to be

root tests (Hylleberg et al., HEGY, 1990, and Beaulieu and Miron, BM, 1992) evidence of unit roots was obtained in the majority of the cases. These results however should be taken with caution, noting that these methods have extremely low power if the alternatives are of a fractional-form. (See, e.g., Diebold and Rudebusch, 1991b; Hassler and Wolters, 1994; Lee and Schmidt, 1996). Therefore, in what follows we employ more general specifications that permit us to test the above models as particular cases of interest.

In order to take into account the two main features of the data, i.e., their degree of dependence across time, and the seasonality, we first consider the following model,

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t; \quad u_t = \rho_s u_{t-12} + \varepsilon_t. \quad (\text{M1})$$

The above model includes the standard cases mentioned above. Thus, for example, if  $d = 0$ , we have a simple seasonal autoregression, while if  $d = 1$ , the classical unit root model. However, allowing  $d$  to be a real value, we can also examine the possibility of fractional integration. In this model, the parameter  $d$  is an indicator of the degree of long range dependence, while the parameter  $\rho_s$  refers to the (short run) seasonal dependence.<sup>4</sup>

Table 2 displays the estimates of the fractional differencing parameter,  $d$ , in (M1) along with the 95% confidence interval of the non-rejection values of  $d$ , using Robinson's (1994) tests, for the three standard cases of no regressors (i.e.,  $\alpha = \beta = 0$  a priori in (M1)), an intercept (i.e.,  $\alpha$  unknown, and  $\beta = 0$  a priori), and an intercept with a linear time trend (i.e.,  $\alpha$  and  $\beta$  unknown).

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statistically insignificant in practically all cases.

**[Tables 2 and 3 near here]**

The first thing we observe in this table is that all except one value of  $d$  (corresponding to “Department stores” with a linear time trend) are in the interval  $(0, 1)$ , implying that the  $I(0)$  and the  $I(1)$  hypotheses are both rejected in favour of fractional integration. The results are very sensitive to the choice of the deterministic terms. In general, the lowest orders of integration are obtained in case of the inclusion of a linear time trend, and the highest values are in all cases with an intercept. The  $t$ -values (though not reported) indicate that the time trend coefficients are statistically significant in all cases implying that the orders of integration reported in the last column of the table should be those to be considered. Finally, we also observe that the results substantially vary from one series to another. The lowest degree of dependence is obtained for the “Department Stores” series. In this case,  $d$  is found to be negative in case of including a linear trend. In all the other cases,  $d$  is positive, and the highest degrees of integration correspond to the cases of “Food retailing” (with  $d = 0.403$ ) and “Cafes, restaurants ...”, with  $d = 0.647$ . Note that in this latter series, the value is above 0.5 implying nonstationarity.<sup>5</sup>

In Table 3 we report the estimated seasonal AR coefficients for each of the reported cases in Table 2. We observe some differences from one series to another implying seasonal heterogeneity. Moreover, all coefficients are very large and close to 1, suggesting that the

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<sup>4</sup> Higher seasonal AR orders were also employed and the results were substantially the same as in the AR(1) case.

<sup>5</sup> In the  $I(d)$  context, if  $d$  belongs to the interval  $(0, 0.5)$ , the series is still covariance stationary, while if  $d$  belongs to the interval  $[0.5, 1)$  the series is nonstationary though mean reverting.

series might contain seasonal unit roots.<sup>6</sup> We see that the lowest coefficients correspond to “Cafes, restaurants ...”, while the largest ones are those referring to “Department stores”. This is precisely the contrary to what we obtained for the long run parameter  $d$ , implying some type of competition between the two parameters ( $d$  and  $\rho_s$ ) in describing the persistence of the series. We can conclude the analysis of these two tables by saying that we observe in all cases long range dependence along with a large degree of seasonal persistence.

**[Figure 2 near here]**

Figure 2 displays for each series the first 120 impulse responses according to the specification in model (M1) with an intercept and a linear time trend. These responses were obtained noting that the equations in model (M1)

$$(1 - L)^d x_t = u_t, \quad \text{and} \quad u_t = \rho_s u_{t-12} + \varepsilon_t.$$

can be expressed as

$$x_t = \sum_{j=0}^{\infty} \psi_j^{(1)} L^j u_t, \quad \text{and} \quad u_t = \sum_{j=0}^{\infty} \psi_j^{(s)} L^{12j} \varepsilon_t,$$

where  $\psi_j^{(1)} = \frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)}$ ,  $\psi_j^{(s)} = \rho_s^j$ , and  $\Gamma(x)$  is the Gamma function.

We observe that in all cases seasonality is important, the values decreasing very slowly. Therefore, we also examine the possibility of seasonal long run dependence, and consider now a model of form:

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<sup>6</sup> In fact, as earlier mentioned, seasonal unit roots were unrejected in the majority of the cases.

$$y_t = \alpha + \beta t + x_t; \quad (1 - L^{12})^{d_s} x_t = u_t; \quad u_t = \rho u_{t-1} + \varepsilon_t, \quad (\text{M2})$$

where  $d_s$  refers now to the seasonal (monthly) long range dependence and  $\rho$  describes the short run dynamics throughout an AR(1) process. In (M2), if  $d_s = 1$ , we have the case of seasonal unit roots, and, if  $d_s = 0$ , a simple (non-seasonal) AR(1) process.

**[Tables 4 and 5 near here]**

Table 4 displays the estimates of  $d_s$  in (M2) and their corresponding 95% confidence intervals, again for the three cases of no regressors, an intercept, and an intercept with a linear time trend. The estimates of  $d_s$  are generally large, being in the majority of cases in the interval  $[0.5, 1)$  implying nonstationarity but still mean reversion and seasonal heterogeneity. The only series where the unit root cannot be rejected are “Food retailing” and “Total Retail Sales”, in the latter for the cases of no regressors and a linear trend. Table 5 reports the associated AR(1) coefficients for each of the cases reported in Table 4. For “Department stores” the values are negative in two of the three cases. For the remaining series, the values are positive, ranging from 0.434 (“Clothing, ...”) to 0.980 (“Cafes, restaurants, ...”).

**[Figure 3 near here]**

The estimated first 120 impulse responses based on the above model (with an intercept and a linear time trend) are displayed in Figure 3. Similar to the previous case, the equations in (M2)



$$(1 - L^s)^{d_s} x_t = u_t, \quad \text{and} \quad u_t = \rho u_{t-1} + \varepsilon_t.$$

can be expressed as

$$x_t = \sum_{j=0}^{\infty} \psi_j^{(2)} L^{12j} u_t, \quad \text{and} \quad u_t = \sum_{j=0}^{\infty} \psi_j L^j \varepsilon_t,$$

where  $\psi_j^{(2)} = \frac{\Gamma(j+d_s)}{\Gamma(j+1)\Gamma(d_s)}$ , and  $\psi_j = \rho^j$ . Again here seasonality appears

important, with values decreasing at a very slow rate.

We finally present the results of model (M3), which is the one combining fractional integration at zero and the seasonal frequencies, i.e.,

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d (1 - L^{12})^{d_s} x_t = u_t, \quad (\text{M3})$$

under the presumption that  $u_t$  is a white noise process. We also estimated the parameters for weakly autocorrelated disturbances, in particular using AR(1) and seasonal AR(1) processes. However, in these cases, the estimates of the fractional differencing parameters were negative in most cases due to the competition with the short run parameters in describing the time dependence.<sup>7</sup> Moreover, several Likelihood Ratio (LR) tests conducted in this context showed strong evidence in favour of the uncorrelated case for the I(0) disturbances  $u_t$ .<sup>8</sup>

**[Table 6 near here]**

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<sup>7</sup> Note that  $d$  and  $\rho$  may compete to describe the non-seasonal dependence, while  $d_s$  and  $\rho_s$  both refer to the seasonal persistence. The main difference across these parameters is that  $d$  and  $d_s$  employ an hyperbolic rate of decay in the autocorrelations, while  $\rho$  and  $\rho_s$  use an exponential decay.

<sup>8</sup> Additionally, we also conducted several tests for serial correlation in the  $(d, d_s)$ -differenced processes (Box-Pierce-type statistics), and we do not find evidence of further need of autocorrelation.

The results based on this model are reported in Table 6. We observe that all estimates are in the interval (0, 1) and the seasonal fractional differencing parameter ( $d_s$ ) is higher than the non-seasonal one ( $d$ ) in the majority of the cases. If we focus on the case with a linear time trend, we notice that for “Food retailing”, “Clothing and soft ...”, “Household goods ...”, “Other retailing” and “Total Retail Sales”,  $d$  ranges between 0.31 and 0.52, while  $d_s$  is in the interval 0.79 and 0.91; for “Cafes, Restaurants, ...”,  $d$  is slightly higher than  $d_s$ ; and finally, for “Department stores”  $d$  is close to 0 (0.05) while  $d_s$  is equal to 0.90 implying once more heterogeneous results across the series.

**[Table 7 near here]**

In Table 7 we present a ranking of the degrees of persistence across the different sectors using the three specifications described above. We built this ranking based on the cumulative first 120 responses, combining thus seasonal and non-seasonal effects. We observe that the results are similar across the three models, with “Cafes, Restaurants, ...”, “Household goods ...” and “Food retailing” displaying the highest degrees of persistence, while “Clothing and soft ...” and “Department stores” displaying the lowest values.

To provide further support to our findings, we also applied all the models described above to several U.S. retail sectors described in Appendix 2B. The results are presented in Appendix 3. Table A1 refers to the estimates of  $d$  in model (M1); Tables A2 to the estimates of  $d_s$  in (M2), and Table A3 to the estimates of  $d$  and  $d_s$  in the model (M3). The two issues claimed in this work are also satisfied for this country: a) Seasonality has

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a strong influence on the data, and b) persistence is highly heterogeneous across sectors. Starting with the estimates of  $d$  in model (M1) reported in Table A1 we observe that all them are in the interval  $(0, 1)$  implying fractional integration. These values range between 0.279 (Clothing with a linear time trend) and 0.864 (Food and beverage with no regressors). If we focus now on the case of seasonal fractional integration (in Table A2) the estimates of the differencing parameter are higher, being in most of the cases in the interval  $(0.5, 1)$ . Allowing for fractional integration simultaneously at zero and the seasonal frequencies (in Table A3) the most interesting feature is that once more the estimates are fractional with values slightly higher for the seasonal fractional differencing parameter. Table A4 displays the ranking of persistence across sectors. It is observed that the highest degrees of persistence are obtained in sectors such as “Furniture, home ...”, “Electronic ...” and “Motor vehicle ...”, and the lowest values occur at “Food and beverage ...” and “Clothing and ...”.

## **6.2 Forecasting of Retail Sales**

We perform here a small in-sample forecasting experiment to check which one of the three models (M1, M2 or M3) better describes the data. Though not reported we first compared the (M1) specification with the  $I(0)$  one based on stationary seasonal autoregressions, and also the (M2) model with the seasonal unit root model, and the results strongly support the (M1) and (M2) specifications in all series. This is not surprising noting that the estimates of  $d$  (in M1) and  $d_s$  (in M2) were found to be fractional in all cases.

Based on the three models, we computed the mean squared errors for the last 24 observations, and the results indicate that the best results are obtained in model (M3) especially over long horizons. Then, we computed the modified Diebold and Mariano (M-DM, 1995) statistic as suggested by Harvey, Leybourne and Newbold (1997).<sup>9</sup> Using this method, we evaluate the relative forecast performance of the different models by making pairwise comparisons. We use the mean squared errors in the computations. The results are displayed in Tables 8 and 9 respectively for 12 and 24-period ahead predictions.

**[Tables 8 and 9 near here]**

For each prediction-horizon we indicate in the tables in bold the rejections of the null hypothesis that the forecast performance of model (Mi) and model (Mj) is equal in favour of the one-sided alternative that model (Mi)'s performance is superior at the 5% significance level. We note that the results are similar for the two time horizons, though they vary across series. In the majority of the cases (M2) and (M3) outperform (M1), implying that a model with a long-memory component exclusively affecting the long-run or zero frequency is inappropriate in all cases. When comparing (M2) with (M3), the results indicate that (M3) outperforms (M2) in several cases, especially at the 12-period horizon.

Given the superiority of (M3) over the (M1) and (M2) models and noting also that the latter models outperform the standard ones based on seasonal and non-seasonal I(0)/I(1)

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<sup>9</sup> Harvey et al. (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic in finite samples, and also that the power of the test is improved when p-values are computed with a Student t-distribution.

models we may conclude this section by saying that incorporating seasonal and non-seasonal long memory models seems to be the most adequate specification for the retail data examined in this work.

Here we also provide further evidence from the U.S. retail sector. Table A4 deals with the forecasting exercise. It is again clear that a model incorporating long memory at both the zero and the seasonal frequencies outperforms models that only use one of the two structures. For instance, if we look at the modified DM statistic in Table A4, the results for the 12-period ahead horizon show that model (M3) outperforms models (M1) and (M2) in the majority of the sectors.

## **7. Discussion and Conclusions**

In this paper we have investigated the time series dependence and other implicit dynamics in retail sales, providing evidence from the Australian and U.S. retail industry. We used a variety of model specifications, including long memory processes at the long run or zero frequency; at the seasonal (monthly) frequencies; and a combination of the two approaches in a single framework. In the latter case, the model contains two differencing parameters, one referring to the long term evolution of the series, and the other one referring to the seasonal structure.

The results first indicated that seasonality is important when modelling these series since in the three specifications seasonality appears as an important issue. Moreover,

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seasonality appears to be heterogeneous across the sectors in the two countries. The results further indicated that shocks affecting the seasonal structure have a transitory effect though taking a very long time to disappear in the long run. Concerning the issue of persistence heterogeneity, it was observed that the degree of persistence substantially changed from one series to another. In general, the orders of integration were found to be lower than one, indicating that the series are mean reverting and thus converge to an average value along time, though taking a very long time to recover and therefore demanding active retailing policies.

The persistence should be allocated to each series separately since distinct measures have to be allocated based on the degree of persistence identified. In Tables 7 and A4 we have summarised our findings, and provided retail practitioners from both Australia and the U.S. with the persistence ranking of each of the retail sectors analyzed. In the Australian case, the lowest degrees of dependence were obtained for “Department stores” and “Clothing and soft ...”, while the largest ones were reported in “Cafes, Restaurants, ...”, “Household goods ...” and “Food retailing”. In the U.S. case, the highest degrees of persistence are observed in “Furniture, home, ...” and “Motor vehicle ...”, while the lowest values correspond to “Pharmacies ...”, “Food and beverage ...” and “Clothing and ...”. The results further indicated that shocks related to the long run evolution of the retail series have a transitory nature disappearing faster than in the seasonal case. However, taking into account the two structures simultaneously throughout a long memory model at zero and the seasonal frequencies, the series appear to be seasonally mean reverting though highly persistent, while the long term evolution presents values above 1 in many cases. Note that in model (M3), the contribution of the zero frequency is not exclusively

based on the fractional differencing parameter  $d$  but also includes the estimate of  $d_s$  since the polynomial  $(1 - L^s)^{d_s}$  can be decomposed into  $(1 - L)^{d_s} S(L)^{d_s}$ , where  $S(L) = (1 + L + L^2 + \dots + L^{s-1})$  is formed exclusively by the seasonal frequencies.<sup>10</sup> Thus, according to the results in Table 6 (with a linear time trend), the contribution to the long run frequency for “Food retailing” is 1.24 (0.42 + 0.82). In fact, it is above 1 in all cases except “Department stores”, which is 0.95 (0.05 + 0.90).<sup>11</sup>

Thus, what are the literature and industry contributions of our research? The study first contributes to the literature by providing more accurate evidence of the persistence of seasonality behaviour of retail sales, while most previous studies have ignored the combined impact of seasonality and persistence on the short and long term dependence of retail sales. Long memory models have also not been implemented previously on retail sales despite the fact that they include as particular cases the standard AR(I)MA models widely employed in the literature. This paper is also the first to adopt a fractional integration model, while most previous papers adopted a traditional integrated model. Models based on fractional integration are more general than the classical models based on integer degrees of differentiation and thus allow for a much richer degree of flexibility in the dynamic specification of the series. Note that an added contribution of this paper is that it provides evidence from various retail sectors, while most previous studies have focused on aggregate retail sales. The use of data from both the Australian and the U.S. retail sectors is also adopted for the first time in this paper.

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<sup>10</sup> Thus, for example,  $(1 - L^4) = (1 - L)(1 + L + L^2 + L^3) = (1 - L)(1 + L)(1 + L^2)$ .

The second contribution to the retail literature relates to the introduction of a new forecasting model that accounts for both persistence and seasonality in retail data. The forecasting comparison showed that a long memory model including both long run and seasonal terms is the most adequate specification for these series in terms of their forecasting properties. Specifically, the long memory model incorporating both the zero and the seasonal frequencies outperformed those models that only use the zero or alternatively the seasonal frequencies, and, since the latter models include as particular cases the standard AR(I)MA and the seasonal AR(I)MA models, the benefits are explicit.

The above contributions to the literature can also directly assist policy makers in the retail industry. In fact, as stated before, when retail authorities or retail businesses have a *priori* knowledge of the persistence and seasonality behaviour of retail sales, they can reap the benefit of positive effects, or avoid being victimized by a negative effect. As we provide evidence from various sectors, the results are also expected to assist retail businesses that operate across multiple retail sectors. Specifically, we expect that the results will most benefit retail businesses that possess a significant market share in the industry, as these are more likely to watch the long trend movement of retail sales. In contrast to small retailers, large retailers are also expected to be more interested in the analysis of industry data given that in most cases they have multiple geographical presences.

The results clearly showed that different retail sectors experience heterogeneous and persistence and seasonality behaviours. Thus policy makers at the industry or store level

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<sup>11</sup> Similarly for the US, the contribution of the long run frequency is above 1 in all cases except “Food and beverage ...” ( $0.35 + 0.61 = 0.96$ ); “Household appl. ...” ( $0.51 + 0.47 = 0.98$ ); and “Pharmacies ...” ( $0.43 + 0.54 = 0.97$ ). See Table A3.



need to distinguish between short term from long term policies depending on the nature of the shock affecting the industry. This is because the consequences are different: in the case of a negative shock, if it is related to the seasonal evolution of the series, short and strong policy measures must be adopted to recover the original level since it will take long time to disappear, however, if the shock is related to the long term evolution, decisive long term measures must be adopted since otherwise the series will tend to remain at a lower level. Some examples of long range policies that can be adopted include 1- the development of retention and retail employees, 2- the improvements of the industry's information base, 3- incentives to mergers and acquisitions in the retail activity.

On the forecasting side, our proposed model is also expected to have direct industry implications. In fact, forecasting accuracy was identified by Peterson (1993) as one of the key priorities for retail stores, especially those operating at a large scale. Agrawal and Schorling (1996, p. 383) also highlighted that “accurate demand forecasting is crucial for profitable retail operations because without a good forecast, either too-much or too-little stocks would result, directly affecting revenue and competitive position”. Note that the persistence results are also expected to assist directly in the forecasting of large stores, given that these stores are more likely to include in their forecasting models assumptions about movements in industry-wide sales and market-share (Peterson, 1993). In other words, by providing persistence results of multiple retail sectors, we have assisted large retailers derive to what extent our long term forecast should be adjusted when short term changes occur in the market.



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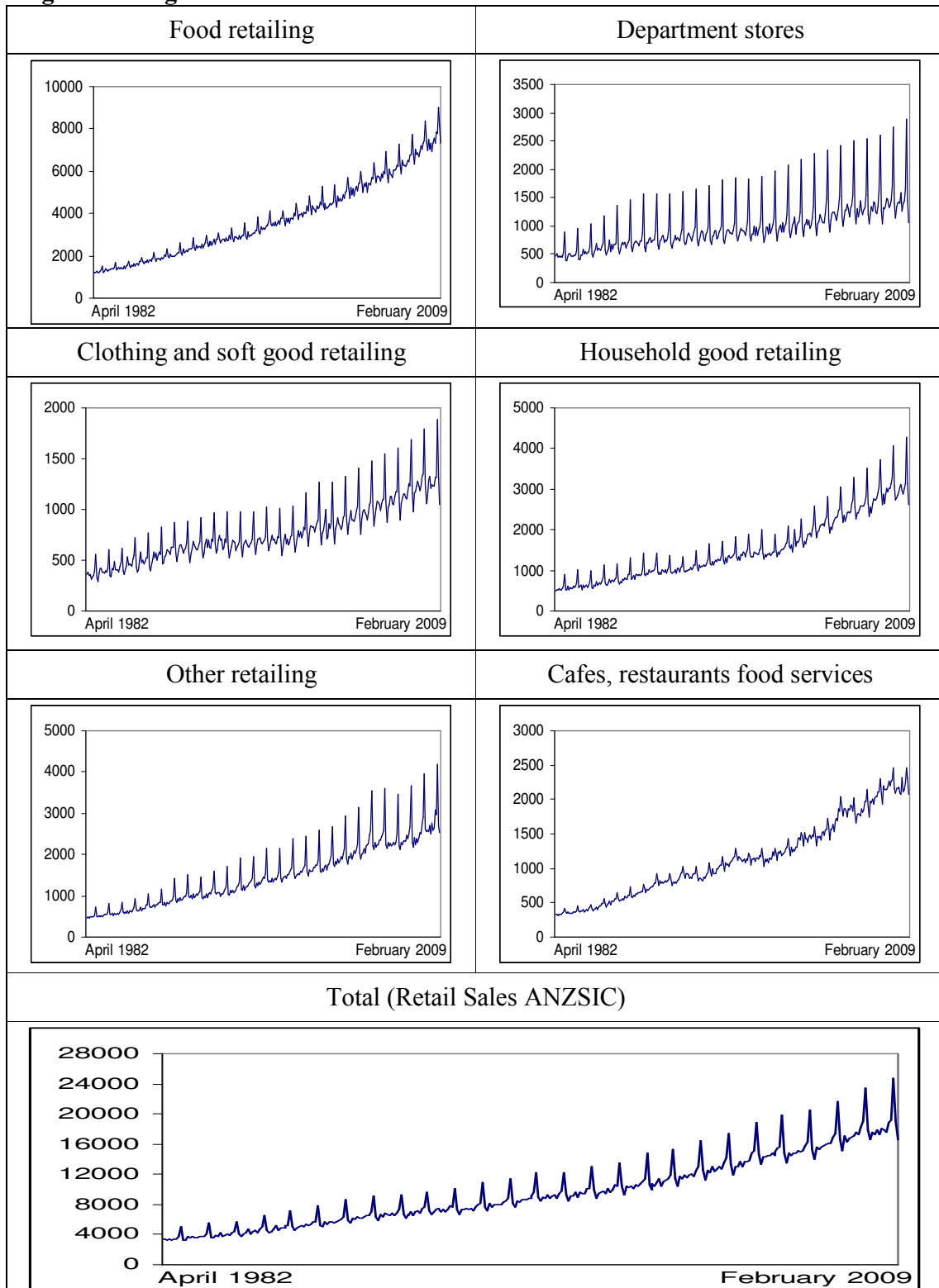
**Table 1: Retail sales (\$ Millions) for retail groups over the period 2002–03 to 2006–07**

	Food	Departm	Clothing	Househ.	Recreat.	Other	Hospit.	Total
2002-03	75283	14528	11498	23344	7199	30180	180636	342668
2003-04	78360	15577	12265	27180	7914	32284	194438	368018
2004-05	80371	16283	13242	29929	8300	31832	201236	381193
2005-06	82334	16305	14002	31689	8172	33091	206089	391682
2006-07	84495	16821	14935	34755	8404	33582	214279	407271

Food: Food retailing; Departm: Department Stores; Clothing: Clothing and soft good retailing; Househ: Household Good retailing; Recreat: Recreational Good retailing; Other: Other retailing; Hospitality and services.



**Figure 1: Original time series data: Australian retail data**



**Table 2: Estimates of the fractional differencing parameter in model (M1)**

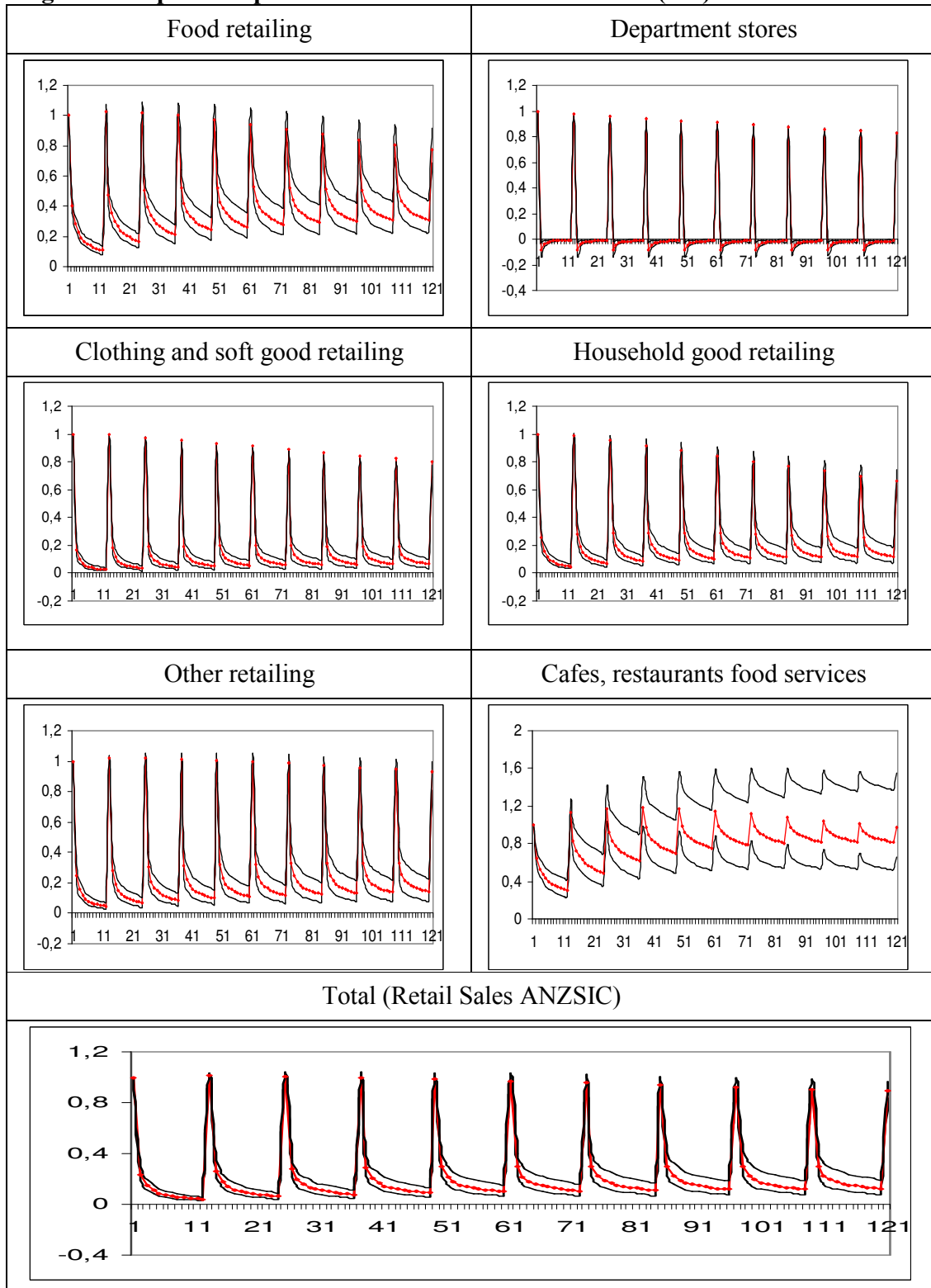
Series	No regressors	An intercept	A linear time trend
Food retailing	0.555 (0.521, 0.590)	0.640 (0.616, 0.664)	<b>0.403</b> (0.356, 0.450)
Department stores	0.204 (0.186, 0.222)	0.383 (0.351, 0.415)	<b>-0.080</b> (-.126, -0.034)
Clothing and soft ...	0.308 (0.285, 0.331)	0.504 (0.475, 0.533)	<b>0.168</b> (0.123, 0.213)
Household goods ...	0.405 (0.380, 0.430)	0.545 (0.519, 0.571)	<b>0.253</b> (0.205, 0.301)
Other retailing	0.411 (0.382, 0.440)	0.542 (0.516, 0.568)	<b>0.249</b> (0.190, 0.308)
Cafes, restaurants ...	0.657 (0.599, 0.715)	0.712 (0.670, 0.754)	<b>0.647</b> (0.576, 0.718)
Total (Retail Sales)	0.438 (0.412, 0.475)	0.580 (0.557, 0.610)	<b>0.234</b> (0.186, 0.282)

In parenthesis, the 95% confidence intervals for the values of  $d$ . In bold, the estimates corresponding to significant deterministic terms.

**Table 3: Estimates of the seasonal autoregressive parameter in model (M1)**

Series	No regressors	An intercept	A linear time trend
Food retailing	0.904	0.930	0.927
Department stores	0.983	0.980	0.983
Clothing and soft ...	0.959	0.959	0.970
Household goods ...	0.944	0.948	0.942
Other retailing	0.974	0.978	0.977
Cafes, restaurants ...	0.807	0.844	0.834
Total (Retail Sales)	0.969	0.976	0.974

**Figure 2: Impulse responses based on the results in model (M1)**



In black the 95% confidence band.

**Table 4: Estimates of the seasonal fractional differencing parameter in model (M2)**

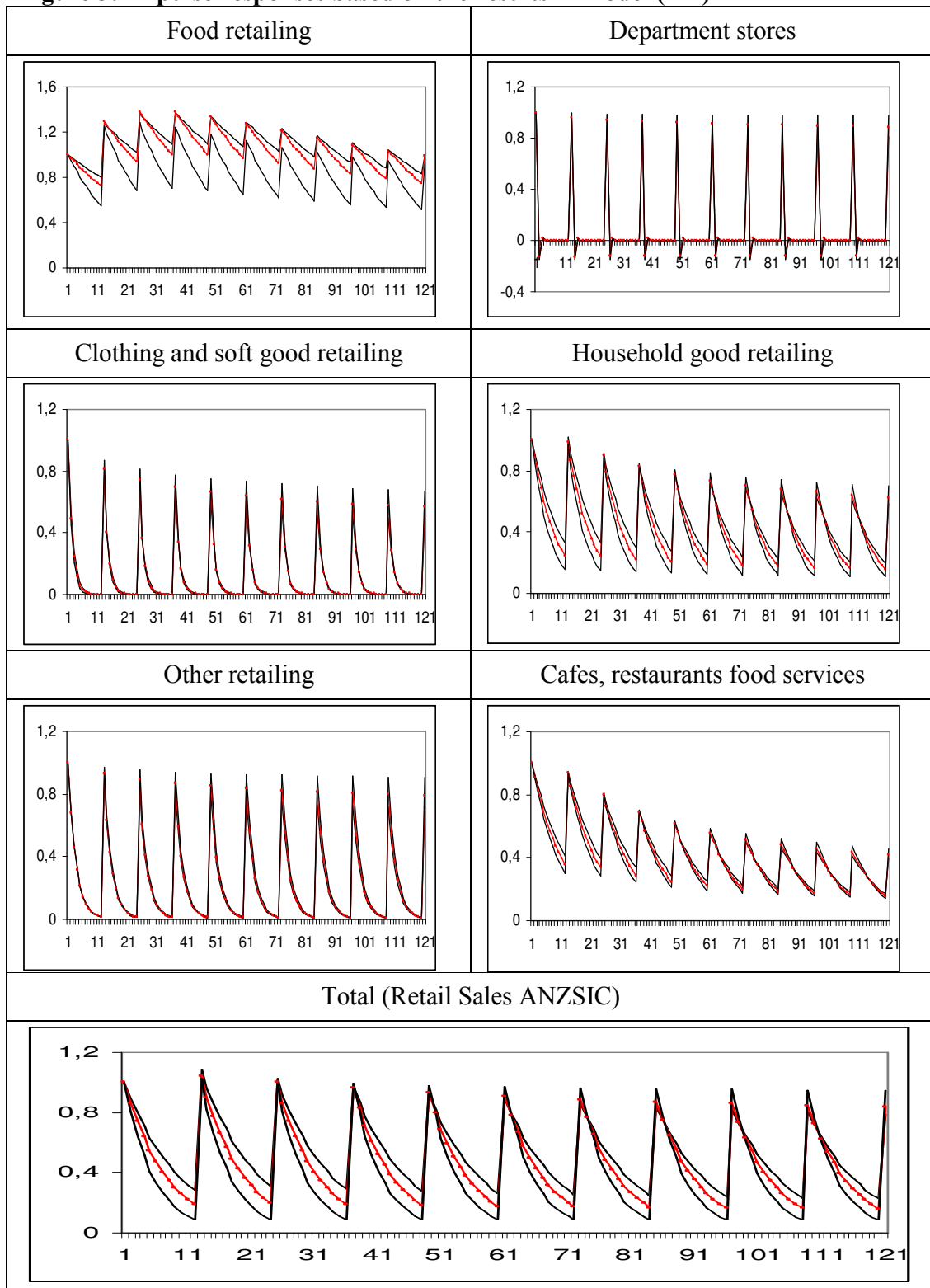
Series	No regressors	An intercept	A linear time trend
Food retailing	0.848 (0.755, 0.950)	0.594 (0.489, 0.700)	<b>0.897</b> (0.800, 1.000)
Department stores	0.958 (0.928, 0.991)	0.919 (0.874, 0.967)	<b>0.951</b> (0.918, 0.990)
Clothing and soft ...	0.821 (0.778, 0.870)	0.776 (0.732, 0.820)	<b>0.816</b> (0.775, 0.865)
Household goods ...	0.802 (0.752, 0.860)	0.770 (0.718, 0.827)	<b>0.791</b> (0.745, 0.847)
Other retailing	0.918 (0.883, 0.960)	0.930 (0.888, 0.970)	<b>0.910</b> (0.873, 0.955)
Cafes, restaurants ...	0.611 (0.569, 0.660)	0.485 (0.394, 0.585)	<b>0.589</b> (0.552, 0.620)
Total (Retail Sales)	0.940 (0.890, 1.000)	0.881 (0.825, 0.950)	<b>0.939</b> (0.889, 1.000)

In parenthesis, the 95% confidence intervals for the values of  $d$ . In bold, the estimates corresponding to significant deterministic terms.

**Table 5: Estimates of the autoregressive parameter in model (M2)**

Series	No regressors	An intercept	A linear time trend
Food retailing	0.761	0.971	0.646
Department stores	-0.135	0.086	-0.136
Clothing and soft ...	0.465	0.704	0.434
Household goods ...	0.748	0.881	0.694
Other retailing	0.677	0.789	0.656
Cafes, restaurants ...	0.912	0.980	0.849
Total (Retail Sales)	0.632	0.860	0.581

**Figure 3: Impulse responses based on the results in model (M2)**



In black the 95% confidence band.

**Table 6: Estimates of the fractional differencing parameters in model (M3)**

Series	No regressors		An intercept		A linear time trend	
	d	d <sub>s</sub>	d	d <sub>s</sub>	d	d <sub>s</sub>
Food retailing	0.53	0.83	0.41	0.81	0.42	0.82
Department stores	0.07*	0.90**	0.07*	0.90**	0.05*	0.90**
Clothing and soft ...	0.35	0.79	0.32	0.78	0.31	0.79
Household goods ...	0.53	0.81	0.46	0.80	0.46	0.80
Other retailing	0.61	0.93**	0.51	0.91**	0.52	0.91**
Cafes, restaurants ...	0.72	0.58	0.69	0.57	0.69	0.57
Total (Retail Sales)	0.59	0.92**	0.38	0.91**	0.39	0.91**

\*: We cannot reject the null hypothesis of I(0) at the 95% level. \*\*: We cannot reject the I(1) null.

**Table 7: Ranking of sectors according to their degree of persistence**

Model (M1)	Model (M2)	Model (M3)
Cafes, restaurants, ...	Cafes, restaurants, ...	Other retailing
Food retailing	Household goods ...	Cafes, restaurants, ...
Household goods ...	Other retailing	Household goods ...
Other retailing	Food retailing	Food retailing
Total (Retail Sales)	Total (Retail Sales)	Total (Retail Sales)
Clothing and soft ...	Clothing and soft ...	Clothing and soft ...
Department stores	Department stores	Department stores

**Table 8: Pairwise comparison using the modified DM statistic (h =12)**

<b>Food retailing</b>				<b>Department Stores</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>3.125(M2)</b>	XXXX	XXXX	(M2)	<b>5.091(M2)</b>	XXXX	XXXX
(M3)	<b>3.098(M3)</b>	2.122	XXXX	(M3)	<b>4.667(M3)</b>	1.980	XXXX
<b>Clothing and soft ...</b>				<b>Household goods ...</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>4.543(M2)</b>	XXXX	XXXX	(M2)	<b>3.444(M2)</b>	XXXX	XXXX
(M3)	<b>4.329(M3)</b>	<b>2.992(M3)</b>	XXXX	(M3)	<b>3.608(M3)</b>	<b>3.088(M3)</b>	XXXX
<b>Other retailing</b>				<b>Cafes, restaurants, ...</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>3.100(M2)</b>	XXXX	XXXX	(M2)	<b>6.512(M2)</b>	XXXX	XXXX
(M3)	<b>3.390(M3)</b>	<b>3.401(M3)</b>	XXXX	(M3)	<b>5.666(M3)</b>	<b>3.367(M3)</b>	XXXX
<b>Total Retail Sales</b>							
	(M1)	(M2)	(M3)				
(M1)	XXXX	XXXX	XXXX				
(M2)	<b>3.791(M2)</b>	XXXX	XXXX				
(M3)	<b>5.431(M3)</b>	2.001	XXXX				

**Table 9: Pairwise comparison using the modified DM statistic (h =24)**

<b>Food retailing</b>				<b>Department Stores</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>2.997(M2)</b>	XXXX	XXXX	(M2)	<b>3.432(M2)</b>	XXXX	XXXX
(M3)	<b>2.112(M3)</b>	1.554	XXXX	(M3)	<b>2.439(M3)</b>	0.997	XXXX
<b>Clothing and soft ...</b>				<b>Household goods ...</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>3.212(M2)</b>	XXXX	XXXX	(M2)	<b>2.511(M2)</b>	XXXX	XXXX
(M3)	<b>3.973(M3)</b>	<b>1.889(M3)</b>	XXXX	(M3)	<b>2.621(M3)</b>	<b>1.118(M3)</b>	XXXX
<b>Other retailing</b>				<b>Cafes, restaurants, ...</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	1.100	XXXX	XXXX	(M2)	<b>2.111(M2)</b>	XXXX	XXXX
(M3)	<b>2.088(M3)</b>	0.402	XXXX	(M3)	1.567	1.113	XXXX
<b>Total Retail Sales</b>							
	(M1)	(M2)	(M3)				
(M1)	XXXX	XXXX	XXXX				
(M2)	<b>1.117(M2)</b>	XXXX	XXXX				
(M3)	1.546	0.823	XXXX				



## Appendix 1: Robinson's (1994) parametric approach for fractional integration

Assuming that  $x_t$  are the errors in a regression model with a linear time trend,

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, \dots, \quad (\text{A1})$$

we suppose that  $x_t$  adopt the form:

$$\rho(L; d) x_t = u_t, \quad t = 1, 2, \dots, \quad (\text{A2})$$

where  $\rho$  is a scalar function that depends on  $L$  and the fractional differencing parameter(s)  $d$ , and that will adopt different forms as shown below, and  $u_t$  is  $I(0)$ . The function  $\rho$  is specified in such a way that all its roots should be on the unit circle in the complex plane, and therefore it includes polynomials of the form  $(1-L)^d$  (as in (M1)),  $(1-L^s)^d$  (as in (M2)), or even more generally,  $(1-L)^d(1-L^s)^{ds}$  (as in (M3)).

Robinson (1994) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_0 : d^* = d_o^*, \quad (\text{A3})$$

in (A1) and (A2), where  $d^*$  is equal to  $d$  in (M1),  $d_s$  in (M2), and a  $(2 \times 1)$  vector  $(d, d_s)^T$  in (M3). Based on  $H_0$  given by (A3), the estimated  $\hat{\gamma} = (\hat{\alpha}, \hat{\beta})^T$  and residuals are:

$$\hat{u}_t = \rho(L; d_o^*) y_t - \hat{\gamma}' w_t, \quad w_t = \rho(L; d_o^*) z_t; \quad \hat{\gamma} = \left( \sum_{t=1}^T w_t w_t' \right)^{-1} \sum_{t=1}^T w_t \rho(L; d_o^*) y_t,$$

with  $z_t = (1, t)^T$ . The functional form of the test statistic is then given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}^T \hat{A}^{-1} \hat{a}, \quad (\text{A4})$$

where  $T$  is the sample size, and

$$\hat{A} = \frac{2}{T} \left( \sum_{j=1}^* \psi(\lambda_j)^2 - \sum_{j=1}^* \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left( \sum_{j=1}^* \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^* \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^* \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \lambda_j = \frac{2\pi j}{T}; \quad \hat{\tau} = \arg \min_{\tau \in T^*} \sigma^2(\tau),$$

and the sums over \* in the above expressions are over  $\lambda \in M$  where  $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_l - \lambda_l, \rho_l + \lambda_l), l = 1, 2, \dots, s\}$  such that  $\rho_l, l = 1, 2, \dots, s < \infty$  are the distinct poles of  $\psi(\lambda)$  on  $(-\pi, \pi]$ . Also,

$$\psi(\lambda_j) = \text{Re} \left[ \log \left( \frac{\partial}{\partial d} \log \rho(e^{i\lambda_j}; d) \right) \right], \quad (\text{A5})$$

and  $I(\lambda_j)$  is the periodogram of  $u_t$  evaluated under the null. Note that in model (M1),

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|. \text{ In model (M2),}$$

$$\begin{aligned} \psi(\lambda_j) = & \log \left| 2 \sin \frac{\lambda_j}{2} \right| + \log \left( 2 \cos \frac{\lambda_j}{2} \right) + \log |2 \cos \lambda_j| + \log \left| 2 \left( \cos \lambda_j - \cos \frac{\pi}{3} \right) \right| + \\ & + \log \left| 2 \left( \cos \lambda - \cos \frac{2\pi}{3} \right) \right| + \log \left| 2 \left( \cos \lambda - \cos \frac{\pi}{6} \right) \right| + \log \left| 2 \left( \cos \lambda - \cos \frac{5\pi}{6} \right) \right|; \end{aligned}$$

while in (M3),  $\psi(\lambda_j) = [\psi_1(\lambda_j), \psi_2(\lambda_j)]^T$ .

The function  $g$  above is a known function coming from the spectral density of  $u_t$ ,

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \quad -\pi < \lambda \leq \pi.$$

Note that these tests are purely parametric, and, therefore, they require specific modelling assumptions about the short memory specification of  $u_t$ . Thus, if  $u_t$  is a white noise, then  $g \equiv 1$ , (and thus,  $\hat{\varepsilon}(\lambda_j) = 0$ ), and if it is an AR process of the form  $\phi(L)u_t = \varepsilon_t$ , then,  $g = |\phi(e^{i\lambda})|^{-2}$ , with  $\sigma^2 = V(\varepsilon_t)$ , so that the AR coefficients are a function of  $\tau$ .

Based on  $H_0$  (A3), Robinson (1994) showed that under certain very mild regularity conditions:

$$\hat{R} \rightarrow_d \chi_p^2, \quad \text{as } T \rightarrow \infty,$$

where  $p$  is the dimension of  $d^*$ .

## **Appendix 2: Retail sectors included in the survey**

### **A. Australian Retail Sectors**

The following retail sectors are included in the analysis

1. Food retailing
2. Department stores
3. Clothing and soft good retailing
4. Household good retailing
5. Other retailing (e.g. Newspaper, book and stationery retailing  
Other recreational goods retailing, etc..)
6. Cafes, restaurants and takeaway food services

### **B. US Retail Sectors**

The following retail sectors are included in the analysis

1. Clothing and clothing accessory stores
2. Department stores
3. Electronic and appliance stores
4. Food and beverage stores
5. Furniture, home furnishing
6. Electronic and appliance stores
7. Household appliance stores
8. Motor vehicle and parts dealers
9. Pharmacies and drug stores

### Appendix 3: Results for the US data

**Table A1: Estimates of the fractional differencing parameter in model (M1)**

Series	No regressors	An intercept	A linear time trend
Clothing and ...	0.555 (0.409, 0.700)	0.379 (0.337, 0.421)	0.279 (0.192, 0.366)
Department stores	0.657 (0.567, 0.747)	0.374 (0.282, 0.466)	0.378 (0.276, 0.480)
Electronic and ...	0.562 (0.383, 0.741)	0.570 (0.513, 0.627)	0.538 (0.449, 0.627)
Food and beverage	0.864 (0.773, 0.955)	0.537 (0.511, 0.563)	0.298 (0.229, 0.367)
Furniture, home ...	0.725 (0.640, 0.810)	0.579 (0.527, 0.631)	0.589 (0.516, 0.662)
Household appl ...	0.751 (0.658, 0.844)	0.507 (0.464, 0.550)	0.518 (0.441, 0.595)
Motor vehicle ...	0.801 (0.713, 0.889)	0.581 (0.513, 0.649)	0.614 (0.539, 0.689)
Pharmacies and ..	0.680 (0.571, 0.789)	0.607 (0.586, 0.628)	0.316 (0.268, 0.364)
Total Retail Sales	0.813 (0.720, 0.906)	0.561 (0.527, 0.595)	0.581 (0.509, 0.653)

In parenthesis, the 95% confidence intervals for the values of  $d$ . In bold, the estimates corresponding to significant deterministic terms.

**Table A2: Estimates of seasonal fractional differencing parameter in model (M2)**

Series	No regressors	An intercept	A linear time trend
Clothing and ...	0.808 (0.751, 0.865)	0.920 (0.861, 0.979)	0.861 (0.806, 0.916)
Department stores	0.813 (0.771, 0.855)	0.867 (0.833, 0.900)	0.867 (0.833, 0.900)
Electronic and ...	0.920 (0.872, 0.968)	0.943 (0.900, 0.986)	0.937 (0.894, 0.980)
Food and beverage	0.549 (0.493, 0.605)	0.533 (0.438, 0.628)	0.674 (0.604, 0.744)
Furniture, home ...	0.775 (0.725, 0.825)	0.842 (0.799, 0.885)	0.826 (0.782, 0.870)
Household appl ...	0.453 (0.423, 0.483)	0.452 (0.411, 0.493)	0.454 (0.416, 0.492)
Motor vehicle ...	0.378 (0.336, 0.420)	0.441 (0.383, 0.510)	0.439 (0.384, 0.507)
Pharmacies and ..	0.594 (0.519, 0.670)	0.170 (0.067, 0.273)	0.471 (0.418, 0.524)
Total Retail Sales	0.547 (0.502, 0.592)	0.636 (0.572, 0.711)	0.603 (0.553, 0.653)

In parenthesis, the 95% confidence intervals for the values of  $d$ . In bold, the estimates corresponding to significant deterministic terms.

**Table A3: Estimates of the fractional differencing parameters in model (M3)**

Series	No regressors		An intercept		A linear time trend	
	d	d <sub>s</sub>	d	d <sub>s</sub>	d	d <sub>s</sub>
Clothing and ...	0.82	0.83	0.41	0.85	0.39	0.84
Department stores	0.89	0.85	0.50	0.84	0.48	0.84
Electronic and ...	0.81	0.88	0.67	0.92	0.65	0.92
Food and beverage	0.88	0.73	0.36	0.63	0.35	0.61
Furniture, home ...	0.83	0.83	0.67	0.83	0.66	0.83
Household appl ...	0.61	0.56	0.52	0.47	0.51	0.47
Motor vehicle ...	0.59	0.50	0.61	0.48	0.61	0.49
Pharmacies and ..	0.66	0.58	0.46	0.53	0.43	0.54
Total Retail Sales	0.82	0.75	0.63	0.62	0.61	0.60

\*: We cannot reject the null hypothesis of I(0) at the 95% level. \*\*: We cannot reject the I(1) null.

**Table A4: Pairwise comparison using the modified DM statistic (h =12)**

<b>Clothing and ...</b>				<b>Department Stores</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>3.453(M2)</b>	XXXX	XXXX	(M2)	<b>3.776(M2)</b>	XXXX	XXXX
(M3)	<b>4.007(M3)</b>	<b>3.567(M3)</b>	XXXX	(M3)	<b>3.111(M3)</b>	<b>3.123(M3)</b>	XXXX
<b>Electronic and ...</b>				<b>Food and beverage</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	1.345	<b>XXXX</b>	XXXX	(M2)	<b>3.991(M2)</b>	XXXX	XXXX
(M3)	1.234	1.109	XXXX	(M3)	1.092	<b>-3.09(M2)</b>	XXXX
<b>Furniture, home ...</b>				<b>Household appliance ...</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>2.151(M2)</b>	XXXX	XXXX	(M2)	1.077	<b>XXXX</b>	XXXX
(M3)	<b>3.500(M3)</b>	1.167	XXXX	(M3)	1.143	1.221	XXXX
<b>Motor vehicle ...</b>				<b>Pharmacies and ...</b>			
	(M1)	(M2)	(M3)		(M1)	(M2)	(M3)
(M1)	XXXX	XXXX	XXXX	(M1)	XXXX	XXXX	XXXX
(M2)	<b>3.452(M2)</b>	XXXX	XXXX	(M2)	<b>2.735(M2)</b>	XXXX	XXXX
(M3)	0.998	<b>-3.77(M2)</b>	XXXX	(M3)	<b>3.009(M3)</b>	<b>2.098(M3)</b>	XXXX
<b>Total Retail Sales</b>							
	(M1)	(M2)	(M3)				
(M1)	XXXX	XXXX	XXXX				
(M2)	<b>3.441(M2)</b>	XXXX	XXXX				
(M3)	<b>3.500(M3)</b>	1.230	XXXX				

**Table A5: Ranking of sectors according to their degree of persistence**

Model (M1)	Model (M2)	Model (M3)
Motor vehicle ...	Furniture, home, ...	Furniture, home, ...
Furniture, home, ...	Total Retail Sales	Electronic and ...
Total Retail Sales	Electronic and ...	Motor vehicle ...
Electronic and ...	Motor vehicle ...	Department stores ...
Household appl.	Household appl.	Total Retail Sales
Department stores ...	Department stores ...	Clothing and ...
Pharmacies and ...	Pharmacies and ...	Household appl.
Food and beverage ...	Clothing and ...	Pharmacies and ...
Clothing and ...	Food and beverage ...	Food and beverage ...



